Mean-Variance and Stochastic Dominance Analysis of Global Exchange-Traded Funds

Zhen-Zhen Zhu\(^a\)
Wing-Keung Wong\(^b\)*
Kok Fai Phoon\(^c\)

Abstract

We employ the stochastic dominance (SD) approach that utilizes the entire return distribution to rank the performance of exchange-traded funds as traditional mean-variance and CAPM approaches may be inappropriate given the nature of non-normal returns. We find second and third-order stochastic dominance relationships amongst the assets and conclude that by investing in higher-order dominant funds, risk-averse investors can maximize their expected utilities but not their wealth. In addition, we find the common characteristic for most pairs of funds is that one fund is preferred to another in the negative domain whereas the preference reverses in the positive domain. We conclude that the stochastic dominance approach is more appropriate compared with traditional approaches as a filter in exchange-traded fund selection. Compared with traditional approaches, the SD approach, not only is assumption free, but also provides greater insights to the performance and risk inherent in an exchange-traded fund’s track record.

Key words: exchange-traded funds, stochastic dominance, risk-averse investors, performance measurement.

JEL Classification: G11, G15

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1 - Introduction

Core-satellite investing has gain adoption among institutional and even private investors. Originally developed as a portfolio management process (Singleton, 2004), with core investments invested in broad market benchmarks and satellite investments in inefficient asset classes, the process has evolved into new variations. After the global financial crisis, many funds have found that their portfolios with diverse asset classes failed to protect on the downside. Further, investments in illiquid assets like private equity and hedge funds suffered significant drawdown during the crisis. One alternative to implementing the core-satellite approach is to consider more liquid satellite investments in managed portfolios. Judicious choice of such liquid funds do provide diversification benefits. Further, exchange-traded funds (ETFs) allow for greater exposure to global and alternative investments, with reduced liquidity risk, though issues like replication and counterparty risk cannot be discounted. In light of the needs of the industry, strategies using ETFs promises investors a passively managed portfolio which can deliver systematic returns from exposure to factor risks, traditionally available only through active management; most importantly, at a much cheaper cost than active management. Since the introduction of the first ETF in the U.S. in 1993, ETFs have captured most of the growth potential of index mutual funds. The popularity of ETFs has increased so rapidly that trading in these funds now accounts for more than half of the daily trading volume on the American Stock Exchange. An ETF can be organized as open-end mutual fund or unit investment trust. An ETF was originally introduced as index-based, in that it consists of a portfolio of securities that is meant to track the investment performance of a specific index. For example, the original US ETFs track equity indexes, such as the S&P500 or the Nasdaq-100. However, with the proliferation of ETFs since the global financial crisis, ETFs have been issued to track indices in other asset classes including commodities, real estate and even hedge funds. Further their structure can include leverage and inverse exposure to indices. As many ETFs track indices like index funds, Rompotis (2005) has compared empirically ETF and index funds, analyzing their performance, their tracking ability and the sources of their expenses. A significant feature of ETF is that, like exchange traded stocks, they trade throughout the trading day, unlike mutual funds do not trade intraday with NAV set at the end of each trading day. Another significant advantage of ETF is their tax efficiency, which derives from their creation and redemption mechanism. In addition, ETFs incur lower costs, given their passive nature. ETFs combine the principal characteristics of classic mutual funds and stocks. Like mutual funds, they offer greater portfolio diversification, with investors
gaining index exposure by a share in an index ETF. Otherwise, ETF are more liquid than mutual funds, with continuous daily trading. Investors are also allowed to short ETFs. In our paper, we employ the stochastic dominance approach that utilizes the entire return distribution to rank the performance of ETFs. We find second and third-order stochastic dominance relationships amongst the assets and conclude that by investing in higher-order dominant funds, risk-averse investors can maximize their expected utilities but not their wealth. In addition, we find the common characteristic for most pairs of funds is that one fund is preferred to another in the negative domain whereas the preference reverses in the positive domain. We conclude that the stochastic dominance approach is more appropriate compared with traditional approaches as a filter in the exchange-traded fund selection. Compared with traditional approaches, the SD approach, not only is assumption free, but also provides greater insights to the performance and risk inherent in an exchange-traded fund’s track record.

The rest of the paper is organized as follows. Section 2 of this paper reviews the literature. The data and methodologies employed are described in Section 3. Empirical results and their implications are presented in Section 4 and Section 5 concludes.

2 - Literature review

Exchange-traded funds or ETFs, as they are commonly known, are tradable securities that derive their value from a pre-defined basket of securities which are constituents of an index. These types of ETF derive their value (and volatility) from the market movements of the underlying securities, which comprise the portfolio, and these funds are similar to index funds managed by institutional portfolio managers.

