

Should Minimum Portfolio Sizes Be Prescribed for Achieving Sufficiently Well-Diversified Equity Portfolios?

Lawrence Kryzanowski^{*1}

Shishir Singh²

Abstract

This paper uses various (un)conditional metrics to measure the benefits of diversification to determine if a minimum portfolio size should be prescribed to achieve a naively but sufficiently well-diversified portfolio for various investment opportunity sets (un)differentiated by cross-listing status and market capitalization. Based on the population of stocks listed on the Toronto Stock Exchange (TSX) for 1975-2003, the study finds that the minimum portfolio size depends upon the chosen investment opportunity set, the metric(s) used to measure the benefits of diversification, and the criterion chosen to determine when the portfolio is sufficiently well diversified.

Keywords: diversification benefits, portfolio size, dispersion, Sharpe and Sortino ratios.

JEL Classification: G11, G23, C15, D81.

* Financial support provided to Kryzanowski from the Concordia University Research Chair in Finance, IFM2 and SSHRC are gratefully acknowledged. We would like to thank Denis Kira, Simon Lalancette and Fassil Nebebe, and participants at the presentations of earlier versions of this paper at Bank of Canada (2006) and the 2006 meeting of the MFS (Edinburgh) for their many helpful comments, and Anas Aboulamer and Kaveh Moradi Dezfouli for excellent research support.

¹ Concordia University Research Chair in Finance, Finance Department, John Molson School of Business, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec, Canada, H3G 1M8. Tel: (514) 8482424, x2782. E-mail: lawrence.kryzanowski@concordia.ca

² 672 Drake Centre, Asper School of Business, University of Manitoba. Tel: (204) 4749783; singh7@cc.umanitoba.ca

1 – Introduction

One of the most troubling implications of the basic CAPM is that the optimal portfolio of risky assets for all investors consists of the market portfolio. Given constant or increasing marginal costs as reported by Bloomfield *et al.* (1977) and decreasing marginal benefits associated with increasing portfolio size (henceforth, PS), Mao (1971), Levy (1978), Kryzanowski and To (1982) and Merton (1987) develop so-called clinical versions of the CAPM for friction-related, constrained investment opportunity sets. In a similar vein, Brennan (1975) shows that a perfectly diversified portfolio is not necessarily optimal for individual investors given transaction costs.

Various studies that are reviewed in the next section of this paper specifically examine the marginal benefits associated with increasing portfolio size. Based on the earlier studies, the average individual investor has traditionally been advised that portfolios of at least 20 and 30 stocks are needed to achieve much of the risk-reduction benefits from increasing portfolio size in the US and Canadian equity markets, respectively [Investopedia (2008); BMO (2001)]. Although no consensus exists on a new portfolio size, more recent research suggests that the “optimal” portfolio size is now larger due to the reduction in trade costs. However, the majority of individual investors only hold one to three stocks in their equity portfolios [Goetzmann and Kumar (2008); Kelly (1995); and Polkovnichenko (2005)]. The average investor holds two to four stocks depending upon the sample [Goetzmann and Kumar (2008); Statman (2004)], and the average holding appears to be increasing over time [from 4 to 7 stocks, according to Goetzmann and Kumar (2008)].

Thus, the major objective of this paper is to answer the following question: Should minimum portfolio sizes be prescribed for achieving sufficiently well-diversified equity portfolios? To this end, the paper examines the impact of increasing portfolio size for portfolios drawn from various investment opportunity (IO) sets using monthly data for Canadian stocks listed on the TSX over the period, 1975-2003 inclusive. The IO sets consist of some of the common mandates of investment managers, such as all firms, (non-)cross listed firms on US trade venues, big/small firms, and IT firms. For each portfolio size and IO set, 5000 portfolios are formed using a naïve diversification strategy of randomly selecting and equally weighting the selected stocks. We provide the rationale for not using sample-based mean-variance portfolio formation strategies in section three of this paper. The

metrics chosen to assess diversification benefits fit into four categories; namely, those that measure risk reduction, those that measure the impact on higher-order return moments, those that measure the impact on reward-to-risk, and those that examine the impact on the probabilities of underachieving various target or lower-bound rates of return. The specific rationales for the choice of these metrics and their implementation are described in section four of this paper.

We make several contributions to the literature. The first contribution is a negative answer to the question: Should minimum portfolio sizes be prescribed for achieving sufficiently well-diversified portfolios? We find that the minimum portfolio size (PS) depends upon the chosen investment opportunity (IO) set, the metric(s) used to measure the benefits of diversification, and the criterion chosen to determine when the portfolio is sufficiently well diversified. The minimums for a fixed investment opportunity set differ both within and across the categories of metrics used for measuring diversification benefits.

The second contribution is that we revisit the portfolio size issue for Canadian stocks for a much more recent period and using a number of recently developed measures of the relative benefits of diversification. We show that the use of these “new” measures introduces considerable ambiguity into the common prescriptions on what constitutes a minimum portfolio size for achieving a sufficiently well-diversified portfolio of domestic equities.

The remainder of the paper is organized as follows. A brief review of the relevant literature is presented in the next section. In section three, the sample and data are described. Section four focuses on test results for the various performance metrics that are used to determine the number of stocks needed to naively achieve a specific level of the potential benefits through diversification. Section five concludes the paper.

2 - Brief literature review

Markowitz (1952) provides a theoretical foundation for portfolio diversification by examining the trade-offs between the means and variances of returns for financial assets. Sharpe (1964), Lintner (1965) and Mossin (1966) use the effect of increasing portfolio size on the diversification of portfolios to derive their versions of the Capital Asset Pricing Model (CAPM). For example, Lintner (1965) capitalizes on the finding of Markowitz (1959) that the total risk of a portfolio is not only less than the weighted sum of the own risks of individual assets, but converges to

systematic or market or nonidiosyncratic risk as the portfolio size tends to infinity.

While researchers agree in principle that the firm-specific risk component of total portfolio risk can be reduced to just systematic risk through diversification, researchers report conflicting evidence on how many assets are needed to achieve a “well-diversified” portfolio. The most distant literature includes the often-cited article by Evans and Archer (1968) who observe that most of the economic benefits of diversification are captured with a PS of eight to ten securities. While Elton and Gruber (1977) find that a PS of eight stocks yields about eighty percent of the benefits from diversification, other studies report that a larger PS is required. According to Latane and Young (1969), a PS of eight only provides 25 percent of potential benefits from diversification. Jennings (1971) and Fama (1965) report PSs of 15 and at least 100 to nearly exhaust potential diversification benefits. Kryzanowski and Rahman (1986) find that a PS of 15 securities produces 95 and 98 percent of the benefits of diversification in terms of the mean and variance of time-varying (MINQUE-estimated) variances, respectively. According to Statman (1987), a PS of at least 30 and 40 stocks are required for borrowers and lenders, respectively, to achieve sufficient portfolio diversification. Wagner and Lau (1971) argue that an investor is better off as measured by the Sharpe ratio by holding a larger number of low quality stocks than a smaller number of high quality stocks.

According to Kryzanowski and Rahman (1986), the minimum PS for a time-varying measure of risk is 15 and 30 for the US and Canadian equity markets, respectively. Cleary and Copp (1999) find that PSs of 50 and 90 stocks achieve total risk reduction benefits of 90% and 95%, respectively, for a sample of Canadian stocks with complete return information for the time period being examined. Due to the introduction of this “look-ahead” bias into their results, their findings may not be useful for making ex ante decisions on the required portfolio size to achieve what the investor considers to be a sufficiently well-diversified portfolio.

More recent studies find that the benefits of increasing PS are more complex than previously thought and provide weaker improvements in diversification benefits [e.g., Surz and Price (2000); Bennett and Sias (2006)]. Due to increases in idiosyncratic volatility, the optimal PS is now 50 stocks [Malkiel and Xu (2006)] or exceeds 300 stocks [Statman (2004)].

The same level of portfolio diversification also is more difficult to achieve in a bearish market as average conditional correlations [Silvapulle and Granger (2001)] and firm-level return dispersions [Demier and Lien (2004)] are higher when market returns are largely negative. Nieuwerburgh

and Veldkamp (2008) attempt to explain the diversification puzzle by arguing that investors hold more specialized and less diversified portfolios because they acquire informational scale economies about a set of highly correlated assets. Bennett and Sias (2006) argue that idiosyncratic risk may be priced because increasing PS results in a decline of the impact of systematic shocks on the time-series variance of a portfolio but not on the cross-sectional portfolio variances. As a result, they argue that the cross-sectional standard deviation of returns is a more intuitive measure of the benefits of diversification.

3 - Samples and data

The sample consists of all TSX-listed firms. The primary source of data is the CFMRC for TSX monthly returns over the period 1975-2003. Due to their similarity with the results presented herein, we do not present the results using US trade data drawn from CRSP for TSX-listed firms that are cross-listed in the US. The risk-free rate is proxied by the 30-day Canadian T-Bill rate, which is obtained from the Bank of Canada.

Portfolios are formed for IOs consisting of all firms, TSX-only listed firms, TSX firms cross-listed in US markets, big/small firms (with firm capitalizations above and below the median, respectively, each year), and IT firms. Our choice of subsamples is based on the conjecture that the preference of investors for certain types of stocks may require different portfolio sizes to avoid material under-diversification. Our choice of the sub-samples of TSX-only listed and TSX cross-listed stocks in the US is motivated by the conjecture that investors might prefer to invest in stocks that they are familiar with (such as local stocks), which has been identified by Grinblatt and Keloharju (2001), among others. This subdivision of our sample also is motivated by the finding of King and Siegel (2003) that cross-listing mitigates some of the value discount associated with TSX-only listed firms relative to their U.S.-listed peers. Our choice of the other three sub-samples is motivated by investors who adopt a style bias based on market capitalization (e.g., small- or large-cap stocks) or are attracted to certain industries (e.g., technology stocks). This is also consistent with the types of products that are available to both retail and institutional investors.

Using a Monte Carlo approach, 5000 equal-weighted portfolios are formed for portfolio sizes (PSs) of 2, 5 through 100 in increments of 5 and all stocks for each of the six IO sets. Checks using 25000 portfolios of some randomly chosen PS/IO combinations suggest that the results reported herein

are robust. Equal weights are used given the findings reported in the literature that no sample-based mean-variance portfolio formation strategy is consistently better in terms of out-of-sample performance than using equal weights. To illustrate, DeMiguel *et al.* (2008) find that none of the 14 models that they evaluate across seven empirical datasets is consistently better than the 1/N rule in terms of Sharpe ratio, certainty-equivalent return, or turnover. They conclude that this indicates that the out-of-sample gain from optimal diversification is more than offset by estimation error.

