Optimal Hedge Ratio Estimation during the Credit Crisis: An Application of Higher Moments

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Abstract

This study aims to investigate and provide further insight into the dynamics of higher moments in the estimation of optimal hedge ratios during the recent credit crisis period by applying the Gram-Charlier expansion series. Furthermore, it compares the performance of the proposed model with conventional hedge ratio methodologies such as: OLS, GARCH and univariate and multivariate GARCH models. The empirical application is performed on spot and futures data on the FTSE 100 and FTSE/ATHEX 20 indices by comparing the in- and out-of-sample hedging effectiveness. The selection of the FTSE/ATHEX 20 index was on the premise that the Greek equity market experienced a prolonged period of extreme movements during the credit crisis period. Overall, the results indicate that the application of the proposed model increases the performance of the hedges in terms of in- and out-of-sample variance reduction.

Keywords: Optimal hedge ratios; conditional volatility; skewness and kurtosis; higher moments; equity indices
JELClassification: G13, G10, C32

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1- Introduction

The objective of hedging is to offset an exposure to adverse price changes in the underlying physical market. A hedge position is determined by a hedge ratio, which reflects the number of futures contracts to buy or sell for each unit of the underlying asset in order to reduce the price risk. Ederington (1979) and Figlewski (1984) are the first to derive the static hedge ratio (minimum-variance hedge) that minimizes the variance of a portfolio. While the assumption of the constant relationship between spot and futures prices provides computational simplicity, Kroner and Sultan (1993) highlight the dynamic behaviour, i.e. time-varying distributions, of financial series. In this respect, Kroner and Sultan (1993), Park and Switzer (1995) and Kavussanos and Nomikos (2000) employ a set of Multivariate General Autoregressive Conditional Heteroskedasticity (MVGARCH) formulations to model the time-varying variances and covariances of spot and futures prices in estimating time-varying hedge ratios. Although the aforementioned studies show that the dynamic hedge ratio outperforms the constant hedge ratio derived from Equation (1), the overall benefits vary across different markets (Lien and Tse, 2002).

Although more sophisticated methodologies have been found to be superior to OLS in terms of hedging performance, there is no consensus in the empirical literature with respect to the most appropriate methodological approach. A particular issue that has yet to receive attention in the optimal hedging analysis, concerns the importance of conditional variations in moments other than the mean and variance. It is widely assumed that financial series and their returns are normally distributed, but empirical evidence shows that they are likely to be skewed and fat-tailed (Campbell and Siddique, 1999) and, if ignored, will have serious implications in risk management, option pricing, trading and hedging activities as well as in portfolio selection and asset allocation.

Lai and Lai (1991), Prakash *et al.* (2003), Sun and Yan (2003) and Jondeau and Rockinger (2006) provide evidence that the incorporation of higher moments in portfolio allocation leads to a superior approximation of expected utility and allows an efficient way to compute optimal portfolios. Burns (2002), Wilhelmsson (2007) and Angelidis *et al.* (2007), among others, illustrate the importance of incorporating distributions that account for skewness and kurtosis in risk management and, particularly, in computation of superior Value-at-Risk estimates. Heston and Nandi (2000) and Christoffersen *et al.* (2006) propose a set of option pricing models that incorporate conditional skewness and find substantial pricing improvements compared with the classical pricing models. Finally, Kostika and Markellos (2007) find that a GARCH model, using asymmetric non-normal distribution, such as Hansen's Skewed t-distribution, provides better hedge rations. However, the literature on modelling the time-varying dynamics of skewness and kurtosis is limited in this field.

The aim of this study is to investigate, for the first time, the time-varying dynamics of higher moments in the estimation of optimal hedge ratio for two equity indices, during the most turbulent period in the financial markets. The UK and Greek markets are selected primarily due to the extreme movements which are depicted in their spot and future equity markets as well as in their higher-moments. For the former, the extreme events are driven by the banking crisis, which has severely impacted UK's economic prospects (Goddard, *et al*. 2009, among others). For the latter, these events are driven by the combined banking crisis and the possibility of default or restructuring on some of Greece's public debt (Edward, 2009; Mahmud, 2010; among others).