With the proliferation of ETFs, especially after the Global Financial Crisis, core-satellite investing, especially selecting ETFs (of differing underlying basket of assets, leverage and valuation (for example, an inverse relation between the ETF price and the underlying index) to include as satellite investments becomes of paramount importance in the investment process. Obviously, we would like to select satellite that provide downside protection and enhance returns of the core portfolio that is selected based on the investors’ objectives and constraints. An important criteria in selecting the satellite ETFs would be performance. Hence, the study that we carry out differs from traditional performance analysis of ETFs. Traditionally, ETFs are evaluated based on the ability to replicate an underlying benchmark (Hassine and Roncalli, 2013). Gallagher and Segara (2005) has examined the
performance by assessing the magnitude of its tracking error of exchange-traded funds in Australia. There are several studies that investigate the performance of ETF relative to traditional mutual funds. Rompotis (2005) presents evidence that the two instruments produce very similar results with respect to average return and mean risk levels as well as tracking ability. Considering German passively managed ETF, the author deems ETF to be hybrids between index funds and equities. Blitz, et al. (2010) find that both European index funds and ETF underperform their benchmarks due to dividend taxes, but not necessarily due to expense ratios. Considering German passively managed ETF, the author deems ETF to be hybrids between index funds and equities. Svetina (2008) on the other hand evaluates domestic equity, international equity as well as fixed income ETF, reports that ETF deliver better performance than retail index funds and similar performance than institutional index funds. Several authors however find that ETF display poorer returns than index funds as their structure and management processes are reluctant not to recapture transaction costs during benchmark changes (Gastineau, 2004; Elton, Gruber and Busse, 2002). Applying the generalized autoregressive conditional heteroskedasticity-in-mean autoregressive moving average (GARCH-M-ARMA) and exponentially GARCH-M-ARMA (EGARCH-M-ARMA) models to analyze the spillover, asymmetric volatility, and leverage effects of financial exchange-traded funds (ETFs), Chen and Malinda (2014) show that bilateral relations in terms of the spillover effects of volatilities and leverage effects exist between financial and non-financial ETFs.

The use of the mean-variance (MV) criterion developed by Markowitz (1952) and the capital asset pricing model (CAPM) statistics developed by Sharpe (1964), Treynor (1965) and Jensen (1969) for portfolio construction and managed funds performance evaluation, respectively, are advocated by contemporary finance. For example, Harper, et al. (2006) report from their cross-country studies ETFs have higher mean risk-adjusted returns, sharp ratios and Jensen’s alpha than than their counterpart closed-end funds. Since the return of ETFs are generally not normal distributions, the results of the MV criterion and the CAPM statistics may mislead the performance of funds. This makes the use of the MV and CAPM measures doubtful. Given that traditional performance measures may not be appropriate to obtain the correct results, many approaches have been offered as alternatives to assess ETFs performance. Hassine and Roncalli (2013), propose a performance measure based on the value-at-risk framework to evaluate the performance of passive management and ETF. However, Ogryczak and Ruszczynski (2002), Leitner (2005) and Ma and Wong (2006) noted that stochastic dominance (SD)
approach is superior as whatever information obtained by applying VaR and CVaR could be obtained by SD approach.

The search for performance measures for exchanged-traded funds is an ongoing process. We recommend the SD approach that allows investors to appropriately rank fund performance without the need for strong assumptions on investors’ utility function or the return distribution of asset returns. SD rules offer superior criteria on which to base investment decisions compared with the traditional MV and CAPM analysis because the assumptions underlying SD are less restrictive. Taylor and Yoder (1999) argued that SD incorporates information on the entire distribution, rather than the first two moments and requires no precise assessment as to the specific form of investors’ risk preference or utility function. In addition, by evaluating European large-cap equity funds, Högholm, et al. (2011) find that a large part of the individual funds significantly underperform the benchmark in the lower tail of the conditional distribution. We note that the SD approach could circumvent the limitation and thus in this paper we recommend to use the SD approach.

The SD approach has been used in the evaluation of performance of mutual funds since the 1970s (Levy and Sarnat, 1970; Porter, 1973). Taylor and Yoder (1999) used the SD approach to compare the performance between load and no-load funds during the 1987 crash. Kjetsaa and Kieff (2003) documented that the SD approach provides a collateral and feasible strategy to reveal relative investment preferences by discriminating among and parsing the universe of mutual fund opportunities. Gasbarro et al. (2007) utilized both the SD approach and the CAPM criterion to compare the performance of 18 country market indices (iShares) and found that SD appears to be both more robust and discriminating than the CAPM in the ranking of iShares.

In this paper, we use the Davidson and Duclos (DD, 2000) or DD test to determine if the difference in the cumulative density functions of the returns of two exchange traded funds are statistically significant. We use the DD test to determine if SD occurred among the 137 ETF during our sample period. We find that over the sample period (January 2004 to December 2014), some ETF dominate others. Conversely, some funds do not dominate any other ETF, but they themselves are not dominated by other ETF as well.
3 - Data and methodology

3.1 Data

The data analysed in this study are the weekly returns of the 137 Exchanged Trade Funds reported by Datastream for the sample period from January 2004 to December 2014. For completeness, we include two traditional equity market indices, the S&P 500 and the Morgan Stanley Capital International Global index (MSCI), and three global bond indices as exhibited in Table 1. We include the S&P 500 in our study because most U.S. based fund managers and investors use the S&P 500 as the equity benchmark. If they invest internationally, diversification benefit can be measured relative to a regional benchmark constructed by Morgan Stanley, like the MSCI. The 3-month U.S. T-bill rate and the MSCI proxy the risk-free rate and the global market index, respectively, are used for CAPM statistics. In addition, in this paper, we select the three ‘most outstanding funds’ with largest or smallest means, standard deviations or Sharpe ratios in our analysis and we display the information of the funds in Table 1.¹