Care should be exercised when comparing portfolio sizes across the six IOs since the IO sets differ in the numbers of stocks included in each IO set population (henceforth ISOP). For example, based on the weighted average number of stocks in each IO set over the studied time period, a portfolio size of 100 represents 13.3% of the IOSP for all firms, 15.9% of the IOSP for TSX-only listed firms, 20.7% of the IOSP for big firms, 37.3% of the IOSP for small firms, 61.0% of the IOSP for cross-listed firms and 91.7% of the IOSP for IT firms. While we follow conventional academic and practitioner practice of defining portfolio sizes in terms of the number of stocks, we also provide the percentage that a specific portfolio size represents of the IOSP to facilitate comparisons across IO sets.

4 - Diversification benefits measured using various metrics

Various metrics for measuring portfolio diversification are examined in this section of the paper. The primary purpose for doing so is to identify the minimum number of stocks (so-called portfolio size or PS) needed, for example, on average to naively diversify a specific percentage of nonsystematic (or idiosyncratic or firm-specific) risk or to capture a specific percentage on average of the reward from bearing risk.

4.1 Correlations of stock returns

Correlations among stocks and between the various investment opportunity (IO) sets are now examined. The average correlation coefficient is used as a standardized measure of the rate and the level of maximum naive risk reduction [De Wit (1998)]. While the average covariance represents the minimum risk level of a portfolio, it is “unbounded” and needs to be standardized. For the US markets during 1962-1997, Campbell *et al.* (2001) find stable market volatility, increasing firm-level volatility, and declining average correlations among individual stock returns. This affects the PS needed to diversify a specified percentage of firm-specific risk. As in

Campbell *et al.* (2001), the monthly correlations are calculated herein using the previous 60 months of returns (i.e., moving window ending with month τ).

Summary statistics for the time-series of conditional equal- and market-weighted mean cross-sectional correlations of returns for the six IO sets of TSX-listed stocks are reported in panels A and B of table 1. To obtain these values, the equal- or market-weighted mean is first calculated for all the correlations between every unique pair of stocks in IO set j for the 60-month moving window ending with month τ that have returns for at least 75% of the months therein.³ Repeating the previous step for each moving window τ for the IO set j generates a time-series of mean conditional cross-sectional correlations for the IO set j . The mean and median of these time-series are highest for the cross-listed IO set for both types of means and lowest for the small firm IO set for the equal-weighted means and for the TSX-only listed IO set for the value-weighted means. The standard deviation is highest for the cross-listed IO set and lowest for the IT IO set. Since the mean, median, minimum and maximum are always lower for a specific IO set when market-weighted, the correlations are, on average, lower for smaller versus larger firms in each IO set. Together, these results suggest that whatever diversification is possible is achievable quicker for small firms and less quickly for cross-listed firms, provided that all else is held equal. Of course, as is subsequently shown, this lower average correlation for small firms is more than offset by greater dispersion in their returns.

Since the time-series plots for the equal- and market-weighted mean correlations for each IO set are similar except for the higher values of the market-weighted mean correlations, only the equal-weighted mean correlations are depicted in figure 1. After rising over the 1980-1983 period, the conditional mean cross-sectional monthly correlations trend downwards with two exceptions. The first is the marked increase in all six IO sets with the October'87 market crash, and the other is the marked increase for the IT set beginning from February 2000.

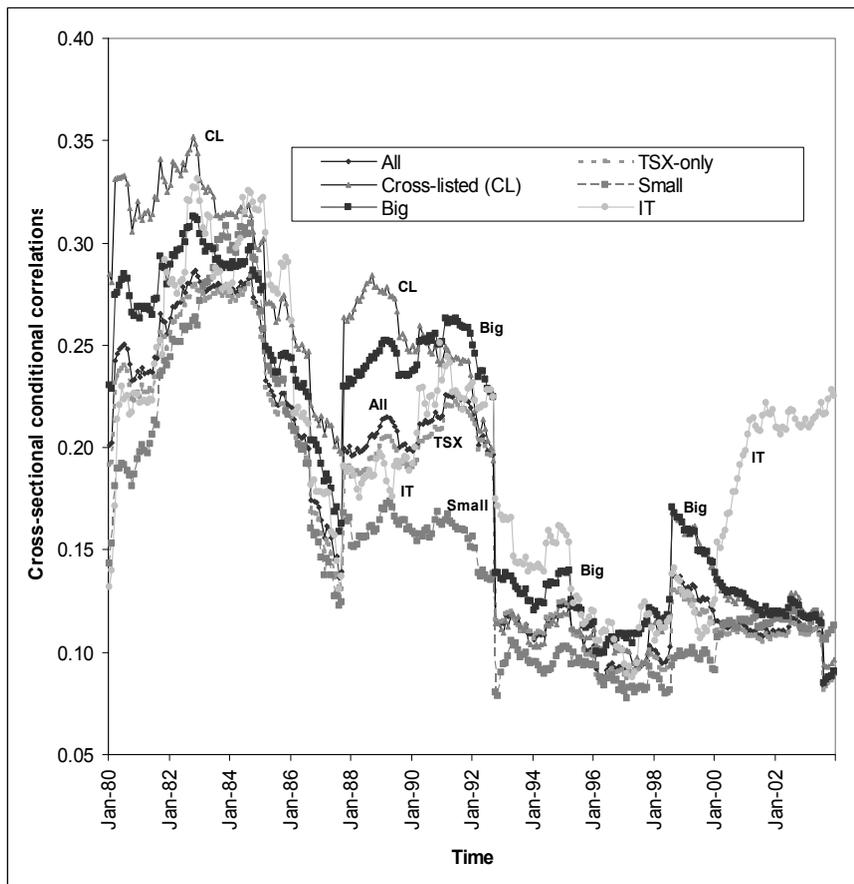
³ The market-weighted (cap) mean correlation is calculated as:

$$\bar{\rho}_{j,\tau}^{cap} = \left(\sum_{i < k} \rho_{i,k,j,\tau} (MV_{i,j,\tau-1} + MV_{k,j,\tau-1}) \right) / \sum_{i < k} (MV_{i,j,\tau-1} + MV_{k,j,\tau-1}), \text{ where } MV_{i,j,\tau-1} \text{ and}$$

$MV_{k,j,\tau-1}$ are the market values of stocks i and k , respectively, in IO set j at the end of month $\tau - 1$; and $\rho_{i,k,j,\tau}$ is the correlation of stocks i and k in IO set j over the 60-month moving window ending at time τ . When calculating the MV , the mid-spread is used in the absence of a closing price in CFMRC.

The correlations between the time-series of conditional equal- and market-weighted mean cross-sectional correlations of returns for the six IO sets of TSX-listed stocks are reported in panels C and D of table 1. All of the time-series are positively and highly correlated in that they range from a high of 0.999 between the TSX-only listed and all IO sets to a low of 0.740 for the IT and cross-listed IO sets when equally weighted, and from a high of 0.954 between small and all IO sets to a low of 0.551 for IT and big IO sets when market weighted. This suggests the existence of one or more factors that determine the time-series variations of the mean correlations of the various IO sets.

Figure 1
Time-series of mean cross-sectional conditional correlations for six IO sets, 1980-2003



Note: This figure depicts the time-series of equal-weighted mean cross-sectional conditional correlations of returns for six investment opportunity (IO) sets of TSX-listed stocks for monthly returns. Each time-series of equal-weighted mean cross-sectional conditional correlations is generated by calculating the mean of the correlations between every unique pair of stocks in IO set j for each 60-month moving window ending with month τ , which consists of the past 60 months and begins from the first month of 1980 through the last month of 2003.

Table 1
Summary statistics for the time-series of conditional mean cross-sectional correlations differentiated by investment opportunity set

Panel A: Summary statistics for the time-series of conditional equal-weighted mean cross-sectional correlations of returns					
IO Set <i>j</i> Firms	Mean	Median	Std Dev	Minimum	Maximum
All	0.172	0.168	0.065	0.083	0.286
TSX-only Listed	0.168	0.161	0.063	0.082	0.284
Cross-listed	0.203	0.208	0.086	0.093	0.352
Big	0.194	0.195	0.070	0.085	0.313
Small	0.154	0.138	0.065	0.078	0.309
IT	0.198	0.199	0.062	0.088	0.331
Panel B: Summary statistics for the time-series of conditional market-weighted mean cross-sectional correlations of returns					
IO Set <i>j</i> Firms	Mean	Median	Std Dev	Minimum	Maximum
All	0.321	0.309	0.116	0.138	0.584
TSX-only Listed	0.290	0.260	0.108	0.093	0.538
Cross-listed	0.442	0.473	0.187	0.174	0.765
Big	0.409	0.412	0.124	0.158	0.593
Small	0.295	0.264	0.118	0.135	0.551
IT	0.439	0.457	0.100	0.174	0.635
Panel C: Correlations between the time-series of conditional equal-weighted mean cross-sectional correlations of returns					
IO Set <i>j</i> Firms	All	TSX-only	Cross-listed	Big	Small
TSX-only Listed	0.999				
Cross-listed	0.977	0.967			
Big	0.992	0.988	0.980		
Small	0.930	0.937	0.890	0.882	
IT	0.810	0.823	0.740	0.758	0.880
Panel D: Correlations between the time-series of conditional market-weighted mean cross-sectional correlations of returns					
IO Set <i>j</i> Firms	All	TSX-only	Cross-listed	Big	Small
TSX-only Listed	0.922				
Cross-listed	0.871	0.833			
Big	0.761	0.718	0.943		
Small	0.954	0.922	0.911	0.793	
IT	0.740	0.700	0.622	0.551	0.714

Table 1 Continued

This table reports various summary statistics and the correlations between the time-series of mean cross-sectional conditional correlations of returns for six investment opportunity (IO) sets of TSX-listed stocks for monthly returns. A time-series of conditional mean cross-sectional correlations for IO set j is generated by calculating the equal- or market-weighted mean of the correlations between every unique pair of stocks in IO set j for each moving window τ . The correlations between the time-series of conditional equal- or market-weighted mean cross-sectional correlations for all the unique combinations of IO set j and j' are then computed and reported herein. Monthly correlations are computed based on the past 60 months, which corresponds to a moving window of 5 years. The 288 moving windows for the monthly returns are from the first month of 1980 through the last month of 2003. “IT” refers to information technology firms.

4.2 Dispersion of stock return metrics

For expositional ease, we use the term “dispersion” herein for both cross-sectional and time-series measures of variability and semi-variability. Dispersion metrics are superior to the correlation metric as a measure of diversification benefits since the dispersion metrics do not depend upon correlations being persistent [Solnik and Roulet (2000)], and they incorporate the effects of both correlations and standard deviations [Statman and Scheid (2007)]. Lower levels of stock return dispersion are typically associated with lower levels of risk and higher correlations [Solnik and Roulet (2000)].

