What's Wrong with Those Duration Measures?

Nothing.

Robin Grieves[a]

J. Clay Singleton[b]

Abstract

In this article, we demonstrate that standard and effective durations for bonds without embedded options are not equal. The two measures are nearly always different. The difference can be large enough to be of consequence in some applications. Additionally, we show that the only consistent measure of duration, one that aggregates correctly when yield curves are not flat, is effective duration.

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[a] Darla Moore School of Business, University of South Carolina, Columbia, SC 29208, robin.grieves@moore.sc.edu

[b] Crummer Graduate School of Business, Rollins College, Winter Park, FL 32789, jcsingleton@rollins.edu

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1 - Introduction

Fixed income investment portfolios include assets of varying maturity and varying duration. Because some investments carry embedded options, assets of the same maturity can have very different durations. Overall portfolio risk management requires the use of sophisticated bond analytic software to measure the portfolios' sensitivity to changes in interest rates.

The portfolio managers' confidence in the software is critical in managing the portfolios' risks. If the managers' expectations about risk measures are not met, confidence in the risk measurement is undermined. One such expectation is that the standard (modified) and effective durations of bonds without embedded options should be the same. Standard duration is a bond's price sensitivity to changes in its own yield. Effective duration is a bond's price sensitivity to a parallel shift in the entire yield curve.

In this article, we demonstrate that the widely held notion that standard and effective durations for bonds without embedded options are equal is mistaken by a large enough amount to be of consequence in some applications. While this article emphasizes duration-neutral bond trades, it is also about yield curve shifts. We show that the relationship between par and spot curves, and their shifts, leads to different standard and effective durations for bonds without embedded options. We consider a sample trade that demonstrates how large the difference in position weighting can be when choosing between standard and effective duration. In addition, we demonstrate that the only consistent measure of duration, one that aggregates correctly when yield curves are not flat, is effective duration.

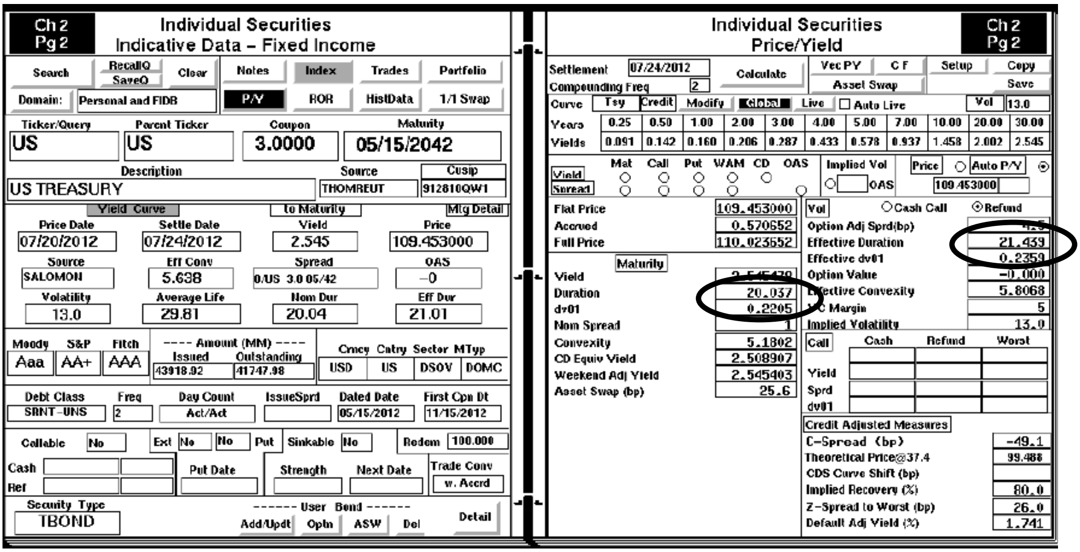
Because the notion is so widely used, the literature on duration is large. For background knowledge and applications of duration, see Fabozzi (2007), a standard text for academics and practitioners alike.

Consider a sample trade that many fixed income portfolio managers use. Suppose that they expect the yield curve to flatten; yields at the long end go down more than yields at the short end. This view is often held when the Fed is expected to hold Fed funds rates constant while inflation expectations decrease. In that case, typically, short rates and the short end of the yield curve decrease less than yields at the long end. The sample trade is to replace a bond with a "barbell," two bonds with durations shorter and longer than the original bond. The barbell has the same market value as the bond and the same duration as the bond, but its key rate durations are different. The barbell has its exposure to curve changes concentrated in the long end.