<table>
<thead>
<tr>
<th>Table 1: Selected Five Market Indices and Exchange-traded Funds</th>
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</thead>
<tbody>
<tr>
<td>Market Index 1 (MI1)</td>
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<tr>
<td>Market Index 2 (MI2)</td>
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<tr>
<td>Market Index 3 (MI3)</td>
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<td>Market Index 4 (MI4)</td>
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<tr>
<td>Market Index 5 (MI5)</td>
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<tr>
<td>ETF 28 (Exchange-traded Funds 28)</td>
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<tr>
<td>ETF 80 (Exchange-traded Funds 80)</td>
</tr>
<tr>
<td>ETF 137 (Exchange-traded Funds 137)</td>
</tr>
</tbody>
</table>

3.2 Methodology

In this paper we will use the mean-variance (MV) and stochastic dominance (SD) approaches to analyze the performance of funds. The former

¹ Readers may refer to Table 2 for more information.
is to use two measures - return and variance or standard deviation – to compare the performance of funds, while the latter is to compare the whole distribution of the returns of assets. The approaches can be used to determine the preference of investors with utility functions $u$ that belongs to $U = \{ u: (-1)^{i+1} u^{(i)} \geq 0, \ i = 1, ..., j \}$ where $u^{(i)}$ is the $i^{th}$ derivative of the utility function $u$. We first discuss the MV approach.

### 3.2.1 Mean-variance criterion

The MV criterion (Markowitz, 1952; Tobin, 1958) states that, for the returns of any two funds $Y$ and $Z$ with means $\mu_Y$ and $\mu_Z$ and standard deviations, $\sigma_Y$ and $\sigma_Z$, respectively, $Y$ is said to dominate $Z$ by the MV approach if $\mu_Y \geq \mu_Z$ and $\sigma_Y \leq \sigma_Z$ with the inequality holds in at least one of the two conditions. Wong (2007) has shown that if $Y$ dominates $Z$ by the MV approach, under some regularity conditions, we have $E[u(X)] \geq E[u(Y)]$ for any $u$ in $U_2$.

### 3.2.2 CAMP statistics

The CAPM risk-adjusted performance measures developed by Sharpe (1964), Treynor (1965), and Jensen (1969) are the Sharpe ratio, Treynor’s index, and Jensen (alpha) index can be considered to be measures for the MV approach because they are using mean and risk as an alternative of variance. The CAPM is a parsimonious general equilibrium model in which the return $R_{i,t}$ of index $i$ at time $t$, is formulated by:

$$\begin{align*}
R_{i,t} - R_{f,t} &= \alpha_i + \beta_i (R_{m,t} - R_{f,t}) + \varepsilon_{i,t} \\
\text{where} \ & \varepsilon_{i,t} \text{ is the residual assumed to be i.i.d.,} \ R_{m,t} \text{ is the return of market portfolio, and} \ R_{f,t} \text{ is the return of risk-free asset at time} \ t.
\end{align*}$$

The Sharpe ratio ($S_i$) is defined as $S_i = (\bar{R}_i - \bar{R}_f)/\bar{\sigma}_i$, which is the ratio of the estimated excess return of an investment to its estimated standard deviation $\bar{\sigma}_i$. The Treynor’s index ($T_i$) is defined as $T_i = (\bar{R}_i - \bar{R}_f)/\hat{\beta}_i$, which is the ratio of the estimated excess return of an investment to its estimated $\hat{\beta}_i$. The Jensen’s index ($J_i$) is defined as $J_i = (\bar{R}_i - \bar{R}_f) - \hat{\beta}_i(\bar{R}_m - \bar{R}_f)$. In addition, the estimate $\hat{\beta}_i$ of $\beta_i$ is commonly used in the comparing the performance of a prospect, where $\beta_i$ is defined in (1). To circumvent the limitation, academics recommend to use the stochastic dominance approach.

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2 Refer to Wong (2007) and Wong and Ma (2008) on the relation between stochastic dominance and mean-variance rules.

3 Readers may refer to Sharpe (1964), Treynor (1965) and Jensen (1969) for the details and discussion of the statistics.
3.2.3 Stochastic dominance test

However, both the MV criterion and CAPM measures are derived assuming the Von Neumann-Morgenstern (1944) quadratic utility function and that returns are normally distributed (Feldstein, 1969; Hanoch and Levy, 1969). Thus, the reliability of performance comparisons using the MV criterion and CAPM analysis depends on the degree of non-normality of the returns data and the nature of the (non-quadratic) utility functions (Fung and Hsieh, 1999b).

Essentially, the most commonly-used SD rules that correspond with three broadly defined utility functions are first-, second- and third-order SD for risk averters, denoted by FSD, SSD, and TSD, respectively. Formally, we let \( F \) and \( G \) be the cumulative distribution functions (CDFs) and \( f \) and \( g \) are the corresponding probability density functions (PDFs) of the returns for two funds \( Y \) and \( Z \), respectively with common support of \([a, b]\) \((a<b)\).