The first “dispersion” metric examined in this section of the paper is referred to as the mean derived dispersion (MDD) or excess standard deviation [Campbell *et al.* (2001)].⁴ The MDD for a fixed portfolio size s and IO set j is given by:

$$MDD_{j,s} = \bar{\sigma}_{j,s} - \sigma_j, \quad (1)$$

where $\bar{\sigma}_{j,s}$ is the mean of the standard deviations for the 5000 randomly selected portfolios with a portfolio size or PS of s for IO set j over the whole time period, and σ_j is the average standard deviation of all the stocks in IO set j .

⁴ Kryzanowski and Rahman (1986) use a conditional variant of the MDD using conditional variances instead of standard deviations where the time-varying variances are estimated using the Minque (Minimum Norm Quadratic Estimator) of Rao (1970).

The mean derived dispersion should be positive and converge monotonically to zero with increasing PS for each investment opportunity (IO) set. The MDD are computed herein and depicted in figure 2 for 22 portfolio sizes (PS) for each of six IO sets using monthly returns for the period, 1975-2003. The MDDs decrease monotonically from a positive value for a PS of 2 to a value of zero for a PS of all for each IO set. The MDDs for PSs of 2 and all also are significantly different at conventional levels for each IO set. If the decision criterion is to achieve at least 90 percent of the potential decline in the MDD by moving from a PS of 2 to all, then the required PS is about 40 (IOSP of 15.1%) for cross-listed firms, 45 (ISOP of 9.3%) for big firms, 95 (ISOP of 15.1%) for TSX-only listed firms and over 100 for the remaining three IO sets. Thus, although the minimum number of stocks is lower for cross-listed firms than for big firms (40 versus 45), it represents a larger percentage of the stocks available for investment in that IO set (15.1% versus 9.3%). If the threshold is increased to 95 percent, then the required PS is about 75 for cross-listed firms, 100 for big firms and over 100 for the remaining four IO sets. Nevertheless, the MDDs for the small-firm IOs are substantially higher for PSs through 100. This is consistent with the finding reported by Bennet and Sias (2006) that smaller firms have relatively higher risk in the US. In addition, about 90 percent of the reduction in the standard deviation of the MDD measures across the 5000 portfolios for each PS and IO set occurs with a PS of between 25 and 40 stocks, where the lower value occurs for big firms and the higher value for all firms.

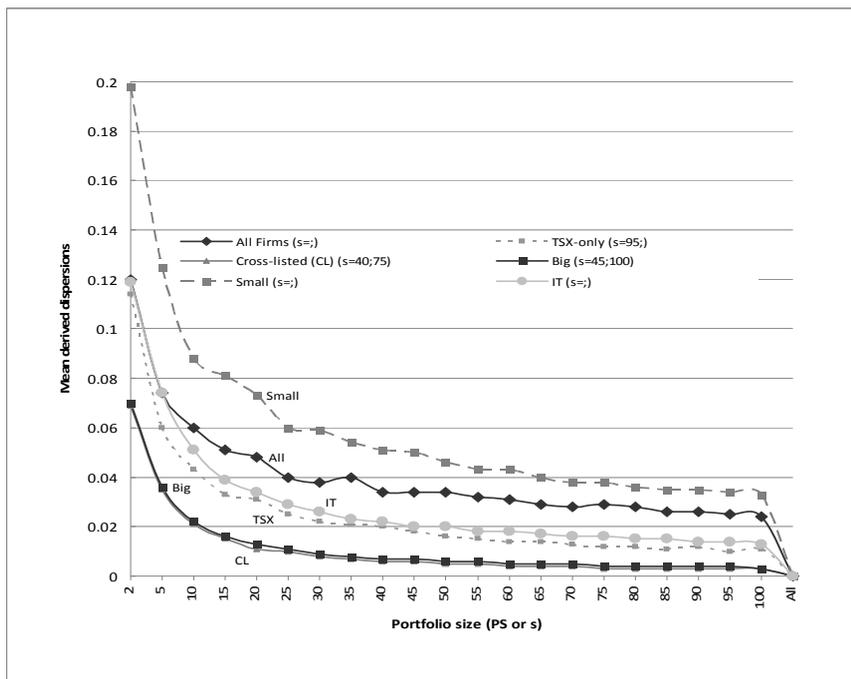
The second dispersion metric examined in this section is referred to as the mean realized dispersion or MRD [de Silva *et al.* (2000); Ankrim and Ding (2002)]. The MRD for a fixed portfolio size s and IO set j is given by:

$$MRD_{j,s} = \frac{1}{N} \sum_{\tau=1}^N \sigma_{j,s,\tau} \quad (2)$$

where $\sigma_{j,s,\tau}$ is the cross-sectional standard deviation for the 5000 randomly selected portfolios for IO set j with a portfolio size of s for month τ ; and N is the number of cross-sections. To obtain the MRD for a portfolio of all stocks, we bootstrap using 5000 samples of $N-1$ firms.

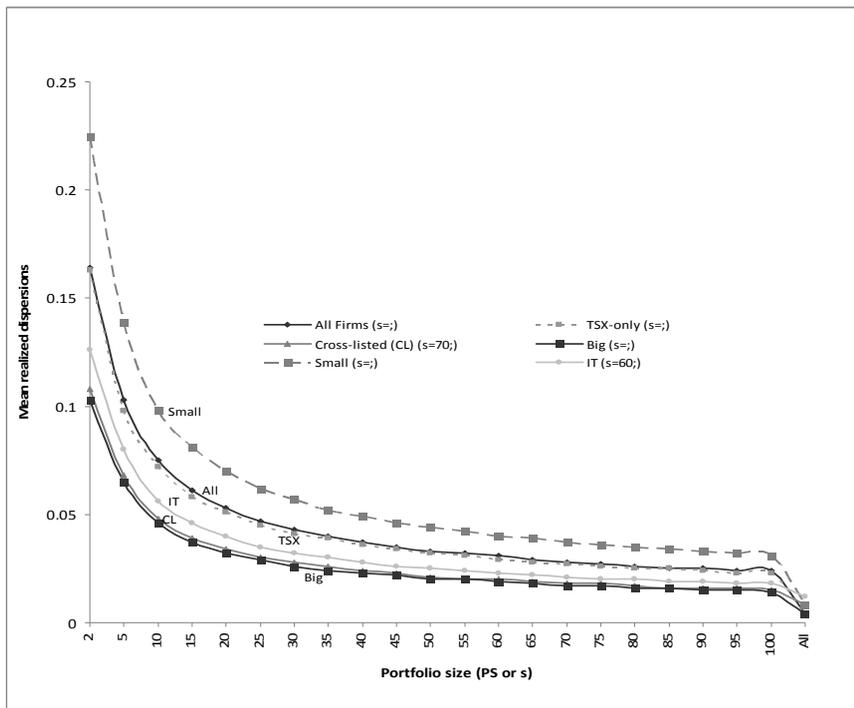
The time-series of conditional MRD are computed herein for 22 portfolio sizes (PS) for the six IO sets and are depicted in figure 3. According to expectations, the MRDs decrease monotonically with increasing PS for each of the IO sets. The MRD metric emits different inferences than those reported above for the MDD metric. For example, the required PS to achieve at least 90 percent of the potential improvement in the MRD by moving from

Figure 2
Mean derived dispersions differentiated by portfolio size and investment opportunity set



Note: This figure plots the mean derived dispersions (MDDs) or excess standard deviations for 22 portfolio sizes (PSs) for six investment opportunity (IO) sets using returns for the 348 months in the period, 1975-2003. The mean excess standard deviation for a fixed portfolio size s and IO set j is given by $MDD_{j,s} = \bar{\sigma}_{j,s} - \sigma_j$, where $\bar{\sigma}_{j,s}$ is the mean of the standard deviations for the 5000 randomly selected portfolios with a portfolio size of s for IO set j , and σ_j is the standard deviation of the equal-weighted portfolio of all the stocks in IO set j . In the legend, (s=x,y) refers to the lowest portfolio sizes of x and y that capture at least 90% and 95%, respectively, of the reduction in MDD from moving from a s of 2 to a s of “All”. A blank for either x or y indicates that $s > 100$. The differences in the means for a s of 2 and “All” are significantly different at the 0.05 level for all IO sets.

Figure 3
Mean realized dispersions differentiated by portfolio size and investment opportunity set



Note: This figure plots the means of the time-series of conditional cross-sectional mean realized dispersions (MRDs) or standard deviations of returns for 22 portfolio sizes (PSs) for six investment opportunity (IO) sets using returns for the 348 months in the period, 1975-2003. The MRD for a fixed portfolio size s and IO set j is given

by $MRD_{j,s} = \frac{1}{N} \sum_{\tau=1}^N \sigma_{j,s,\tau}$, where $\sigma_{j,s,\tau}$ is the cross-sectional standard deviation across

the 5000 randomly selected portfolios for IO set j with a portfolio size of s for month τ ; and N is the number of cross-sections. In the legend, $(s=x,y)$ refers to the lowest portfolio sizes of x and y that capture at least 90% and 95%, respectively, of the reduction in MRD from moving from a s of 2 to a s of “All”. A blank for either x or y indicates that $s > 100$. The differences in the means for a s of 2 and “All” are significantly different at the 0.05 level for all IO sets. “All” is proxied by the MRD for a PS of $N-1$ for all the IO sets.

a PS of 2 to all is under 100 for only two IO sets. The two PSs below 100 are 70 (IOSP of 42.7%) for cross-listed firms and 60 (IOSP of 55.0%) for IT firms. The decrease in the cross-sectional standard deviation of returns of individual firms with increasing portfolio size may help to explain the findings of MacDonald and Shawky (1995) that using this measure instead of a time-series measure reduces market volatility by about 50%, and results in a significantly positive relation between risk and return. This time-varying measure should be superior in capturing the impact of a changing investment opportunity set on conditional risk.

However, the MRD inferences implicitly assume that the cross-sectional distribution of portfolio standard deviations across the 5000 randomly selected portfolios for each unique combination of PS and IO set is relatively normal. If the distributions are highly positively skewed with high positive kurtosis at low PSs and these higher-order moments become more like those of a normal distribution as the PS increases (as is subsequently shown), then the required PS for an investor who dislikes both positive variance and kurtosis and likes positive skewness is likely to be lower than the over 100 stocks based on a consideration of the MRD metric in isolation. The MRD also is consistently higher for the small-firm IO set relative to the other IO sets for all PSs.