In our case, we will replace the on-the-run 30-year bond with a combination of cash and STRIPS with the same maturity as the bond. In essence, the barbell moves some of the bond's annuity to the maturity date and the rest of the annuity into cash. This reduces exposure to yield changes in the middle of the curve and increases exposure to changes in the long end. If our portfolio managers' expectation of flattening is realized, the trade should be profitable. The profits come from capital gains in the long end compared with the original bond. If a small parallel shift occurs, because the two positions have the same duration, the bond and the barbell will have similar price changes.

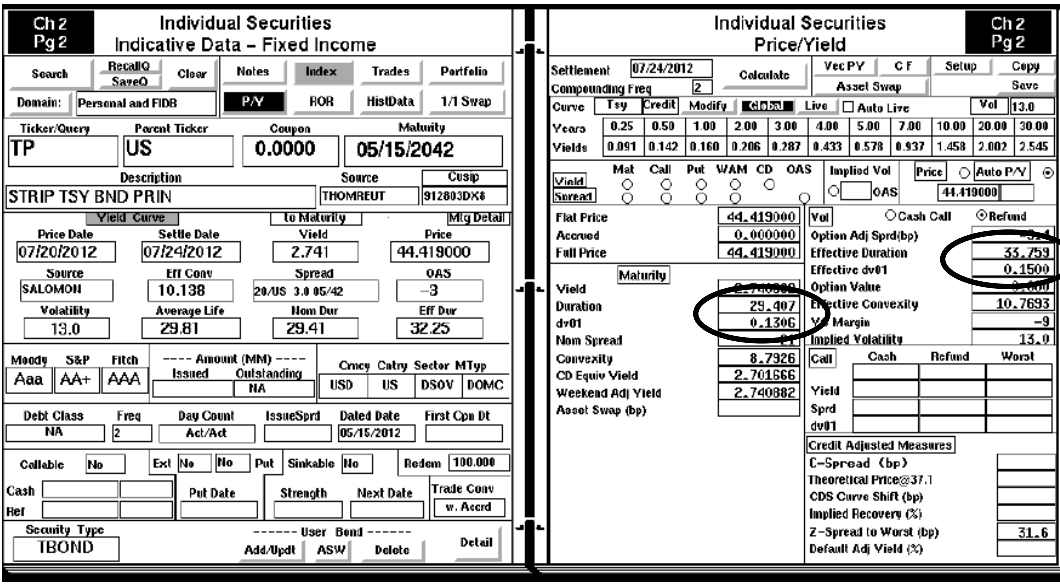
Duration-neutral trades require that the DV01 (dollar value of a 0.0001 change in yields) or PVBP (price value of a basis point change in yields) of the buy and sell sides be equal. Figure 1 shows the Yield Book Price/Yield page for the on-the-run bond. Notice that the bond's standard DV01 is 0.2205, which means that its value increases or decreases by $2205 per million of par value for each basis point increase or decrease in its yield to maturity. The bond's effective DV01 is 0.2359.

Figure 1. Yield Book page for the on-the-run bond and maturity matched principal STRIPS



If we want to sell the bond and replace it with a duration-neutral amount of the Principal STRIPS of 5/15/2042 and cash, we need to know the DV01 of the STRIPS. Cash, which has zero duration, has a DV01 of zero. Figure 2 displays price-sensitivity information for the Treasury Principal STRIPS of 5/15/2042. The DV01 based on standard duration is 0.1306 and the effective DV01 is 0.1500.