Define \( H_0 = h \) and \( H_j(x) = \int_a^x H_j(t)dt \) for \( h = f, g, H=F, G \) and \( j=1,2,3 \).

\[
\text{Fund } Y \text{ dominates Fund } Z \text{ by FSD if and only if } F_1(x) \leq G_1(x); \text{ by SSD if and only if } F_2(x) \leq G_2(x); \text{ and finally, by TSD if and only if } F_3(x) \leq G_3(x); \text{ for all } x, \text{ and the strict inequality holds for at least an interval of } x; \text{ and } Y \text{ has not lower expected return than } Z \text{ (Chan, et al., 2012).}
\]

The existence of FSD (SSD, TSD) implies that the expected wealth (utilities) of investors are always higher when holding the dominant fund than holding the dominated fund and, consequently, the dominated fund should not be chosen. Investors exhibit non-satiation (more is preferred to less) under FSD; non-satiation and risk aversion under SSD; and non-satiation, risk aversion, and decreasing absolute risk aversion (DARA) under TSD. It is well-known that hierarchical relationship exists in SD (Levy 1992, 2015; Sriboonchitta et al. 2009): FSD implies SSD, which in turn implies TSD. However, the converse is not true. Thus, only the lowest dominance order of SD is reported in practice.

Recent advances in SD empirical techniques allow the statistical significance of SD to be determined. To date, the SD tests for risk averters have been well developed, see, for example, Davidson and Duclos (DD, 2000), Barrett and Donald (BD, 2003) and Linton et al. (LMW, 2005). The DD test has been found to be one of the most powerful, but yet less conservative in size (Wei and Zhang, 2003; Tse and Zhang, 2004 and Lean et al., 2008); while the BD

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\(^4\) Refer to Wong and Li (1999) for the discussions on the definitions.
test is another powerful test instrument and the LMW is useful as it is extended from Kolmogorov-Smirnov test for FSD and SSD by relaxing the iid assumption. We report the results of the DD test and skip reporting those of BD and LMW tests as the former can test the SD relationship up to the third-order and use the grids points to determine any portion of the dominance and the results of both BD and LMW tests are consistent with those of the DD test. For any two funds Y and Z with CDFs F and G, respectively, and for a grid of pre-selected points \( x_1, x_2, \ldots, x_k \), Bai et al. (2015) modify the DD statistic to obtain the following \( j^{th} \)-order DD test statistic, \( T_j(x) \) \((j = 1, 2, and 3)\):

\[
T_j(x) = \frac{\hat{F}_j(x) - \hat{G}_j(x)}{\sqrt{\hat{V}_j(x)}},
\]

where

\[
\hat{V}_j(x) = \hat{V}_F^j(x) + \hat{V}_G^j(x) - 2\hat{V}_{F,G}^j(x),
\]

\[
\hat{H}_j(x) = \frac{1}{N_h(j-1)!} \sum_{i=1}^{N_h} (x - h_i)^{j-1}, H = F, G;
\]

\[
\hat{V}_F^j(x) = \frac{1}{N_h} \left[ \frac{1}{N_h((j - 1)!)^2} \sum_{i=1}^{N_h} (x - h_i)^{2(j-1)} - \hat{H}_j(x)^2 \right], h = f, g;
\]

\[
\hat{V}_{F,G}^j(x) = \frac{1}{N_h} \left[ \frac{1}{N_h((j - 1)!)^2} \sum_{i=1}^{N_h} (x - f_i)^{j-1}(x - g_i)^{j-1} - \hat{F}_j(x)\hat{G}_j(x) \right].
\]

in which \( F_j \) and \( G_j \) are defined in (1).

It is empirically impossible to test the null hypothesis for the full support of the distributions. Thus, Bishop et al. (1992) recommend testing the null hypothesis for a pre-designed finite number of \( x \). Specifically, the following hypotheses are tested:

\[
H_0 : F_j(x_i) = G_j(x_i), \text{ for all } x_i, i = 1, 2, \ldots, k;
\]

\[
H_A : F_j(x_i) \neq G_j(x_i) \text{ for some } x_i;
\]

\[
H_{AI} : F_j(x_i) \leq G_j(x_i) \text{ for all } x_i, F_j(x_i) < G_j(x_i) \text{ for some } x_i;
\]

\[
H_{A2} : F_j(x_i) \geq G_j(x_i) \text{ for all } x_i, F_j(x_i) > G_j(x_i) \text{ for some } x_i.
\]

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We note that in the above hypotheses, $H_A$ is set to be exclusive of both $H_{A1}$ and $H_{A2}$, which means that if either $H_{A1}$ or $H_{A2}$ is accepted, we will not say $H_A$ is accepted. Under the null hypothesis, Bai et al. (2011) suggest using a simulation approach to generate the simulated critical values. In this paper, we follow their recommendation.

Accepting either $H_0$ or $H_A$ implies non-existence of any SD relationship, non-existence of any arbitrage opportunity between any two exchange-traded funds and neither of the two exchange-traded funds is preferred to one another. However, if $H_{A1}$ or $H_{A2}$ of order one is accepted, a particular exchange-traded fund stochastically dominates another exchange-traded fund at first-order. In this situation, any non-satiated investor will increase her/his expected wealth if s/he switches from the dominated exchange-traded fund to the dominant one. On the other hand, if $H_{A1}$ or $H_{A2}$ is accepted for order two or three, a particular exchange-traded fund stochastically dominates the other at second- or third-order. In this situation, arbitrage opportunity does not exist and switching from one exchange-traded fund to another will only increase investors’ expected utilities, but not expected wealth.