The third “dispersion” metric examined in this section is referred to as the normalized portfolio variance (NV) in the literature [e.g., Goetzmann and Kumar (2008)]. This measure is based on the law of average covariances by Markowitz (1976), which states that the variance of a portfolio approaches the average covariance among the stock returns as the PS increases. This measure is developed from the following simplified expression for the variance of a portfolio drawn from IO set j for a PS of s :

$$\sigma_{j,s}^2 = \sum_{i=1}^s w_i^2 \sigma_i^2 + \sum_{j=1}^s \sum_{i=1}^s w_i w_k \text{cov}(r_i, r_k) \quad \text{for } k \neq i \quad (3)$$

Since the weights for stocks i and k in an equal-weighted portfolio of s stocks are $w_i = w_k = 1/s$, the variance of the portfolio can be re-written as:

$$\sigma_{j,s}^2 = \frac{1}{s} \sum_{i=1}^s \frac{1}{s} \sigma_i^2 + \sum_{k=1}^s \sum_{i=1}^s \frac{1}{s^2} \text{Cov}(r_i, r_k) \quad \text{for } k \neq i \quad (4)$$

Defining average variance and covariance as $\bar{\sigma}_{j,s}^2 = \frac{1}{s} \sum_{i=1}^s \sigma_i^2$ and

$\overline{cov}_{j,s} = \frac{1}{s(s-1)} \sum_{k=1}^s \sum_{i=1}^s cov(r_i, r_k)$, respectively, and substituting these expressions into equation (4) yields:

$$\sigma_{j,s}^2 = \left(\bar{\sigma}_{j,s}^2 / s \right) + (s-1) \overline{Cov}_{j,s} \quad (5)$$

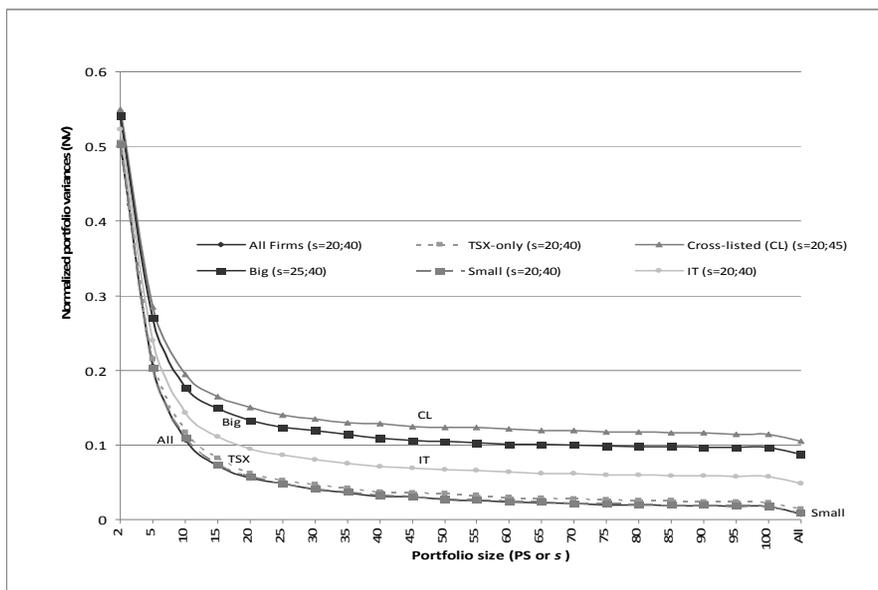
Normalizing both sides of the above expression by the average variance yields the following normalized variance for the i -th randomly selected and equal-weighted portfolio of size s for IO set j :

$$NV_{j,s,i} = \sigma_{j,s,i}^2 / \bar{\sigma}_j^2 = (1/s) + [(s-1)/s] \left(\overline{cov}_{j,s,i} / \bar{\sigma}_j^2 \right) + [(s-1)/s] \overline{corr}_{j,s,i} \quad (6)$$

where $\sigma_{j,s,i}^2$ is the variance of returns for the i -th randomly selected portfolio of size s for IO set j over the full period; $\bar{\sigma}_j^2$ is the average cross-sectional variance of returns for all the stocks in IO set j over the full period; $\overline{cov}_{j,s,i}$ is the average cross-sectional covariance of returns for the i -th randomly selected portfolio of size s for IO set j over the full period; $\overline{corr}_{j,s,i}$ is the average cross-sectional correlation of returns for the i -th randomly selected portfolio of size s for IO set j over the full period; and $i = 1, \dots, 5000$.

Cross-sectional mean NV values for 22 portfolio sizes for six IO sets using monthly returns over the period, 1975-2003, are depicted in figure 4. The means of the NV display a gradual asymptotic decline as the PS increases and the mean NVs are significantly different for portfolio sizes of 2 and all. If the decision criterion is to achieve at least 90 percent of the potential reduction of moving from a PS of 2 to all, then the required PSs are about 20 to 25 stocks for the six IO sets. This represents an ISOP that ranges from a low of 2.7% for all firms to a high of 18.3% for IT firms. If the threshold is increased to 95 percent, then the required PSs are higher at about 40 for all IO sets. Furthermore, 90 and 95 percent of the convergence towards the steady-state standard deviation of the NVs is obtained with approximately 20 and 30 stocks, respectively, for the six IO sets. These findings differ from those presented above for the MDD metric but are similar to those reported for an earlier period for Canadian markets by Kryzanowski and Rahman (1986).

Figure 4
Normalized portfolio variances (NVs) differentiated by portfolio size and investment opportunity set

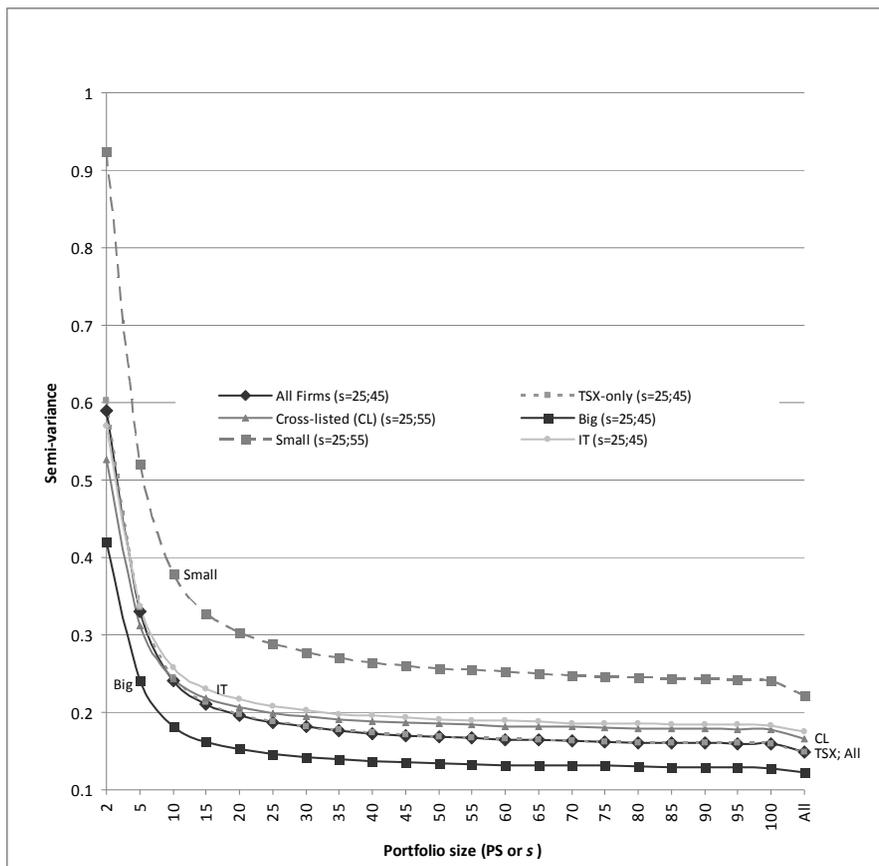


Note: This figure depicts the mean values of the normalized portfolio variance (NV) metric for 22 portfolio sizes for 6 investment opportunity (IO) sets using returns for the 348 months in the period, 1975-2003. The mean NV for investment opportunity (IO) set j and portfolio size s is given by:

$$\mu_{NPV_{j,s}} = (1/5000) \sum_{i=1}^{5000} \sigma_{j,s,i}^2 / \bar{\sigma}_j^2 = (1/5000) \sum_{i=1}^{5000} (1/s) + [(s-1)/s] (\overline{\text{cov}}_{j,s,i} / \bar{\sigma}_j^2),$$

where $\sigma_{j,s,i}^2$ is the standard deviation of returns for the i -th randomly selected portfolio of size s for IO set j over the full period; $\bar{\sigma}_j^2$ is the average cross-sectional standard deviation of returns for all the stocks in IO set j over the full period; and $\overline{\text{cov}}_{j,s,i}$ is the average cross-sectional covariance of returns for the i -th randomly selected portfolio of size s for IO set j over the full period. In the legend, (s=x,y) refers to the lowest portfolio sizes of x and y that capture at least 90% and 95%, respectively, of the reduction in NV from moving from a s of 2 to a s of "All". A blank for either x or y indicates that $s > 100$. The differences in the means for a s of 2 and "All" are significantly different at the 0.05 level for all IO sets.

Figure 5
Semi-variance measured relative to the risk-free rate multiplied by 100
differentiated by portfolio size and investment opportunity set



This figure depicts the semi-variances measured relative to the risk-free rate based on 5000 random drawings for each unique combination of the 22 PSs and six IO sets using monthly returns over the period, 1975-2003. In the legend, (s=x,y) refers to the lowest portfolio sizes of x and y that capture at least 90% and 95%, respectively, of the reduction in semi-variance from moving from a s of 2 to a s of “All”. A blank for either x or y indicates that s > 100. All of the means for a s of 2 and “All” are significantly different at the 0.05 level.

The last measure of stock diversification benefits examined in this section is the semi-variance. Cross-sectional mean values of the semi-variance measured in reference to the risk-free rate and the market return for 22 portfolio sizes for each of the 6 IO sets using monthly returns over the period, 1975-2003, are calculated, and are depicted in figure 5 for those in reference to the risk-free rate to conserve valuable journal space. As for the NVs, both measures of semi-variance decline asymptotically as the PS increases and all their means are significantly different for portfolio sizes of 2 and all. For a risk-reduction threshold of 90% percent, the required PS is marginally higher than for the NVs at approximately 25 for all IO sets, which represents an ISOP range of 3.3% (big firms) to 22.9% (IT firms). For a risk-reduction threshold of 95%, the required PSs are also higher than for the NVs and range between about 45 and 55 stocks. Thus, the required PS is higher for all IO sets when risk is measured by semi-variance instead of variance. Also, as shown earlier, while adequate diversification may require the same number of stocks, for example, for the all firm and IT IO sets, it requires a much greater proportion of the stocks in the IT versus the all-firm IO set.