Figure 2. Yield Book page for the maturity matched principal STRIPS

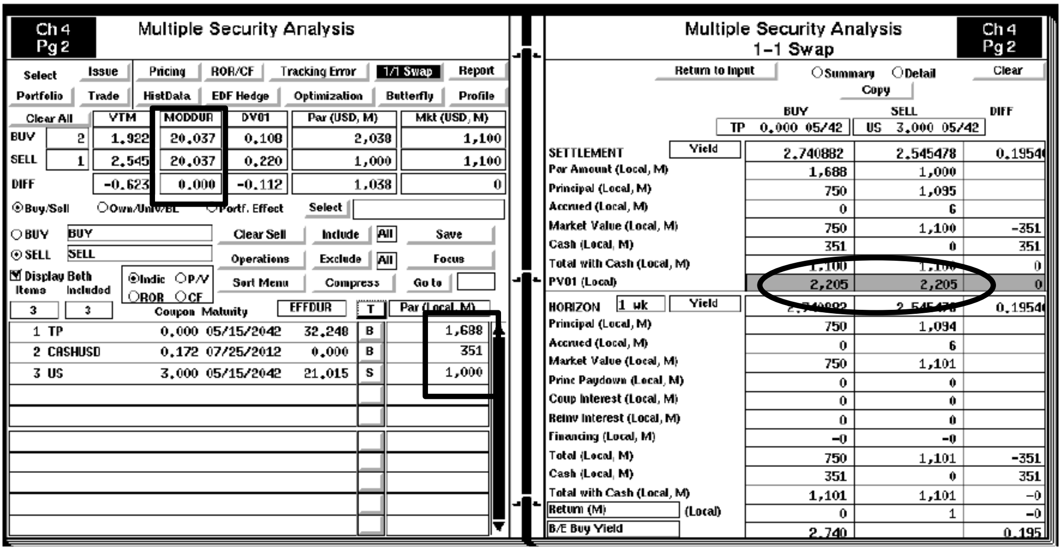


A duration-neutral trade using standard duration, then, requires us to buy \$2205 / \$1306 = 1.688 x \$1 million par of STRIPS for every \$1 million par of bonds that we sell. That is, we need to buy \$1.688 million par of STRIPS for every million dollars par of the bond sold. When we weight the trade in that manner, if the yield of the bond and the yield of the STRIPS move in the same direction by the same amount (the normal assumption), the change in the value of the bond sold will exactly match the change in the value of the barbell purchased. Changes in the overnight, or very short-term money market, rate on cash do not matter to the price change of the trade. The trade is duration neutral. Getting this weighting right requires measuring duration and, consequently, DV01, correctly.

Why would we do such a trade? Again, this trade is a flattener; we make money if the yield curve flattens. While we have an opinion on the future shape of the yield curve, we do not necessarily have an opinion on the level. Consequently, we want a duration-neutral trade that takes advantage of our market view.

Figure 3 displays the Yield Book's solution to a duration-neutral one-for-one swap, selling the bond and buying the STRIPS. Notice that the standard (modified) duration of the buy side, STRIPS and cash, equals the standard duration of the sell side.

Figure 3. Yield Book page for standard duration weighted swap



In sum, duration-neutral trades are constructed with individual bond DV01s. Non-callable bond trades are (almost always) structured using standard duration analysis, while trades involving callables or other bonds with embedded options are structured with option-adjusted (or effective) durations.

When the yield curve is steep, the difference between standard and effective duration for STRIPS and other bonds without embedded options can be large. First, we will see how much difference exists for weighting trades with effective duration instead of standard duration. Then, we will look at total returns for parallel shifts of the yield curve and for flattening curves. Finally, we will discuss how option-adjusted (effective) duration and standard duration can differ for bonds without embedded options.

Figure 4 displays the trade weighted by effective DV01. The price sensitivity of each side matches at $2,359, but the standard duration of the two sides no longer matches. The weighting difference is nearly 7% of the original position, which is clearly a significant difference in trade weighting.

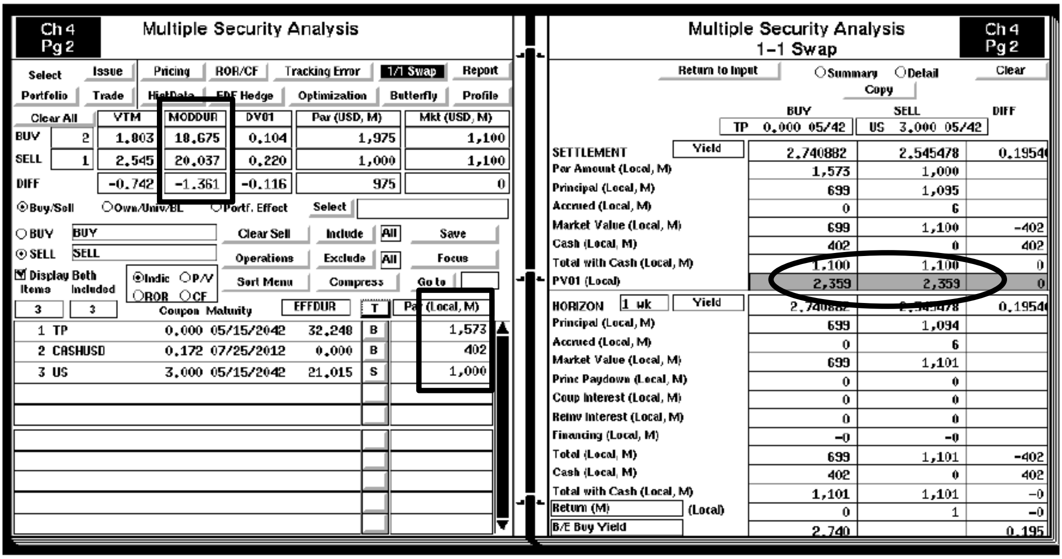
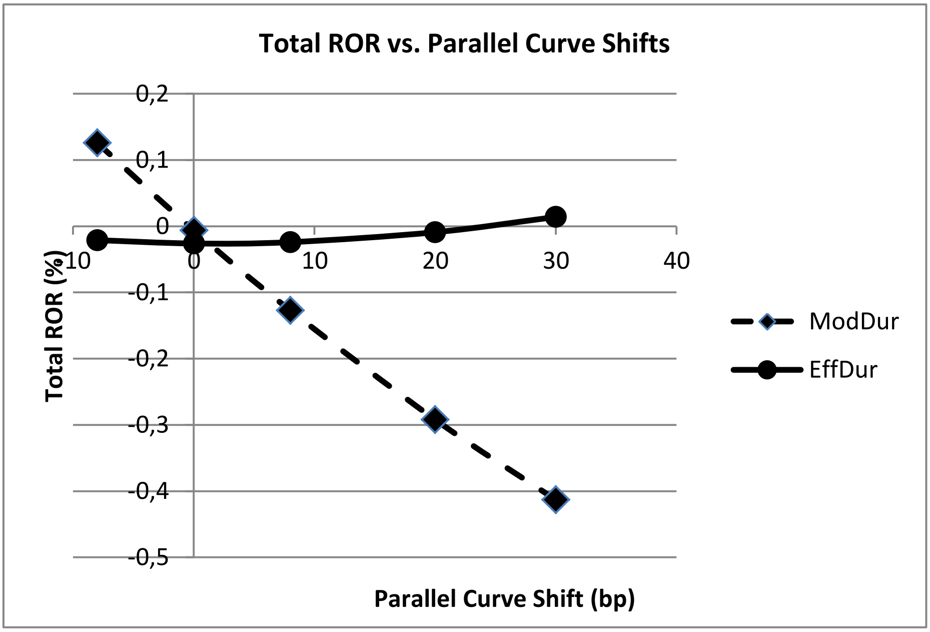