The DD test compares the distributions of $Y$ and $Z$ at a finite number of grid points, and various studies, see, for example, Tse and Zhang (2004) and Lean, et al. (2008) have examined the choice of these points. Too few grids will miss information on the distributions between any two consecutive grids (Barrett and Donald 2003). To make more detailed comparisons, we follow Fong, et al. (2005) and Gasbarro, et al. (2007) to make 10 major partitions with 10 minor partitions within any two consecutive major partitions in each comparison and to make the statistical inference. Bai et al. (2015) improve the SD test by deriving the limiting process of the SD statistic $T_j (x)$ so that the SD test can be performed by max $|T_j (x)|$ to take care of the dependency of the partitions. We follow their recommendation in our analysis.

4 - Empirical findings and implications

4.1 Mean-Variance criterion and CAPM statistics results

We first apply the MV criterion to conduct our empirical analysis of the Exchange-traded Funds (ETFs) and plot in Figure 1 the means and the standard deviations of the returns for all 137 exchange-traded funds studied in this paper. From the figure, we find that, in general, the means and the

\[ \text{Refer to our conclusion for the discussion.} \]
standard deviations of the returns for all exchange-traded funds move together. This finding is consistent with portfolio theory that higher risk associates with higher mean.

We exhibit in Table 2 the summary statistics of the five indices (MI1 to MI5) and the three ‘most outstanding funds’ (ETF 28, ETF 80, and ETF 137) with largest or smallest means, standard deviations, or Sharpe ratios. The means and standard deviations vary widely across 137 exchange-traded funds. For example, ETF 28 acquires the largest weekly mean return (0.003297) and the largest standard deviation (0.075045) among the 137 exchange-traded funds and the five market indices while both ETF 137 and ETF 80 exhibit the lowest weekly mean returns (-0.000161) and smallest standard deviation (0.001866), respectively. Interestingly, using the MV criterion, we have found a fund, ETF 28, possessing the largest mean that does not dominate any other fund, including ETF 80 and ETF 137. Thus, we conclude that, using the MV criterion, a fund with the largest mean return may not be a good investment choice. On the other hand, ETF 80 is found to have significant higher mean and smaller standard deviation than ETF 137. Therefore, ETF 80 dominates ETF 137 by the MV criterion.

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6 Refer to Table 1 for the full names of MI 1 to MI 5, ETF 28, ETF 80 and ETF 137 and the characteristics of the funds.
Next, we investigate the CAPM measures for all indices and funds and present all the results in Table 3. From Table 3, ETF 28 exhibits the largest Sharpe ratio (-0.1412) while ETF 80 has the lowest (-7.3806). Furthermore, from the table, we find that sometimes a fund dominates another fund by a CAPM statistic but the dominance relation can be reverse if measured by other CAPM statistic(s). For example, ETF 80 dominates ETF 137 by Jensen’s index whereas ETF 137 dominates ETF 80 by both Sharpe ratio and Treynor’s index. Thus, we conclude that, in general, different funds are chosen using different CAPM measures. In addition, our results show that some of the return distributions are non-normal and exhibit both negative skewness and excess kurtosis. Therefore, the results drawn by both the MV and CAPM statistics can be misleading.

Nonetheless, from our analysis using the MV criterion and CAPM statistics, we observe some consistent outcomes. We find, for example, that ETF 80 (fund with the smallest standard deviation) dominates ETF 137 (fund with the smallest mean) by the MV rule while ETF 137 dominates ETF 80 (fund with the smallest standard deviation and Sharpe ratio) by using CAPM statistics. Thus, one may ask whether the fund with the largest Sharpe ratio and the fund
with the smallest standard deviation are the best choices while the fund with the smallest Sharpe ratio is the worst choice. To explore this question and to examine alternative measures to choose funds, we utilize the SD approach.

### Table 2: Summary Statistics of the Exchange-traded Funds and the Five Market Indices

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std dev</th>
<th>SR</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI1</td>
<td>0.0013</td>
<td>0.0235</td>
<td>-0.5363</td>
<td>-0.5193***</td>
<td>7.522***</td>
<td>1378.89***</td>
</tr>
<tr>
<td>MI2</td>
<td>0.0011</td>
<td>0.0233</td>
<td>-0.5473</td>
<td>-0.8080***</td>
<td>8.513***</td>
<td>1795.67***</td>
</tr>
<tr>
<td>MI3</td>
<td>0.0006</td>
<td>0.0088</td>
<td>-1.5112</td>
<td>0.1619</td>
<td>0.430**</td>
<td>6.93**</td>
</tr>
<tr>
<td>MI4</td>
<td>0.0004</td>
<td>0.0089</td>
<td>-1.5147</td>
<td>0.1426</td>
<td>0.410**</td>
<td>5.96*</td>
</tr>
<tr>
<td>MI5</td>
<td>0.0000</td>
<td>0.0035</td>
<td>-3.9567</td>
<td>-0.3638***</td>
<td>1.771***</td>
<td>87.71***</td>
</tr>
<tr>
<td>ETF 28</td>
<td>0.0033</td>
<td>0.0750</td>
<td>-0.1412</td>
<td>16.6731***</td>
<td>361.607***</td>
<td>3153928.5***</td>
</tr>
<tr>
<td>ETF 80</td>
<td>0.0001</td>
<td>0.0019</td>
<td>-7.3806</td>
<td>-0.2194**</td>
<td>3.700***</td>
<td>332.09***</td>
</tr>
<tr>
<td>ETF 137</td>
<td>-0.0002</td>
<td>0.0099</td>
<td>-1.413</td>
<td>-2.8095***</td>
<td>64.601***</td>
<td>100566.85***</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>0.0033</th>
<th>0.0750</th>
<th>-0.1412</th>
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</thead>
<tbody>
<tr>
<td>Maximum (ETF)</td>
<td>-0.0002</td>
<td>0.0019</td>
<td>-7.3806</td>
</tr>
</tbody>
</table>