4.3 Higher-order moments of stock return metrics

Since the higher-order moments of stock returns are priced,⁵ this section examines the impact of increasing portfolio size on the third (skewness) and fourth (kurtosis) moments of stock returns. The time-series mean of the cross-sectional skewness and kurtosis are given by:

$$\mu_{Skew_{j,s}} = \frac{1}{N} \sum_{\tau=1}^N Skew_{j,s,\tau} \quad \text{and} \quad \mu_{Kurt_{j,s}} = \frac{1}{N} \sum_{\tau=1}^N Kurt_{j,s,\tau} \quad (7)$$

where $Skew_{j,s,\tau}$ and $Kurt_{j,s,\tau}$ are respectively the cross-sectional skewness and kurtosis for the 5000 randomly selected portfolios for IO set j with a portfolio size of s for month τ ; and N is the number of cross-sections.

The time-series means of the conditional skewness estimates for the cross-sectional distributions of portfolio returns are computed herein for 22 portfolio sizes (PSs) for each of the six IO sets and are depicted in figure 6. Our expectation is that the skewness of the cross-sectional distribution of

⁵ Kraus and Litzenberger (1976) and Harvey and Siddique (1999) find that unconditional and conditional skewness in returns, respectively, are priced. Fang and Lai (1997) find that the expected excess rate of return is related not only to the systematic variance but also to the systematic skewness and systematic kurtosis with a positive, negative and positive premium, respectively, for each.

returns across the 5000 randomly generated portfolios for each unique combination of PS and IO set should, on average, be highly right skewed at a PS of 2, and should decrease with increasing portfolio size. As expected, the mean cross-sectional skewness is highly positive at a PS of 2 and decreases monotonically as the PS increases from 2 to all stocks for all IO sets. Furthermore, between 35 to 40% of the cross-sectional right skewness is foregone when the PS increases from 2 to 5. Since skewness is positively priced by investors, this suggests that the required PS to not lose over 10% of the benefits of right cross-sectional skewness is about 2 stocks (i.e., a highly concentrated portfolio). This is consistent with the lower PSs of portfolio managers (such as hedge funds) who aggressively attempt to achieve high returns.

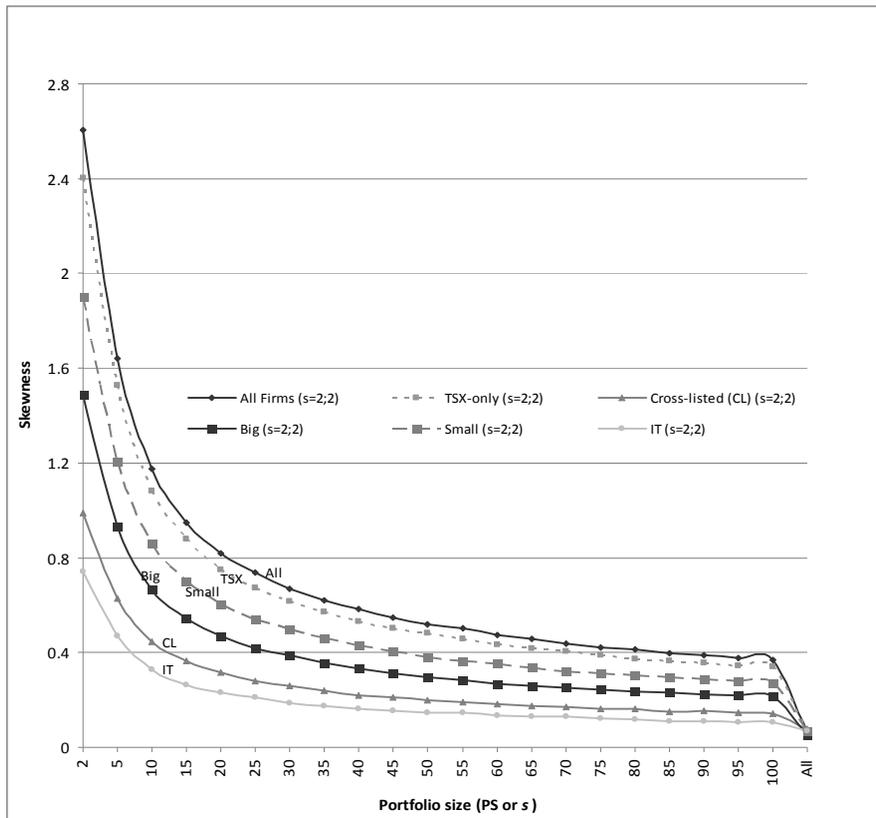
The time-series means of the conditional kurtosis of the cross-sectional distributions of portfolio returns are computed herein for 22 PSs for each of the six IO sets and are depicted in figure 7. Our expectation is that the kurtosis of the cross-sectional distribution of portfolio returns for the 5000 randomly generated portfolios for each unique combination of PS and IO set should, on average, be highly peaked at a PS of two, and that the excess kurtosis should decline with increasing portfolio size.⁶ As expected, the time-series means of the cross-sectional kurtosis begin with their largest positive values at a PS of two for the six IO sets and decrease monotonically in value towards 3 but still remain positive for a PS of all for all six IO sets. This suggests that the cross-sectional distributions of portfolio returns are quite peaked for low PSs, and become increasing less peaked (i.e., flatter) as the PS increases and have almost the same peakedness as a normal distribution when all (N-1) stocks in each IO set are considered. Since kurtosis is negatively priced by investors, this suggests that the required PS to achieve at least 90 percent of the potential reduction in the mean cross-sectional kurtosis by moving from a PS of 2 to a PS of all is, on average, about 20 to 25 for all six IO sets.

4.4 Composite return and risk metrics

This section examines metrics that capture the various types of tradeoffs between the first (mean) and second (standard deviation or semi-standard deviation) moments of return distributions.

⁶ Excess kurtosis is a useful measure obtained by subtracting 3 (i.e., the kurtosis of a normal distribution) from the kurtosis measure. Common interpretations are that positive excess kurtosis indicates a "peaked" distribution and negative excess kurtosis indicates a "flat" distribution. However, kurtosis not only measures the peakedness of a distribution but it also measures the tail heaviness of a distribution relative to that of the normal distribution [Ruppert (1987)] and is very sensitive to outliers in the data [Bryson *et al.* (2006)].

Figure 6
Skewness differentiated by portfolio size and investment opportunity set

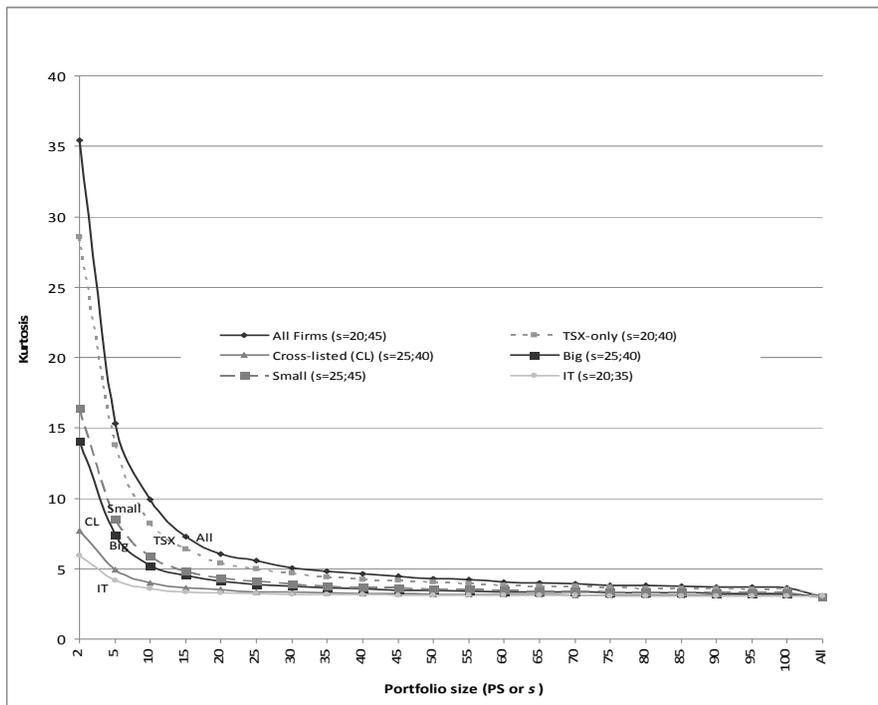


This figure depicts the time-series means of the cross-sectional skewness for each unique combination of the 22 PSs and six IO sets using monthly returns over the period, 1975-2003. The time-series mean of the cross-sectional skewness is given by

$$\mu_{Skew_{j,s}} = \frac{1}{N} \sum_{\tau=1}^N Skew_{j,s,\tau}, \text{ where } Skew_{j,s,\tau} \text{ is the cross-sectional skewness for the 5000}$$

randomly selected portfolios for IO set j with a portfolio size of s for month τ ; and N is the number of cross-sections. In the legend, $(s=x,y)$ refers to the lowest portfolio sizes of x and y that avoid foregoing respectively more than 10% and 5% of the skewness from moving from a s of 2 to a s of "All". A blank for either x or y indicates that $s > 100$. All of the means for a s of 2 and "All" are significantly different at the 0.05 level. "All" is proxied by the skewness for a PS of $N-1$ for all the IO sets.