Figure 4. Yield Book page for effective duration weighted swap

Alternative risk measures are, of course, used in hedging bond portfolios and structuring trades. See, for example, Martillini, Priaulet, Fabozzi, and Luo (2006).

Which approach to weighting the trade works better? Which trade is closer to being duration neutral? Figure 5 displays the total return over a one-month horizon for trades weighted with standard duration and trades weighted with modified duration. The returns are for parallel shifts of the yield curve of -8 basis points, 0 basis points, +8 basis points, +20 basis points, and +30 basis points. The downward shift is limited by low yields at short maturities, currently.

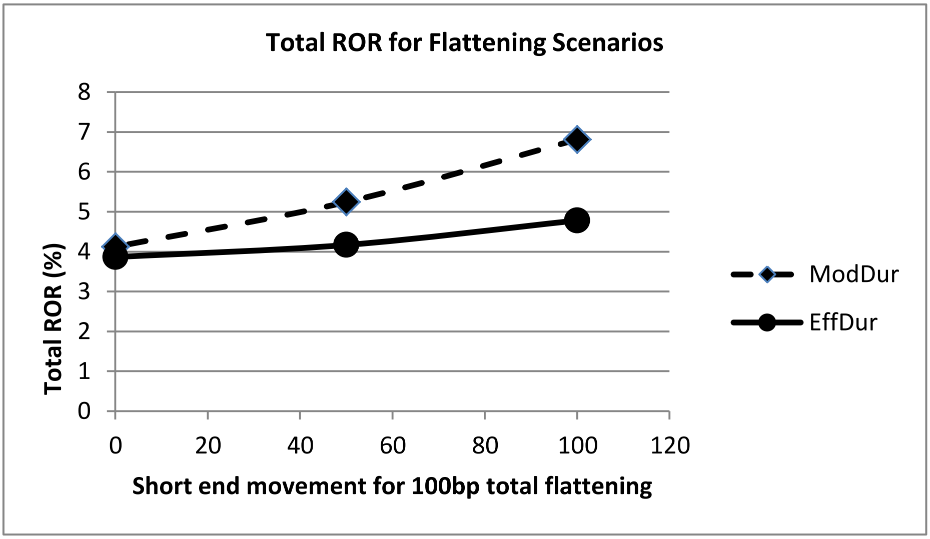
Figure 5. Total rate of return for parallel curve shifts



The evidence is stark. Effective DV01 is superior in constructing duration-neutral trades.

How about for flattening scenarios? Figure 6 displays total one-month returns for three flattening scenarios. In the first, three-month yields are unchanged and the bond's yield declines by 100 basis points, with the change allocated across the rest of the maturities using linear interpolation. This scenario is a bull flattening. The second scenario increases three-month yields by 50 basis points and decreases the bond's yield by 50 basis points, representing a curve rotation. The third scenario increases the three-month yield by 100 basis points and leaves the bond's yield unchanged. This is a bear flattening.

Figure 6. Total rate of return for 100 basis points of flattening



At first glance, it appears that weighting the trade with standard duration is superior, but that is simply the result of having the trade be sensitive to parallel shifts, as well as flattening. If we were wrong about the curve's change and it steepened instead of flattening, losses would be greater with the standard duration weighting. The more stable, but significant, gains from effective DV01 weighting represent superior trade weighting.