Note: ETF 28, ETF 80 and ETF 137 are the ‘most outstanding funds’ in which ETF 28 possesses the largest weekly mean return (0.003297) and the largest standard deviation (0.075045); ETF 80 exhibits the lowest weekly standard deviation (0.001866) and the smallest Sharpe ratio (-7.3806); ETF 137 exhibits the lowest weekly mean return (-0.000161). Results in bold are extreme values. The symbols *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. SR and JB denote Sharp ratio and Jarque-Bera test, respectively. MIi is the abbreviation of Market Index i, where i=1, 2, 3, 4 and 5.
Table 3: CAPM Statistics

<table>
<thead>
<tr>
<th></th>
<th>Beta</th>
<th>Sharpe Ratio</th>
<th>Treynor's Index</th>
<th>Jensen's Index</th>
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<tbody>
<tr>
<td>Market Index 1 (MI1)</td>
<td>0.9869</td>
<td>-0.5363</td>
<td>-0.0128</td>
<td>0.000005</td>
</tr>
<tr>
<td>Market Index 2 (MI2)</td>
<td>1</td>
<td>-0.5473</td>
<td>-0.0128</td>
<td>0</td>
</tr>
<tr>
<td>Market Index 3 (MI3)</td>
<td>0.3603</td>
<td>-1.5112</td>
<td>-0.0370</td>
<td>-0.008729</td>
</tr>
<tr>
<td>Market Index 4 (MI4)</td>
<td>0.3731</td>
<td>-1.5147</td>
<td>-0.0362</td>
<td>-0.008761</td>
</tr>
<tr>
<td>Market Index 5 (MI5)</td>
<td>0.3523</td>
<td>-3.9567</td>
<td>-0.0395</td>
<td>-0.009426</td>
</tr>
<tr>
<td>ETF 28</td>
<td>0.2303</td>
<td>-0.1412</td>
<td>-0.0460</td>
<td>-0.007658</td>
</tr>
<tr>
<td>ETF 80</td>
<td>0.3530</td>
<td>-7.3806</td>
<td>-0.0390</td>
<td>-0.009264</td>
</tr>
<tr>
<td>ETF 137</td>
<td>0.3645</td>
<td>-1.4130</td>
<td>-0.0386</td>
<td>-0.009403</td>
</tr>
</tbody>
</table>

Note: We use the MSCI world index as the market portfolio and 3 month treasury bill as the risk-free asset. See equation (1) for the formula of the listed statistics.

4.2 Stochastic dominance results

DD stated that the null hypothesis of equal distribution could be rejected if any value of the test statistic, $T_j$ ($j=1,2,3$; see eqn (3)), is significant. In order to minimize the Type II error and to accommodate the effect of almost SD (Leshno and Levy, 2002; Guo, et al., 2013, 2014), we follow Gasbarro, et al. (2007) to use a conservative 5% cut-off point for the proportion of test statistics in statistical inference. Using a 5% cut-off point, we conclude fund Y dominates fund Z if we find at least 5% of $T_j$ to be significantly negative and no portion of $T_j$ is significantly positive. The reverse holds if the fund Z dominates fund Y.
Table 4: Pair-wise Comparison of the Exchange-traded Funds by the Davidson-Duclos (DD) Tests

<table>
<thead>
<tr>
<th></th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>ETF28</th>
<th>ETF80</th>
<th>ETF137</th>
<th>Domination</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>=</td>
<td>&lt;2,3</td>
<td>&lt;2,3</td>
<td>=</td>
<td>&lt;2,3</td>
<td>&lt;2,3</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>=</td>
<td>&lt;2,3</td>
<td>&lt;2,3</td>
<td>=</td>
<td>&lt;2,3</td>
<td>&lt;2,3</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>=</td>
<td>&lt;2,3</td>
<td>&lt;2,3</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>&lt;2,3</td>
<td>=</td>
<td>&lt;2,3</td>
<td>&lt;2,3</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M5</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>=</td>
<td>&lt;2,3</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETF28</td>
<td>=</td>
<td>=</td>
<td>=</td>
<td>=</td>
<td>=</td>
<td>=</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETF80</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>=</td>
<td>&gt;2,3</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETF137</td>
<td>&gt;2,3</td>
<td>&gt;2,3</td>
<td>=</td>
<td>=</td>
<td>&lt;2,3</td>
<td>&lt;2,3</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dominated</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Note: The results in this table are read based on rows-versus-column basis. For example, the cell in the fourth row MI4 and the first column MI1 tells us that MI4 stochastically dominates MI1 at second and third-order while the cell in the second row MI2 and the first column MI1 inform readers that MI1 does not stochastically dominate MI2. The five indices MI1 – MI5 and the “three most outstanding funds”, ETF 28, ETF 80 and ETF 137, are defined in Table 1.

We depict in Table 4 the DD test statistics for the pair-wise comparison of the 5 market indices and the 3 most outstanding exchange-traded funds and display in Table 5 the summary results of their DD test for risk averters. From the tables, we first find that, in general, one could not conclude that risk averters may not always prefer investing in funds than indices and vice versa because there exist SSD relationships from funds to indices as well as from indices to funds.