Figure 7
Kurtosis differentiated by portfolio size and investment opportunity set



This figure depicts the time-series means of the cross-sectional kurtosis for each unique combination of the 22 PSs and six IO sets using monthly returns over the period, 1975-2003. The time-series mean of the cross-sectional kurtosis is given by

$$\mu_{Kurt_{j,s}} = \frac{1}{N} \sum_{\tau=1}^N Kurt_{j,s,\tau}$$

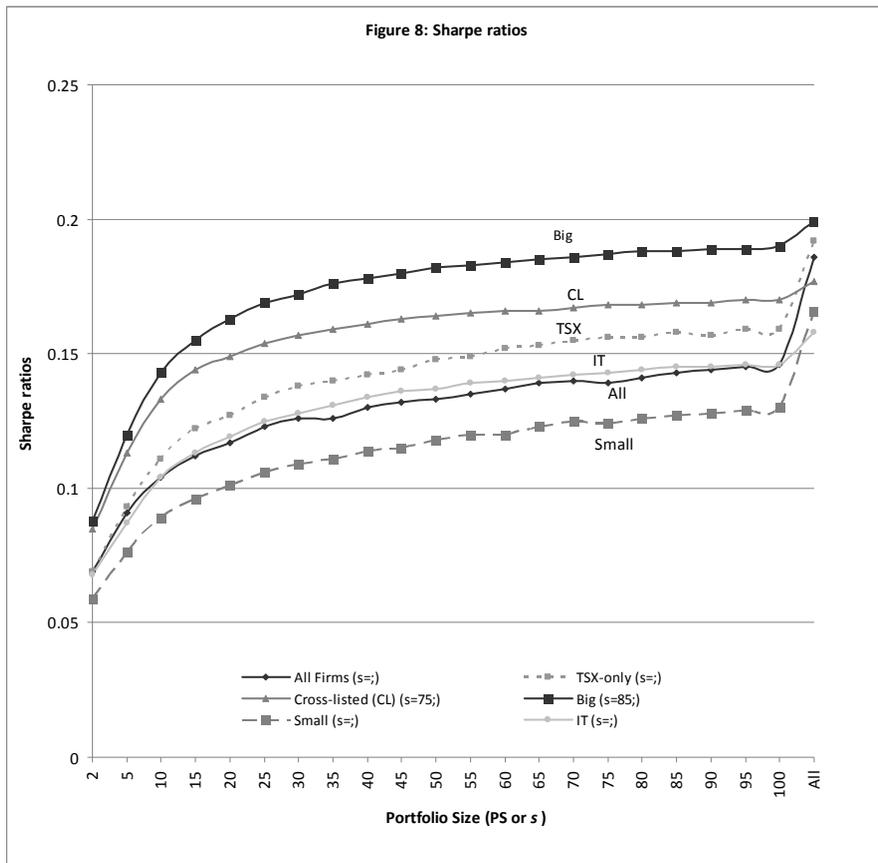
where $Kurt_{j,s,\tau}$ is the cross-sectional kurtosis for the 5000

randomly selected portfolios for IO set j with a portfolio size of s for month τ ; and N is the number of cross-sections. In the legend, $(s=x,y)$ refers to the lowest portfolio sizes of x and y that capture respectively 90% and 95% of the reduction in positive kurtosis from moving from a s of 2 to a s of “All”. A blank for either x or y indicates that $s > 100$. All of the means for a s of 2 and “All” are significantly different at the 0.05 level. “All” is proxied by the kurtosis for a PS of $N-1$ for all the IO sets.

The first and second measures of diversification benefits examined in this section are the Sharpe [Sharpe (1994)] and the Sortino [Sortino and Price (1994)] ratios. The Sharpe ratio is an excess return to total risk measure given by $Sh_{j,s} = (\bar{r}_{j,s} - r_f) / \sigma_{j,s}$, and the Sortino ratio is an excess return to semi-standard deviation risk measure given by $Sor_{j,s} = (\bar{r}_{j,s} - r_f) / \sigma_{j,s}^-$. Thus, while the Sharpe ratio accounts for any volatility in the returns of an asset, the Sortino ratio only accounts for deviations below the mean since deviations above the mean are not considered to be a component of risk.

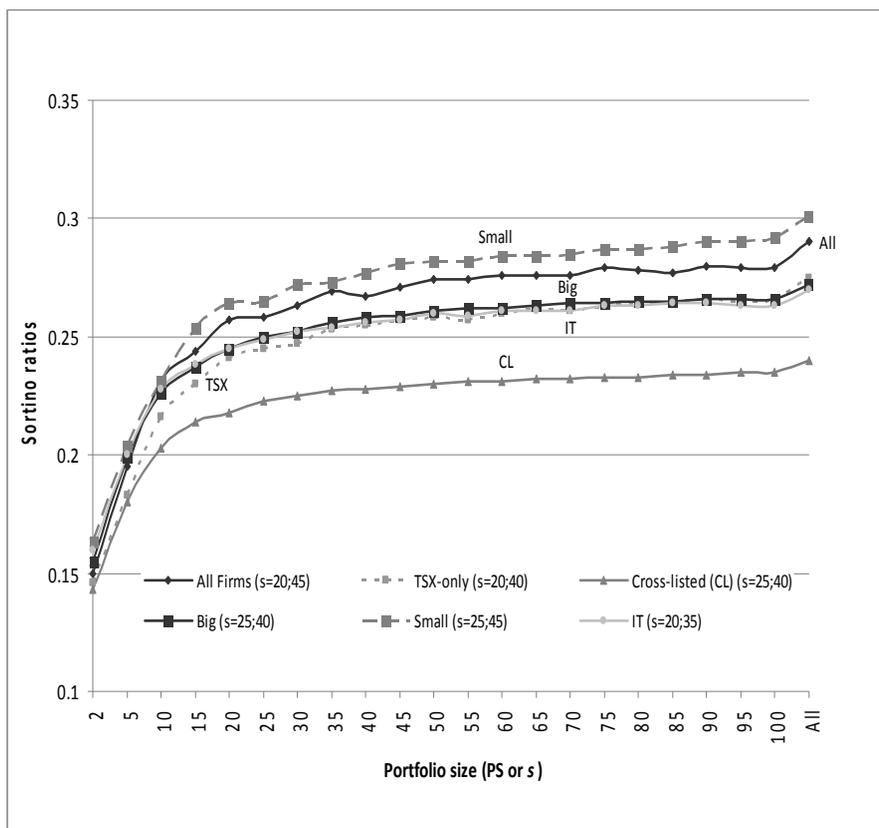
The mean values of the Sharpe and Sortino ratios are depicted respectively in figures 8 and 9 for 5000 randomly generated portfolios for each unique combination of j and s . As expected given the theoretical efficiency of holding the “market”, both Sh and Sor increase with increasing portfolio size. If the decision criterion is to achieve on average at least 90 percent of the potential increase in the Sharpe ratio from moving from a PS of 2 to all, then the required PS is about 75 (ISOP of 45.7%) for cross-listed firms, 85 (ISOP of 17.6%) for big firms, and over a hundred for the other four IO sets. None of the IO sets has a PS less than 100 for the Sharpe ratio if the threshold is increased to 95 percent. In contrast and as expected given the positive skewness identified earlier, the required PSs are much lower for all IO sets based on the Sortino metric in order to achieve at least 90 percent of the potential increase in the Sortino ratio from moving from a PS of 2 to all (also see figure 3). Specifically, the required PSs on average are about 50 for cross-listed firms (ISOP of 30.5%) and IT firms (ISOP of 45.9%), 55 for big firms (ISOP of 11.4%), 60 for TSX-only listed firms (ISOP of 9.5%) and all firms (ISOP of 8.0%), and 75 for small firms (ISOP of 28.0%). This once again illustrates the greater risk inherent in the small firm and IT sectors. Interestingly, the mean values for the small firm IO set depicted in figures 8 and 9 are higher [lower] for each portfolio size based on the Sharpe [Sortino] ratios compared to those for the all IO set.

Figure 8
Sharpe ratios differentiated by portfolio size and investment opportunity set



This figure depicts the mean Sharpe ratios based on 5000 random drawings for each unique combination of the 22 PSs and six IO sets using monthly returns over the period, 1975-2003. The Sharpe ratio is a mean excess return to total risk measure given by $Sh_{j,s} = (\bar{r}_{j,s} - r_f) / \sigma_{j,s}$. In the legend, (s=x,y) refers to the lowest portfolio sizes of x and y that capture respectively 90% and 95% of the increase in the Sharpe ratio from moving from a s of 2 to a s of “All”. A blank for either x or y indicates that s > 100. All of the means for a s of 2 and “All” are significantly different at the 0.05 level.

Figure 9
Sortino ratios differentiated by portfolio size and investment opportunity set



This figure depicts the mean Sortino ratios based on 5000 random drawings for each unique combination of the 22 PSs and six IO sets using monthly returns over the period, 1975-2003. The Sortino ratio is a mean excess return to semi-standard deviation risk measure given by $Sor_{j,s} = (\bar{r}_{j,s} - r_f) / \sigma_{j,s}^-$. In the legend, (s=x,y) refers to the lowest portfolio sizes of x and y that capture respectively 90% and 95% of the increase in the Sortino ratio from moving from a s of 2 to a s of “All”. A blank for either x or y indicates that s > 100. All of the means for a s of 2 and “All” are significantly different at the 0.05 level.

The third composite metric examined herein is the Sharpe-ratio-adjusted excess-return measure (ER) used by Xu (2003), which is given by:⁷

$$ER_{j,s} = \bar{R}_{j,s} - (\sigma_{j,s}/\sigma_j) \bar{R}_j \quad (8)$$

where $\bar{R}_{j,s}$ and $\sigma_{j,s}$ are the average return and standard deviation of returns for IO set j for a PS of s , and \bar{R}_j and σ_j are the average return and standard deviation of returns for the equal-weighted portfolio of all the stocks in IO set j .

The mean values for ER are calculated for 5000 randomly generated portfolios for each unique combination of j and s . As expected given the theoretical efficiency of holding the “market”, the undepicted ER values are negative at a PS of two and move monotonically towards zero with increasing portfolio size. If the decision criterion is to achieve at least 90 percent of the potential reduction in the shortfalls in Sharpe-ratio-adjusted excess returns of moving from a PS of 2 to all, then the required PS is about 40 for IO sets consisting of cross-listed firms (ISOP of 24.4%) and of big firms (ISOP of 8.3%), about 55 (ISOP of 50.5%) for the IO set of IT firms and over a hundred for the other three IO sets. Only the IO set of cross-listed firms has a PS less than 100 (i.e., 75) if the threshold is increased to 95 percent. Furthermore, 90 percent of the convergence towards the steady-state standard deviation of zero for both of these measures is obtained with approximately 40, 40 and 45 stocks for the IO sets of cross-listed, big and IT firms, respectively.

4.5 Probability of underperforming a target or lower-bound rate of return

The metrics examined in this section of the paper are based on the intuition of reducing the relative importance of idiosyncratic volatility with

⁷ The ER metric becomes the M^2 metric of Modigliani and Modigliani (1997) if excess returns are used for the portfolio and the market, respectively. Xu (2003) also uses a relative return measure (θ), which is given by $\theta_{j,s} = |\bar{R}_j|^{-1} \{ \bar{R}_{j,s} - (\sigma_{j,s}/\sigma_j) \bar{R}_j \}$, where $|\bar{R}_j|$ is the average absolute return for the equal-weighted portfolio of all the stocks in IO set j , and all the other terms are as defined earlier. This metric is a large multiple of ER for all j for a fixed s , since $\theta_{j,s} = |\bar{R}_j|^{-1} ER_{j,s}$ and $|\bar{R}_j|^{-1} \square 1$. Thus, the nontabulated θ values emit the same inferences as ER in terms of minimum portfolio size for all six IO sets.

respect to total portfolio variance. Based on the approach by Xu (2003), these measures deal with determining the number of stocks required for the probability (likelihood) to underperform a target or lower-bound return over various investment holding periods. Three target or lower-bound rates of return are used herein; namely, the market return (as proxied by an equal-weighted average of all the stocks available for investment in each IO set in each month over the studied period), a zero return and a lower-bound return of -25%.