2 - Standard duration

Standard duration, the standard measure, is a bond's relative price sensitivity to an instantaneous change in its yield to maturity. Formulas for computing standard duration can be found in virtually any fixed income textbook, for example, Fabozzi (2007). Consequently, we will not discuss calculating standard duration in this paper. If we are simply interested in measuring the interest rate risk of one option-free bond at a time, standard duration is up to the task. However, as we will demonstrate below, because contemporaneous movements in bond yields are rarely equal along the term structure, effective duration is a superior measure of price sensitivity for structuring and monitoring portfolios of bonds.

3 - Effective duration

Effective duration is estimated from modeled changes in a bond's price given predetermined interest rate shifts rather than from price-yield mathematics. Shifts in a reference yield curve are simulated and the bond's model price change between an upshift and downshift in rates is divided by the movement in rates to find the bond's sensitivity to rate changes. The expression for effective duration is given by

$$\boldsymbol{EFF = \frac{-\Delta P/\Delta y_r}{P}}, \tag{1}$$

where

EFF = effective duration,

P = the bond's price, and

y_r = the reference yield curve.

Equation (1) simply tells us that effective duration is a bond's percentage price change based on a particular yield curve change.

The linkage between effective and standard duration is revealed through an alternative calculation of effective duration:

$$EFF = \frac{-\Delta P/\Delta y_r}{P} = \frac{-\Delta P/\Delta y}{P} \bullet \frac{\Delta y}{\Delta y_r} = MOD \bullet \beta_y, \tag{2}$$

where β_y is the yield beta that represents the change in the bond's yield to maturity (Δy) with respect to changes in the reference yield (Δy_r) [see, e.g., Fabozzi (1995)].[1] Equation (2) depicts the relationship between standard and effective duration. For a bond whose yield changes one-for-one as the reference yield changes, the yield beta is one, so the effective duration and standard duration are the same. Differential contemporaneous yield changes create differences between effective and standard duration for bonds without embedded options.

[1] Yield betas can be estimated using a linear regression where the dependent variable is the bond's yield and the independent variable is the reference bond's yield.

4 - Differences in duration: The case of zero coupon bonds

Let us now compare coupon and zero coupon bonds. Coupon bonds are priced relative to the par curve[2] and zero coupon bonds are priced relative to the spot curve. The relationship between these curves is mathematical; it is neither a matter of market opinion nor does it vary with risk preferences. To be sure, zero coupon bonds can be mispriced with respect to the spot curve, just as coupon bonds can be mispriced with respect to the par curve. Aside from the possibility of mispricing, we know how zero coupon bonds' yields and coupon bonds' yields change with respect to one another and it is *not* one-for-one.

A simple example will illustrate this point. The numbers in the example are exaggerated, but the principles hold with equal force using realistic data. Table 1 presents par and spot rates for a simple (and short) yield curve. Table 2 contains cash flows for three annual pay par bonds. From the price and the one-year cash flow, we can easily deduce that the one-year spot rate is 5% just as the one-year par bond rate is 5%. Because a coupon bond can be thought of as a portfolio of zero coupon bonds, we use the bootstrap methodology[3] to uncover the two-year and the three-year spot rates. These numbers appear unrealistic, but only because they are exaggerated to make the differences we are discussing apparent in a three-year term structure. With only a slight upward slope from, say, 20 to 21 years, the difference between the par and spot rates can be quite large.

Table 1. Stylized upward sloping yield curve

Term	Par rate	Spot rate
1 Year	5.00%	5.00%
2 Years	10.00%	10.263%
3 Years	15.00%	16.157%

[2] The par curve is the name for the standard yield curve for coupon-paying bonds. Less frequently, it is also used to mean the yields on par-priced bonds constructed from the zero coupon or spot curve. Typically, which meaning is appropriate will be clear from the context in which "par curve" is used.
[3] See pp. 135–140, Fabozzi (2007) for a discussion of bootstrapping.

Table 2. Stylized par bond prices and cash flows

Price	Year 1	Year 2	Year 3
100	105		
100	10	110	
100	15	15	115

Now, let us shift the par curve upward by 100 basis points and bootstrap the new par curve to uncover the implied spot rates. Table 3 presents the new par and spot rates as a result of the upward parallel shift in the par curve. Table 4 presents the differences between the par/spot curves after the upward shift and initial par/spot curves. Note that a parallel 100-basis-point upward shift in the par curve engenders a non-parallel shift in the spot curve with the three-year spot rate moving up by 108 basis points in response. The yield beta for the three-year zero-coupon bond is 1.08 and its effective duration would be 1.08 times its standard duration. Again, because the standard duration of a three-year zero is relatively small, a difference of 8% between standard and effective duration may not matter much, but for 25- or 30-year STRIPS an 8% difference would be material.[4]

Table 3. Stylized par/spot curves shifted upward by 100 basis points

Term	Par	Spot
1 Year	6.00%	6.00%
2 Years	11.00%	11.289%
3 Years	16.00%	17.238%

[4] For a 100-basis-point downward shift in the par curve, the results are the same. The spot curve moves more than the par curve. The yield betas for downward shifts equal the yield betas for upward shifts.