Secondly, from Table 4, we find that in our sample period MI 5 is the most favourable index and MI 2 is the least favourable index as the former dominates 132 other indices/funds but is dominated only by ETF 80 at SSD whereas the latter is dominated by 29 indices/funds but dominates only 79 other indices/funds at SSD. On the other hand, ETF 80 is the most favourable fund as it dominates 137 other indices/funds at second order and is not dominated by any other index/fund. Similarly, from the tables, one could conclude that ETF 137 is the second most favourable fund. The least favourable fund is ETF 28 as it dominates none of the funds and is not dominated by any other indices/funds.

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One may refer to the notes in the tables on how to read the tables.
Figure 2: DD Statistics of S&P 500 – ETFs 80 and their Cumulative Distribution Functions

Note: \( T_j \) is the Davidson and Duclos (DD) statistic for risk averters with \( j = 1, 2 \) and 3 defined in equation (2) with \( F = \text{S&P 500} \) and \( G = \text{ETF 80} \). S&P 500 (ETF 80) refers to the cumulative S&P 500 (ETF 80).
Figure 3: DD Statistics of MI 4 – ETFs 137 and their Cumulative Distribution Functions

Note: $T_j$ is the Davidson and Duclos (DD) statistic for risk averters with $j = 1, 2$ and $3$ defined in equation (2) with $F = $ MI 3 and $G = $ ETF 137. MI 4 (ETF 137) refers to the cumulative MI 4 (ETF 137).

Now, we come back to our conclusion drawn using the MV criterion and CAPM statistics. By using the MV rule, we find that ETF 28 does not dominate or is dominated by any other fund, but ETF 80 dominates ETF 137. However, we can’t get any preference by using the CAPM statistics since by using different method we draw different conclusion. Using the SD approach, we demonstrate that ETF 80 is the most favourable fund whereas ETF 28 is the least favourable fund. This finding leads us to conjecture that the SD approach could exploit more information to decide on fund choice than its MV and CAPM counterparts. Based on this conjecture, we further investigate whether we can acquire additional information by adopting the SD approach to obtain the percentage of significant DD statistics for pairs of these funds in details, namely, ETF 28 (largest mean) versus ETF 137 (smallest mean), ETF 80 (smallest standard deviation and smallest Sharpe ratio) versus ETF 28 (largest standard deviation and largest Sharpe ratio), ETF 80 (the most favourable fund under SD) versus ETF 137 (the second most favourable fund under SD). The results are reported in Table 6.

We apply equation (3) with $F$ to be the first one and $G$ to be the second one stated in the first column in Table 5. To compare the performance of different
assets, we first examine the CDFs of the returns for different assets and their corresponding first three orders of the DD statistics, $T_j$, for risk averters in Figures 2 and 3. We use Figure 2 as an example to illustrate the comparison between MI 1 and ETF 80. The figure shows that the CDF of MI 1 lies below that of ETF 80 in the downside risk, and their positions reverse in the upside profit. This indicates that there is no FSD between the two assets, and that MI 1 dominates ETF 80 in the downside risk while ETF 80 dominate MI 1 in the upside profit. In order to verify this finding formally, we use the first three orders of the DD statistics, $T_j (j=1, 2, 3)$, for the two portfolios, with the results reported in Table 6.

The percentage of significant values reported in Table 6 show that 20 percent of $T_1$ is significantly positive, whereas 17 percent of $T_1$ is significantly negative. However, in Figure 2, $T_2$ and $T_3$ are positive over the entire range of the distribution, with 22 percent of $T_2$ (36 percent of $T_3$) being significantly positive and no $T_2$ ($T_3$) being significantly negative at the 5 per cent level. Thus, Table 5 and Figure 2 together lead up conclude that 1) there is no FSD between MI 1 and ETF 80, 2) MI 1 dominates ETF 80 significantly in the downside risk while their dominance relationship reverse in the upside profit, 3) ETF 80 SSD (TSD) dominate MI 1 for risk averters. The analysis for other pairs of assets could be conducted in the same way and we now draw conclusion from the figures (we skip some figures for simplification and the figures are available on request) and tables in the following: 1) ETF 80 is the most favourable asset, as it dominates 137 other indices/funds at second order and is not dominated by any other index/fund; 2) MI 2 (MSCI index) is the second least favourable asset as it is dominated by 20 other indices/funds at second order and dominates only 79 other index/fund; 3) ETF 28 is the least favourable asset as it dominates no or is not dominated by any other indices/funds.

We note that most of the SD comparisons for assets in the literature behave as in the above comparison: one asset dominates another asset at SSD or TSD (see for example, Seyhun, 1993). Applying the DD technique, we could obtain more information than the usual SD comparison as we state in the above example: one asset dominates another asset on the downside while the reverse dominance relationship can be found on the upside. This finding is in line with the direction of research in Post and Levy (2005) who investigate the behaviours of investors in bull and bear markets.
Table 5: Summary of the Davidson-Duclos (DD) Test Statistics

<table>
<thead>
<tr>
<th>Index / Fund</th>
<th>SSD</th>
<th>TSD</th>
<th>SSD</th>
<th>TSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI 1</td>
<td>85</td>
<td>86</td>
<td>21</td>
<td>23</td>
</tr>
<tr>
<td>MI 2</td>
<td>79</td>
<td>82</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>MI 3</td>
<td>126</td>
<td>126</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>MI 4</td>
<td>124</td>
<td>125</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>MI 5</td>
<td>132</td>
<td>133</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ETF 28</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ETF 80</td>
<td>137</td>
<td>137</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ETF 137</td>
<td>125</td>
<td>125</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: The values indicate the number of indices/funds for each index/fund dominates or the number of indices/funds that it is dominated by.