The mean values of the average probabilities over implicit holding periods of one month, one year and three years of not achieving these three target or lower-bound rates of return criteria are calculated for 5000 randomly generated portfolios for various unique combinations of j and s . They are neither tabulated nor depicted to conserve valuable journal space. The results for the shortest holding period of one month should capture the trading behavior of an active noise trader, and the successively longer one and three year holding periods should capture the trading behavior of progressively more value-oriented investors.

The average probabilities of obtaining (compound) returns that are inferior to the all-firm (“market”) returns for each IO set over the three holding periods are examined first. The average probabilities for the least diversified portfolios consisting of two stocks are always above 0.5 (ranging between 0.543 for cross-listed firms for the one-month holding periods to 0.683 for TSX-only listed firms for the three-year holding periods), and the average probabilities decline monotonically with movement towards the most diversified portfolios consisting of all stocks in each IO set for all three holding periods. The reduction in the average probabilities from a PS of two to a PS of all is significant for all IO sets for the three holding periods. The average probabilities for the IO set of small firms are higher than for the other five IO sets for the less diversified PSs for all three holding periods. For a fixed PS and IO set, most of the average probabilities decline as the holding period gets longer. The major exceptions are the slightly higher average probabilities for big firms and IT firms for more diversified portfolios when the holding period goes from one to three years. These results strongly show that holding portfolios that tend to mimic the market will be favored if the investor is concerned about the probability of underperforming the market. This may explain the behavior of many managed funds (such as mutual funds) to behave as if they were closet indexers.

The average probabilities of obtaining negative (compound) returns over the three holding periods are examined next. All but one of the average probabilities for the least diversified portfolios consisting of two stocks is below 0.5, and average probabilities decline monotonically as we move to the most diversified portfolios consisting of all stocks in each IO set for all three holding periods. The reduction in the average probabilities from a PS of two to a PS of all is significant for all IO sets for the three holding periods. Not surprisingly, the average probabilities for the IO set of small firms are higher than for the other five IO sets for all PSs for all three holding periods. For a fixed PS and IO set, the average probabilities also decline as the holding period gets longer. If the decision criterion is to achieve on average at least 90 percent of the potential decrease in the average probabilities from moving from a PS of 2 to all, then the required PS is dependent on both the IO set being examined and the length of the holding period. Specifically, the required PSs at 90% are approximately 70 stocks for cross-listed and IT firms, 60 for big firms and over 100 for the other three IO sets for the one-month holding periods. The required PSs at 90% are approximately 55 stocks for the all-firm IO set, 60 for the TSX-only listed and big firm IO sets, about 75 stocks for cross-listed firms, about 95 for IT firms and over 100 for small firms for a one-year holding period. The required PSs at 90% are 50 for big firms, 70 for IT firms and over 100 for the other four IO sets for the three-year holding periods.

Finally, the average probabilities of obtaining (compound) losses of more than 25% over the three holding periods are examined. For the least diversified portfolio size of 2, the average probabilities of achieving a loss of more than 25% is lowest for big firms and highest for small firms for all three holding periods. For PSs of 5 or less, this probability generally increases with an increase in the holding period for each of the six IO sets. For PSs of 10 or greater, this probability generally increases from a one month to a one year holding period and then generally decreases from a one year to a three year holding period. If the decision criterion is to achieve on average at least 90 percent of the potential decrease in the average probabilities from moving from a PS of 2 to all, then the required PS is dependent on both the IO set being examined and the length of the holding period. Specifically, the required PSs at 90% are approximately 5 stocks for big firms, 10 stocks for all, TSX-listed only and small firms, 15 for cross-listed firms and 20 for the IT firms for the one-month holding periods. The required PSs at 90% are approximately 25 stocks for the IT IO set, about 35 for the all, TSX-only

listed and cross-listed firm IO sets, about 40 for big firms, and over 100 for small firms for a one-year holding period. The required PSs at 90% are about 15 for big firms, about 25 for cross-listed and IT firms, about 30 for all and TSX-only listed firms and about 85 for small firms for the three-year holding periods. Thus, with the exception of portfolios drawn from the small or IT IO sets, a portfolio of 40 stocks will have, on average, a probability of up to about 6% of obtaining a compound loss of more than 25% for holding periods of 1 month, 1 year or 3 years.

These findings provide mixed evidence for the refutation by various authors [e.g., Samuelson (1963, 1989); Kritzman (1994, 1997); Fisher and Statman (1999)] that investors suffer from the cognitive error that losses are reduced over longer holding periods. Specifically, while the probability of not earning a positive return is reduced with a longer holding period, the probability of not earning the market return is increased with a longer holding period. Thus, the existence of cognitive error depends upon the choice of anchor or benchmark return.

5 - Concluding comments

This paper used various metrics to determine the minimum portfolio size required to achieve a sufficiently well diversified portfolio for various investment opportunity sets. Based on a summary of these results for the achievement, on average, of 90% of the benefits of diversification presented in table 2, we found that this minimum portfolio size is very sensitive to the performance metric used to measure such benefits and is somewhat less sensitive to the investment opportunity set from which the portfolio selection is made. Whether or not these findings are applicable to future time periods depends upon whether or not the investment opportunities represented by the time period studied herein are applicable to future time periods.

Thus, if the conventional measures of risk (such as the time-series variance, semi-variance and NV) are used, then about 20 to 25 stocks are required on average to achieve about 90% of the potential benefits from diversification. If average time-series measures of the excess standard deviation of nonmarket portfolios over that of the market portfolio (such as the MDD) are used, then the number of stocks required to achieve about 90% of the potential benefits from diversification is about 40 and 45 stocks for cross-listed and big firms, respectively, and about 95 or greater for the other four investment opportunity sets. As noted repeatedly in the text, a similar

portfolio size in number represents quite different proportions of the total number of stocks available in each investment opportunity set (24.4% and 9.3% for cross-listed and big firms in this case). If instead time-series averages of the standard deviations of the cross section of portfolio returns (such as the MRD) are used, then the required number of stocks on average is about 70 for cross-listed firms, 60 for IT firms and over 100 for the other four IO sets. However, this inference must be tempered since it ignores higher-order moments of the cross-sectional distributions of portfolio returns, especially for investors who prefer positive skewness and dislike positive kurtosis. If skewness and kurtosis are considered in isolation, then the minimum number of stocks required to achieve about 90% of the potential benefits from diversification (or not diversifying in the case of skewness) is about 2 stocks for skewness and about 20 to 25 stocks for kurtosis.

If the investor is concerned about the impact of diversification on reward for bearing risk (as measured by *ER*, Sharpe ratio or Sortino ratio), then the minimum number of stocks required to achieve about 90% of the potential benefits from diversification ranges from 40 to over 100 stocks depending upon the performance metric used and the investment opportunity set considered. To illustrate, most likely due to the skewness and kurtosis in the various return distributions considered, such diversification benefits are achieved with from about 50 stocks to about 75 stocks depending upon the investment opportunity set based on the Sortino ratio that uses downside risk. And finally, if the investor is interested in achieving about 90% of the potential benefits from diversification in meeting three rates of return targets or lower bounds over holding periods of one month, one year or three years, then portfolio sizes of over 100 are required for a market rate of return target, portfolio sizes of 55 to over 100 are required for a zero rate of return target, and portfolio sizes of 5 to 40 (up to 100 for small firms) are required for a lower-bound return target of better than -25%.

The ambiguity in what is the optimal portfolio size also illustrates the need for financial advisors to carefully implement the “know your client” rules. Specifically, it raises the standard of care required in assessing the tolerance and attractiveness of various moments of the return distributions of various potential portfolio sizes for investors.

Much scope exists for further research on this theme. This includes an examination of other investment opportunity sets that are consistent with the objectives of open-end mutual funds or the mandates of pension fund managers such as value and growth, industry-specific and multi-asset classes.

It also includes the consideration of the costs associated with increasing portfolio size, particularly trade costs, and the lack of liquidity. Other interesting avenues of research include an examination of potentially asymmetric benefits of diversification in up and down markets or in expansionary or recessionary economies, and an examination of whether the ambiguity in the minimum portfolio size extends to other asset classes and markets.

Table 2
Summary of the minimum portfolio sizes required to achieve 90% of the benefits of diversification based on the various performance metrics

Performance Metric	IO Set j For Firms					
	All	TSX-only Listed	Cross-listed	Big	Small	IT
Panel A: Dispersion of stock returns						
Mean derived dispersion (MDD)	>100 [≥13.3%]	95 [15.1%]	40 [24.4%]	45 [9.3%]	>100 [≥37.3%]	>100 [≥91.7%]
Mean realized dispersion (MRD)	>100 [≥13.3%]	>100 [≥15.9%]	70 [42.7%]	>100 [20.7%]	>100 [≥37.3%]	60 [55.0%]
Normalized portfolio variance (NV)	20 [2.7%]	20 [3.2%]	20 [12.2%]	25 [5.2%]	20 [7.5%]	20 [18.3%]
Semi-variance (risk-free or mean return as target)	25 [3.3%]	25 [4.0%]	25 [15.2%]	25 [5.2%]	25 [9.3%]	25 [22.9%]
Panel B: Higher-order moments of stock returns						
Skewness	2 [0.3%]	2 [0.3%]	2 [1.2%]	2 [0.4%]	2 [0.7%]	2 [1.8%]
Kurtosis	20 [2.7%]	20 [3.2%]	25 [15.2%]	25 [5.2%]	25 [9.3%]	20 [18.3%]
Panel C: Composite return and risk						
Sharpe-ratio-adjusted excess-return (ER)	>100 [≥13.3%]	>100 [≥15.9%]	40 [24.4%]	40 [8.3%]	>100 [37.3%]	55 [50.5%]
Sharpe ratio	>100 [≥13.3%]	>100 [≥15.9%]	75 [45.7%]	85 [17.6%]	>100 [37.3%]	>100 [91.7%]
Sortino ratio	60 [8.0%]	60 [9.5%]	50 [30.5%]	55 [11.4%]	75 [28.0%]	50 [45.9%]