Table 4. Difference between initial and upshifted curves

Term	Par rates	Spot rates
1 Year	1.000%	1.000%
2 Years	1.000%	1.026%
3 Years	1.000%	1.080%

Our simple numerical example demonstrated that parallel shifts in an upward-sloping par curve lead to greater movement in the associated spot curve, making standard duration less useful for measuring interest rate risk. We shifted the par curve because, for most bond analytic software, that is the default option. It is, also, more intuitive.

Before we generalize the simple numerical result, let us first change the shifts in Tables 3 and 4 to a parallel shift in the spot curve to see the associated shift in the par curve. We are doing this because, as shown below, moving from spot shifts to par shifts gives much simpler mathematics, although, they are, admittedly, still messy. We will report the result but not produce additional figures.

In Table 3 and Table 4, instead of having the par yields be 5%, 10%, and 15% shifting to 6%, 11%, and 16%, let those be spot rates. Because the spot rates moved farther than the par yields, we should, now, expect the two- and three-year par yields to move less than 100 basis points. That is exactly what we get with par yields moving by 100 basis points for one-year, 97.7 basis points for two-year, and 94.2 basis points for three-year maturities. The underlying point remains: par yields and spot rates move by different amounts when the yield curve is not flat.

Every bond is a portfolio of zero coupon bonds. Alternatively, every bond is a two-bond portfolio, an annuity, and a zero coupon bond. Because the portfolio duration is the market-weighted average of the constituent bonds, every bond's actual price movements will be a combination of the parallel shift of the spot curve and, if the spot curve is upward sloping, the smaller shift in the annuity curve. We see this for linear curves, but it holds, also, for curves with multiple bends. That follows from the annuity being built from a portfolio of STRIPS.

The distinction between standard and effective matters most when the yield curve is sharply upward or downward sloping, at longer maturities, and when looking at zero coupon bonds. These become significant differences when structuring portfolios with specific duration targets.

More complex models of spot curve construction and dynamics exist. For example, see Martellini and Priaulet (2001). They are not tractable for our purposes.

5 - Aggregation

A desirable characteristic for a duration measure is that a portfolio's duration be a consistent aggregate of the individual security's durations. An example of this requirement is that a Treasury bond should have a standard duration equal to the duration of a portfolio of STRIPS that match its cash flows exactly when the cash flows of both are priced from the same yield curve.

To demonstrate that standard duration does not meet this criterion, let us extend our short and steep yield curve by two more years and continue to make the curve impossibly steep to demonstrate the mathematical result with just a few periods. Table 5 prices a five-year bond by (mistakenly) using the bond's yield to maturity for each cash flow. That yield is also used to calculate the standard duration for each cash flow, treating it as a zero coupon bond. The bond's payments have a present value of par, and both the standard and effective durations equal the standard duration found from entering the bond's characteristics into Excel's MDURATION function (2.689).

Table 5. Standard and effective duration with spot rates equal to yield to maturity

Year	Par	Spot	CashFlow	ModDur	PV	ModDur2	YldBeta	EffDur
1	5.00	~~25.00~~	25	0.8	20.00	16.00	1	16.00
2	10.00	~~25.00~~	25	1.6	16.00	25.60	1	25.60
3	15.00	~~25.00~~	25	2.4	12.80	30.72	1	30.72
4	20.00	~~25.00~~	25	3.2	10.24	32.77	1	32.77
5	25.00	~~25.00~~	125	4.0	40.96	163.84	1	163.84
					100.00	268.93		268.93
						2.689		**2.689**

Table 6 repeats all of the calculations in Table 5 using the correct spot rates. The price remains at par, as it must, but the standard durations for the zero coupon bonds are different because of different yields, and the weights for the portfolio's (the five-year bond) standard duration are different, also because of different yields. Here the standard duration no longer matches the MDURATION result (2.439 vs. 2.689). Effective duration does not match the MDURATION result either (3.075 vs. 2.689).