Table 6: Results of Davidson-Duclos (DD) Test for Risk Averters

<table>
<thead>
<tr>
<th>Sample</th>
<th>FSD</th>
<th>SSD</th>
<th>TSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI 1 – MI 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MI 1 – MI 3</td>
<td>0</td>
<td>17</td>
<td>0</td>
</tr>
<tr>
<td>MI 1 – MI 4</td>
<td>19</td>
<td>17</td>
<td>0</td>
</tr>
<tr>
<td>MI 1 – MI 5</td>
<td>19</td>
<td>17</td>
<td>0</td>
</tr>
<tr>
<td>MI 1 – ETF 28</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>MI 1 – ETF 80</td>
<td>20</td>
<td>17</td>
<td>0</td>
</tr>
<tr>
<td>MI 1 – ETF 137</td>
<td>13</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>MI 2 – MI 3</td>
<td>19</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>MI 2 – MI 4</td>
<td>17</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>MI 2 – MI 5</td>
<td>20</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>MI 2 – ETF 28</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>MI 2 – ETF 80</td>
<td>20</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>MI 2 – ETF 137</td>
<td>13</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>
Note: DD test statistics are computed over a grid of 100 on weekly exchange-traded fund returns. The table reports the percentage of DD statistics which is significantly negative or positive at the 5% significance level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution. $T_j$ is the Davidson and Duclos (DD) statistic for risk averters with $j=1, 2$ and 3 defined in equation (3) with $F$ to be the first one and $G$ to be the second one stated in the first column. The five indices MI1 – MI5 and the “three most outstanding funds”, ETF 28, ETF 80 and ETF 137, are defined in Table 1.

### 5 - Conclusion

An exchange-traded fund (ETF) is an investment fund traded on exchanges, much like stocks traded in a stock exchange. An ETF consists of assets such as stocks, commodities, or bonds, and trades close to its net asset value over the course of the trading day. While many ETFs track indices, such as stocks, bonds, real estate, and commodity indices, the proliferation of ETFs since the global financial crisis have seen complex ETFs being issued including inverse and leveraged ETFs. ETFs provide useful core as well as satellite investments because of their low costs, tax efficiency, and stock-like
liquidity features. ETFs are the most popular type of exchange-traded product. With its increasing complexity and non-normality returns distribution, additional investor skill is required to evaluate the quality of exchange-traded fund and how an exchange-traded fund fits into an investor’s portfolio. In our paper, we find that sometimes the traditional MV criterion and CAPM statistics are ambiguous in their evaluation of the exchange-traded funds. At other times, some MV and CAPM measures can identify the dominant funds. However, such MV and CAPM measures fail to inform of the dominance relationship nor the preferences of investors. This paper introduces a powerful SD test to present a more complete picture for ETF performance appraisal and to draw inference on the preference of investors on the funds. The SD approach that is basically free of assumption is used to investigate the characteristics of the entire distribution of returns and test whether rational investors benefit from any exchange-traded fund to maximize their expected utilities and/or expected wealth. An advantage of this approach is that it alleviates the problems that can arise if exchange-traded fund returns are non-normally distributed. Our approach also allows for a meaningful economic interpretation of the results. Based on a sample of the 137 exchange-traded funds from the Datastream database, we find the existence of second-order SD relationship among other funds/indices; indicating that the non-satiated and risk-averse investors would maximize their expected utilities, but not their expected wealth by switching from the SSD dominated exchange-traded funds to their corresponding SSD dominant ones. In addition, by applying the DD technique, we also discover that in most SD relationships, one fund dominates another fund in the negative domain while the reverse dominance relationship can be found in the positive domain. Besides, the normality assumption in the traditional measures, the difference may also come from the traditional measures definition of an abnormal return as an excess return adjusted to some risk measures, while the SD tests employ the whole distribution of returns. The SD measure is an alternative that is superior to the traditional measures to help investors and fund managers in managing their investment portfolios.

We conclude that the stochastic dominance approach is more appropriate compared with traditional approaches as a filter in exchange-traded fund selection. Compared with traditional approaches, the SD approach, not only is assumption free, but also provides greater insights to the performance and risk inherent in an exchange-traded fund’s track record. In this paper, we study the preference of risk averters on the ETFs. We note that one could apply the approach used in Qiao, et al. (2012) to extend the work to study the preference of risk seekers on the ETFs and employ the approach used in Fong, et al. (2008), Gasbarro, et al. (2012), and Clark, et al. (2015) to study the preference
of investors with S-shaped and revised S-shaped utility functions on the ETFs. One could also incorporate portfolios optimization and other mean-risk models like MVaR and Omega ratios with stochastic dominance analysis in the analysis (Hoang, et al., 2015). De Jesús and Ortiz (2011) apply Value-at-risk (VaR) and conditional VaR (CVaR) based on a generalized extreme value distribution to analyze the performance of different stock markets. All these approaches could be used to analyse the performance of ETFs. We leave this to the further study of our paper.

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