Table 2 Continued

Performance Metric	IO Set j For Firms					
	All	TSX-only Listed	Cross-listed	Big	Small	IT
Panel D: Probability of underperforming a target return						
Prob. [$\bar{R}_{j,s} < 0$], 1-month <i>HP</i>	>100 [$>13.3\%$]	>100 [$>15.9\%$]	70 [42.7%]	60 [12.4%]	>100 [$>37.3\%$]	70 [64.2%]
Prob. [$\bar{R}_{j,s} < 0$], 1-year <i>HP</i>	55 [7.3%]	60 [9.5%]	75 [45.7%]	60 [12.4%]	>100 [$>37.3\%$]	95 [87.2%]
Prob. [$\bar{R}_{j,s} < 0$], 3-year <i>HP</i>	>100 [$>13.3\%$]	>100 [$>15.9\%$]	>100 [$>61.0\%$]	50 [10.4%]	>100 [$>37.3\%$]	70 [64.2%]
Prob. [$(\bar{R}_{j,s} - \bar{R}_j) < 0$], all 3 <i>HP</i> s	>100 [$>13.3\%$]	>100 [$>15.9\%$]	>100 [$>61.0\%$]	>100 [$>20.7\%$]	>100 [$>37.3\%$]	>100 [$>91.7\%$]
Prob. [$\bar{R}_{j,s} \leq -25\%$], 1-month <i>HP</i>	10 [1.3%]	10 [1.6%]	15 [9.1%]	5 [1.0%]	10 [3.7%]	20 [18.3%]
Prob. [$\bar{R}_{j,s} \leq -25\%$], 1-year <i>HP</i>	35 [4.7%]	35 [5.6%]	35 [21.3%]	40 [8.3%]	100 [37.3%]	25 [22.9%]
Prob. [$\bar{R}_{j,s} \leq -25\%$], 3-year <i>HP</i>	30 [4.0%]	30 [4.8%]	25 [15.2%]	15 [0.31%]	85 [31.7%]	25 [22.9%]

This table reports the minimum portfolio sizes (PS or s) that are indicated by each of the tests reported earlier to achieve 90% of the benefits of diversification achieved from moving from a PS of 2 to a PS of all for each of the six IO sets examined herein. $\bar{R}_{j,s}$ is the average return for investment opportunity set j for a portfolio size (PS) of s , \bar{R}_j is the mean return on the market as proxied by the equal-weighted average return of all the stocks available for investment during each month in IO set j , and *HP* is holding period. The values in the brackets represent the proportion of the time-series weighted average population of stocks in each IO set j represented by the indicated portfolio size.

References

- Ankrim, E. M. and Z. Ding, 2002. Cross-sectional volatility and return dispersion. *Financial Analysts Journal* 58 (5), 67-73.
- Bennett, J. A. and R. W. Sias, 2006. Why company-specific risk changes over time? *Financial Analysts Journal* 62 (5), 89-100.
- Bloomfield, T., R. Leftwich and J. B. Long, Jr., 1977. Portfolio strategies and performance. *Journal of Financial Economics* 5 (2), 201-218.
- BMO, 2001. Benefits of diversification, *Perspectives*, BMO Nesbitt Burns Private Client Division (Winter), 3.
- Brennan, M. J., 1975. The optimal number of securities in a risky asset portfolio when there are fixed costs of transacting. *Journal of Financial and Quantitative Analysis* 10 (3), 483-496.
- Brys, G., H. Mia and A. Struyf, 2006. Robust measures of tail weight. *Computational statistics and data analysis* 50 (3), 733-759.
- Campbell, J. Y., M. Lettau, B. Malkiel and Y. Xu, 2001. Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *Journal of Finance* 56 (1), 1-43.
- Cleary, S. and D. Copp, 1999. Diversification with Canadian stocks: How much is enough? *Canadian Investment Review* 12 (3), 7-16.
- de Silva, H., S. Sapra and S. Thorley, 2001. Return dispersion and active management. *Financial Analysts Journal* 57 (5), 29-42.
- Demirer, R. and D. Lien, 2004. Firm-level return dispersion and correlation asymmetry: Challenges for portfolio diversification. *Applied Financial Economics* 14 (6), 447-456.
- DeMiguel, V., L. Garlappi and R. Uppal, 2008. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? forthcoming in *The Review of Financial Studies*.
- De Wit, D. P. M., 1998. Naive diversification. *Financial Analysts Journal* 54 (4), 95-100.
- Elton, E. J. and M. J. Gruber, 1977. Risk reduction and portfolio size: An analytical solution. *Journal of Business* 50 (4), 415-437.
- Evans, J. L. and S. H. Archer, 1968. Diversification and the reduction of risk: An empirical analysis. *Journal of Finance* 23 (5), 761-767.
- Fama, E. F., 1965. Portfolio analysis in a stable paretian market. *Management Science* 11 (3), 404-419.
- Fang, H. and T.-Y. Lai, 1997. Co-kurtosis and capital asset pricing. *Financial Review* 32 (2), 293-307.

Fisher, K. L. and M. Statman, 1999. A behavioral framework for time diversification. *Association for Investment Management and Research* (May/June), 88-97.

Goetzmann, W. N. and A. Kumar, 2008. Equity portfolio diversification, *Review of Finance* 12 (3), 433-463.

Grinblatt, M. and M. Keloharju, 2001. How distance, language, and culture influence stockholdings and trades. *Journal of Finance* 56 (3), 1053–1073.

Harvey, C. and A. Siddique, 1999. Autoregressive conditional skewness. *Journal of Financial and Quantitative Analysis* 34 (4), 465–487.

Investopedia, 2008. Available at: <http://www.investopedia.com/ask/answers/05/optimalportfoliosize.asp>.

Jennings, E. H., 1971. An empirical analysis of some aspects of common stock diversification. *Journal of Financial and Quantitative Analysis* 6 (2), 797-813.

Kelly, M., 1995. All their eggs in one basket: Portfolio diversification of US households. *Journal of Economic Behavior and Organization* 27 (1), 87–96.

King, M. and D. Segal, 2003. Corporate governance, international cross listing and home bias. *Canadian Investment Review* 16 (4), 8-18.

Kraus, A. and R. Litzenberger, 1976. Skewness preference and the valuation of risk assets. *Journal of Finance* 31 (4), 1085-1100.

Kritzman, M., 1994. What practitioners need to know about time diversification. *Financial Analysts Journal* 50 (1), 14-18.

Kritzman, M., 1997. Time diversification: An update. *Economics and Portfolio Strategy*. (P.L. Bernstein, Inc., New York).

Kryzanowski, L. and A. Rahman, 1986. Abstract: Diversification and the reduction of stochastic dispersion. *Financial Review* (August), 49.

Kryzanowski, L. and M. C. To, 1982. Asset pricing models when the number of securities held is constrained: A comparison and reconciliation of the Mao and Levy models. *Journal of Financial and Quantitative Analysis* 17 (1), 63-73.

Latane, H. A. and W. E. Young, 1969. Test of portfolio building rules. *Journal of Finance* 24 (4), 595-612.

Levy, H., 1978. Equilibrium in an imperfect market: A constraint on the number of securities in the portfolio. *American Economic Review* 68 (4), 643-658.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics* 47 (1), 13-37.

MacDonald, J. A. and H. Shawky, 1995. On estimating stock market volatility: An explanatory approach. *Journal of Financial Research* 18 (4), 449–463.

Malkiel, B. G. and Y. Xu, 2006. Idiosyncratic risk and security returns. Working Paper, School of Management, University of Texas at Dallas, Texas.

Mao, J. C. T., 1971. Security pricing in an imperfect capital market. *Journal of Financial and Quantitative Analysis* 6 (4), 1105-1116.

Markowitz, H. M., 1952. Portfolio selection. *Journal of Finance* 7 (1), 77-91.

Markowitz, H. M., 1959. Portfolio selection: Efficient diversification of investments. (John Wiley and Sons, Inc., New York).

Markowitz, H. M., 1976. Markowitz revisited. *Financial Analysts Journal* 32 (5), 47-52.

Merton, R. C., 1987. A simple model of capital market equilibrium with incomplete information. *Journal of Finance* 42 (3), 483-510.

Modigliani, F. and L. Modigliani, 1997. Risk-adjusted performance: How to measure it and why. *Journal of Portfolio Management* 23 (2), 45-54.

Mossin, J., 1996. Equilibrium in a capital asset market. *Econometrica* 34 (4) (October), 768-783.

Nieuwerburg, S. V. and L. L. Veldkamp, 2008. Information acquisition and under-diversification. Working Paper, Stern School of Business, New York University.

Polkovnichenko, V., 2005. Household portfolio diversification: A case for rank-dependent preferences. *Review of Financial Studies* 18 (4), 1467-1502.

Rao, C.R., 1970. Estimation of heteroscedastic variances in linear models, *Journal of the American Statistical Association* 65 (329), 161-172.

Ruppert, D., 1987. What is kurtosis? An influence function approach, *The American Statistician* 41, 1–5.

Samuelson, P.A., 1963. Risk and uncertainty: The fallacy of the law of large numbers. *Scientia* 98, 108-113.

Samuelson, P.A., 1989. The judgment of economic science on rationale portfolio management: Indexing, timing, and long-horizon effects. *Journal of Portfolio Management* 16 (1), 3-12.

Sharpe, W. F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 19 (3), 425-442.

Sharpe, W. F., 1994. The Sharpe ratio. *Journal of Portfolio Management* 21 (1), 49-58.

Silvapulle, P. and C. W. J. Granger, 2001. Large returns, conditional correlation and portfolio diversification: A value-at-risk approach. *Quantitative Finance* 1 (2), 542-551.

Solnik, B. and J. Roulet, 2000. Dispersion as cross-sectional correlation. *Financial Analysts Journal* 56 (1), 54-61.

Sortino, F. A. and L. N. Price, 1994. Performance measurement in a downside risk-framework. *Journal of Investing* 3 (3), 59–65.

Statman, M., 1987. How many stocks make a diversified portfolio. *Journal of Financial and Quantitative Analysis* 22 (3), 353-363.

Statman, M., 2004. The diversification puzzle. *Financial Analysts Journal* 60 (4), 44-53.

Statman, M. and J. Scheid, 2007. Correlation, return gaps and the benefits of diversification. Working Paper, Santa Clara University (November). Available at SSRN: <http://ssrn.com/abstract=995844>.

Surz, R. J. and M. Price, 2000. The truth about diversification by the numbers. *Journal of Investing* 9 (4), 1-3.

Wagner, W. H. and S. C. Lau, 1971. The effect of diversification on risk. *Financial Analysts Journal* 27 (6), 48-53.

Xu, Y., 2003. Diversification in the Chinese stock market. *Working Paper*, School of Management, University of Texas at Dallas, Dallas.

Xu, Y. and B. Malkiel, 2003. Investigating the behavior of idiosyncratic volatility. *Journal of Business* 73 (4), 613-644.