Table 6. Standard and effective duration with spot rates bootstrapped from yields

Year	Par	Spot	CashFlow	ModDur	PV	ModDur2	YldBeta	EffDur
1	5.00	5.00	25	0.95	23.81	22.68	1.00	22.68
2	10.00	10.26	25	1.81	20.56	37.30	1.03	38.28
3	15.00	16.16	25	2.58	15.95	41.20	1.08	44.51
4	20.00	23.41	25	3.24	10.78	34.94	1.19	41.68
5	25.00	34.03	125	3.73	28.90	107.80	1.49	160.40
					100.0	243.91		307.53
						2.439		**3.075**

Table 7 prices the five-year bond for a 50-basis-point increase in yields (with new spot rates calculated) and for a 50-basis-point decrease in yields. The percentage price change for a one-percentage-point change in yields (i.e., the usual definition of duration) is 3.076, nearly identical to the effective duration in Table 6.

Table 7. Effective duration for +/- 50-basis-point shifts

Year	Spot	CashFlow	PV	Spot	CashFlow	PV	
1	5.50	25	23.70	4.50	25	23.92	
2	10.7	25	20.37	9.75	25	20.76	
3	16.7	25	15.73	15.6	25	16.18	
4	24.0	25	10.57	22.8	25	10.99	
5	34.7	125	28.11	33.2	125	29.71	
			98.48			**101.55**	**3.07**

What duration do we find is we treat the bond as a portfolio of STRIPS? Using regular (modified) duration for each of the cash flows, the market-value-weighted average duration is 2.439, which is substantially shorter than either the whole bond effective duration of 3.075 or the formula duration of 2.689. The aggregation of regular duration fails. The portfolio of STRIPS effective duration is 3.075, exactly the effective duration of the bond, priced with spot rates.

6 - Conclusion

When measuring the price sensitivity of a bond with an embedded option, a forceful argument can be made for using effective duration. For option-free bonds, many argue that standard duration will suffice because the bond's cash flows will not change in response to shifts in the appropriate yield curve. The purpose of this article is to demonstrate that effective duration will not necessarily equal standard duration even if bonds are option-free. Portfolio managers, who might doubt their bond analytic software because it reports this difference, can now understand the underlying logic.

Weighting bond trades with standard duration exclusively will subject some portfolios to unnecessary interest rate risk. The differences are most important for bonds with very different coupon rates (in the limit, zero coupon bonds) and when yield curves are sharply sloping upward or downward.

Appendix

Now, let us consider the general case, albeit in a very stylized fashion. We will consider a two-period spot curve and a two-period annuity curve. Every bond is a portfolio comprised of an annuity (the coupons) and a zero coupon bond for the face amount. This portfolio's duration is a market-weighted average of the component durations. If we demonstrate that STRIPS and annuities have different standard and effective durations, that also demonstrates that STRIPS and bonds have different standard and effective durations.

Consider a simple linear annual spot curve with terms to maturity 't':

$$S_t = \alpha + bt, \qquad \text{(A.1)}$$

where S_t is the zero coupon yield for discounting cash flows maturing t years; hence, *a* is the intercept, so that changes in *a* will result in parallel shifts in the spot curve, and *b* is the slope of the spot curve.

The one-year spot rate is

$$S_1 = a + b \qquad \text{(A.2)}$$

The two year spot rate is

$$S_2 = a + 2b \qquad \text{(A.3)}$$

Given those spot yields, the prices of the one-year and two-year zero coupon bonds are as follows:

$$P_{0,1} = \frac{100}{(1+a+b)} \qquad \text{(A.4)}$$

$$P_{0,2} = \frac{100}{(1+a+2b)^2} \qquad \text{(A.5)}$$

In addition, the derivatives of these prices with respect to the intercept, *a*, which measures a parallel shift in spot yields, and the standard duration for each are as follows:

$$\frac{\partial P_{0,1}}{\partial a} = -\frac{-100}{(1+a+b)^2} \qquad \text{(A.6)}$$

$$\frac{\partial P_{0,2}}{\partial a} = -\frac{-200}{(1+a+2b)^3} \qquad \text{(A.7)}$$

$$dur_1 = -\frac{\frac{\partial P_{0,1}}{\partial a}}{P_{0,1}} = \frac{1}{(1+a+b)} \qquad \text{(A.8)}$$

$$dur_2 = -\frac{\frac{\partial P_{0,2}}{\partial a}}{P_{0,2}} = \frac{2}{(1+a+2b)} \tag{A.9}$$

A one-year annuity has a single cash flow and its yield is equal to the one-year spot rate:

$$\Upsilon_1 = (a+b) \tag{A.10}$$

The two-year annuity yield to maturity can be approximated to a high degree of accuracy with the duration-dollar-weighted average of the two spot rates (see Fabozzi and Mann, 2010):

$$\Upsilon_2 = \frac{dur_1 P_{0,1}(a+b) + dur_2 P_{0,2}(a+2b)}{dur_1 P_{0,1} + dur_2 P_{0,2}} \tag{A.11}$$