The novelty of this study is that it assesses and captures the time-varying dynamics of the shape of the distribution of the underlying indices by presenting a GARCH model, which to the best of our knowledge, has not been previously applied in the estimation of the optimal hedge ratios. This particular GARCH model, the GARCH with skewness and kyrtosis model (GARCH-SK), accounts for time-varying skewness and kurtosis model and it is based on the Gram-Charlier expansion series. The hedging effectiveness of the GARCH-SK model is compared with its counterpart obtained by OLS, Error-Correction and MVGARCH models. The effectiveness is evaluated in-sample and out-of-sample using the minimum variance approach.

The remainder of this study is structured as follows. Section 2 presents the different methodologies utilised, Section 3 describes the data-set considered, Section 4 presents the empirical results and the hedging effectiveness of the different models employed, and, finally, Section 5 concludes.

2- Methodology

Let ΔS_t and ΔF_t denote the logarithmic changes in the spot and futures prices of the equity indices. Ederington (1979) and Figlewski (1984) derive the variance-minimizing hedge ratio as the ratio of the unconditional covariance between spot and futures price changes, which is the slope coefficient, γ_1, presented in the following regression:

$$\Delta S_t = \gamma_0 + \gamma_1 \Delta F_t + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0, \sigma^2\right) \tag{1}$$

The higher the reported R^2 of the estimated regression, the greater the power of the minimum-variance hedge. However, Ghosh (1993) and Lien (1996), among others, have shown that ignoring the long-run relationship between spot and futures price may lead to biased estimates of the minimum-variance hedge. Therefore, it is estimated through the following error correction model:

$$\Delta S_t = \gamma_0 + \gamma_1 \Delta F_t - \gamma_2 u_{t-1} + \sum_{i=1}^{p} b_i \Delta S_{t-1} + \sum_{j=1}^{q} c_j \Delta F_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0, \sigma^2\right) \tag{2}$$

where u_{t-1} denotes the lagged errors from the cointegration regression and ε_t is the error correction term. While the assumption of the constant relationship between spot and futures prices provides computational simplicity, Kroner and Sultan (1993) highlight the time-varying dynamics of moments of financial series and, along with the studies of Park and Switzer (1995), Kavussanos and Nomikos (2000) and Alizadeh and Nomikos (2004), argue in favour of the estimation of a dynamic hedge ratio. This ratio is more efficient than the conventional static hedge ratio, since it is updated to the arrival of new information in the market and is estimated as follows:

$$\gamma_1 = \frac{Cov\left(\Delta S_t, \Delta F_t\right)}{Var\left(\Delta F_t\right)}$$

Equation (3) describes the minimum-variance hedge ratio as the ratio of the covariance of spot and futures price changes over the variance of futures price changes. The variance of the series is estimated using the GARCH model proposed by Bollerslev (1986). It models the variance of a financial time series by conditioning the variance of the series on the square of the lagged error terms and lagged variances. The specification is given as:

: $$\sigma_t^2 = \beta_o + \sum_{i=1}^{q} \beta_{1,i} \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_{2,j} \sigma_{t-j}^2, \quad \varepsilon_t \sim iid\left(0, \sigma_t^2\right) \qquad (4)$$

However, financial time series exhibit leptokurtosis and leverage effects, and, therefore, the GARCH model is extended to capture the dynamics of the financial series. In this respect, Hansen (1994) proposes an Autoregressive Conditional Density (ARCD) model, which allows for time-variation in variance, skewness and kurtosis. The standard setup of the model is based on a generalization of the Student-t distribution with zero mean and unit variance, which allows for asymmetry and fat-tailedness to vary over time. Similar approaches have been developed by Theodosiou (1998), Rockinger (2003), Bera and Kim (2002) and Wilhelmsson (2007), among others, as they offer computational advantages. The major disadvantage of these approaches lies within their computational efficiency which is a result of imposing several restrictions on the parametric values of these types of models to allow for convergence. Another disadvantage is that these formulations do not allow for any conditional time-varying dynamics in the estimated residuals. Although Campbell and Siddique (1999) are the first to propose a Generalized Autoregressive Heteroskedasticity Variance and Skewness (GARCH-K) model, they assume kurtosis to be constant. It is Leon, *et al.* (2005) who propose a GARCH-type model that allows simultaneously for time-varying volatility, skewness and kurtosis (GARCH-SK). The model is estimated assuming a Gram-Charlier expansion series of the normal density function for the error term in the mean equation, which is more computationally efficient than the aforementioned formulations and it is given as:

$$
\begin{aligned}
& r_t = a_0 + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0, \sigma_t^2\right) \\
& \varepsilon_t = \eta_t \sqrt{\sigma_t}, \quad \eta_t \sim (0,1) \\
& \sigma_t = \beta_0 + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}, \\
& s_t = \gamma_0 + \sum_{i=1}^{q} \gamma_i \eta_{t-i}^3 + \sum_{j=1}^{q} \gamma_q s_{t-q}, \\
& k_t = \delta_0 + \sum_{i=1}^{q} \delta_i \eta_{t-i}^4 + \sum_{j=1}^{p} \delta_j k_{t-j},
\end{aligned} \tag{5}
$$

where s_t is the conditional skewness on time *t* and k_t conditional kurtosis on time *t*. The density function for the standardized residuals η_t conditioned on the information available in *t-1* is given as:

$$f\left(\eta_t\right) = \varphi\left(\eta_t\right) \psi^2\left(\eta_t\right) / \Gamma_t \tag{6}$$

where,

$$
\begin{aligned}
& \varphi\left(\eta_t\right) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{\eta_t^2}{2}}, \\
& \psi\left(\eta_t\right) = \left[1 + \frac{s_t}{3!}\left(\eta_t^3 - 3\eta_t\right) + \frac{k_t - 3}{4!}\left(\eta_t^4 - 6\eta_t^2 + 3\right)\right], \\
& \Gamma_t = 1 + \frac{s_t^2}{3!} + \frac{\left(k_t - 3\right)^2}{4!}
\end{aligned}
$$

and $\varphi(\cdot)$ denotes the probability density function of the standard normal distribution and $\psi(\cdot)$ is the fourth order of the Gram-Charlier polynomial.

3- Data description

The data set comprising this study is daily and includes the prices of FTSE-100 and ATHEX-20 spot and future indices spanning the period from January 3, 2005 to November 30, 2010. All data is obtained from Datastream, while non-trading days were removed, resulting a total of 1497 pair of observations. Data for the period January 3, 2005 to August 31, 2010 (1432 observations) is used for the in-sample analysis, whereas data from September 1, 2010 to November 30, 2010 (65 observations) is applied for the out-of-sample analysis. The delivery months for FTSE-100 and ATHEX-20 contracts are March, June, September and December; however, the large size of the markets allows for continuous contract rollover. The descriptive statistics for the levels and returns of the equity indices are reported in Table 1. The coefficients of skewness are negative for the levels and returns of the spot and futures indices. Negative skeweness indicates a bias towards downside exposure, while, positive skewness is desirable, since it allows for the possibility of large positive returns (Campbell and Siddique, 2000). Coeffiecients of excess kurtosis above three suggest that the distribution of the returns of the spot and futures indices is leptokurtic. This means that their distribution has a more acute peakedness and fatter tails to account for large deviations in the levels and returns of the spot and futures indices. The largest coefficient of excess kurtosis is reported for the returns of the FTSE-100 with a value of 11.624, followed by ATHEX-20 with a value of 6.474. The Jargue-Bera test reveals significant departures from normality for all series at a 1% significance level for both levels and returns of spot and futures indices. Finally, the Dickey and Fuller (1979) and Phillips and Perron (1978) unit root tests reveal that the series are first-differenced stationary at the 1% significance level.