Substituting equations (A.2) and (A.3) for prices and equations (A.6) and (A.7) for durations, after some algebraic manipulation we find

$$\Upsilon_2 = \frac{(a+b)(1+a+2b)^3 + 2(a+2b)(1+a+b)^2}{(1+a+2b)^3 + 2(1+a+b)^2} \tag{A.12}$$

For a parallel shift in the spot curve (Δa), each spot rate changes by

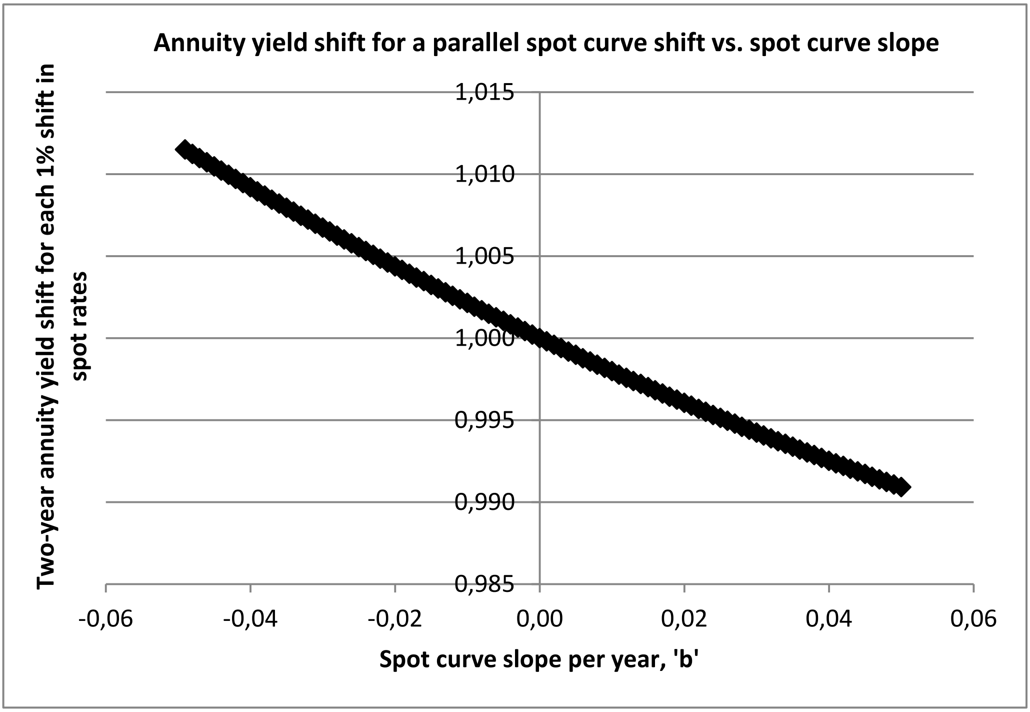
$$\frac{\partial S_1}{\partial a} = \frac{\partial S_2}{\partial a} = 1 \tag{A.13}$$

For the same parallel shift in the spot curve, the one-year annuity yield changes exactly as the one-year spot rate. The two-year annuity's yield change depends on the slope of the spot curve, *b*,:

$$\frac{\partial \Upsilon_2}{\partial a} = \frac{\left\{\begin{array}{c} [(1+a+2b)^3 + 2(1+a+b)^2] * \\ [(a+b)3(1+a+2b)^2 + (1+a+2b)^3 + 2(a+2b)2(1+a+b) + 2(1+a+b)^2] \\ -[(a+b)(1+a+2b)^3 + 2(a+2b)(1+a+b)^2] * \\ [3(1+a+2b)^2 + 4(1+a+b)] \end{array}\right\}}{[(1+a+2b)^3 + 2(1+a+b)^2]^2} \tag{A.14}$$

Equation (A.12) does not simplify very far or to terms with intuitive meaning. Consequently, in Figure A., we plot the derivative of annuity yield with respect to *a* versus values for *b*. That is, we show that the yield beta between spot curve shifts and annuity yields and, consequently, par yields depends on the slope of the spot curve. Alternatively, if our analysis had been in par space, we would have shown the yield beta of spot curves with respect to par shifts.

Figure A.1. Annuity yield shift for a parallel spot curve shift vs. spot curve slope



The yield betas that come out of our analysis are absolutely small. Notice that the y-axis values only range from 0.985 to 1.015. That is an artifact of having only a two-year annuity. The point remains: if the yield curve is not flat, annuity yields move more or less than spot yields when there is a parallel shift of the spot curve. Notice, also, that the annuity yields move less than spot yields for upward-sloping spot curves. This is exactly the flip side of spot yields moving more than par yields when the par curve is upward sloping. The upshot is that the price change of the annuity will differ from its standard duration times the shift in spot rates. Standard duration fails as a weight for duration neutral trades because it does not measure expected price changes correctly.

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