Table 1: Descriptive statistics and unit root tests for spot and futures prices of FTSE-100 and ATHEX-20 indices

	FTSE 100				ATHEX-20			
	Log Levels		% Returns		Log Levels		% Returns	
	Spot	Futures	Spot	Futures	Spot	Futures	Spot	Futures
Mean	3.735	3.734	0.002	0.002	3.084	3.199	-0.017	-0.022
St. Dev.	0.059	0.060	0.601	0.602	0.145	0.183	0.882	0.907
Skewness	-0.752	-0.756	-0.109	-0.177	-0.597	-0.558	-0.157	-0.082
Kurtosis	3.066	3.099	11.194	11.624	2.344	1.992	5.821	6.474
JB	141.458	143.026	4009.015	4445.414	115.867	141.031	480.655	721.884
DF	0.229	0.241	-40.683	-40.333	-0.874	-1.073	-35.247	-36.941
PP	0.228	0.240	-40.682	-40.332	-0.873	-1.072	-35.246	-36.940

This table reports the descriptive statistics and unit root tests for both the log levels and continuous returns of the spot and future prices of the equity indices. The sample period is from January 3, 2005 to August 31, 2010, with a total of 1432 observations. JB is the Jarque-Bera (1980) test for normality, and is chi-square asymptotic with two degrees-of-freedom. DF is the Dickey and Fuller (1979) and PP is the Phillips and Perron (1978) unit root tests. The 1%, 5% and 10% critical values for these tests are: -3.962, -3.411 and -3.127.

4- Empirical results

The first model to be examined is the GARCH model - Equation (6) - which is estimated over the period of 3rd January 2005 to 31st August 2010. The estimation results and diagnostics tests are reported in Table 2. The Schwarz Bayesian Information Criterion (SBIC) is used to select the appropriate number of lags in the mean model. The diagnostics tests, including 1st and 10th order ARCH test and Ljung and Box test for autocorrelation, do not indicate the presence of autocorrelation and ARCH effects in the standardised residuals of all models. The coefficients of the lagged squared error, β_1, and of lagged conditional variance, β_2 are both significant in all models, indicating that the model is capable of capturing the dynamics of the second moment. In-sample volatility of the equity indices for the spot changes is presented in Figure 1. ATHEX-20 index exhibits larger conditional volatility than the FTSE-100 index. The Greek market experienced a number of extreme movements during the credit crisis, which is reflected in its variance.

Table 2: Estimation of the GARCH model

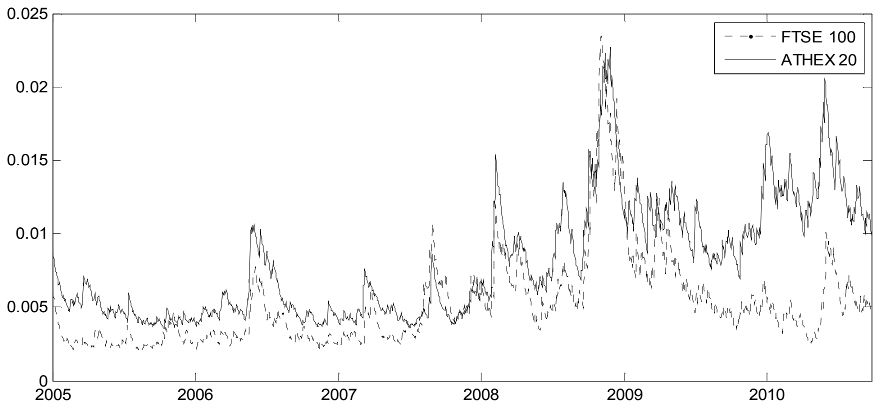
$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid\left(0, \sigma_t^2, \nu\right)$$

$$\sigma_t^2 = \beta_o + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-2}^2$$

	FTSE-100		ATHEX-20	
	SPOT	FUTURES	SPOT	FUTURES
α_0	$2.70*10^{-04}$	$2.79*10^{-04}$	$3.29*10^{-04}$	$4.22*10^{-04}$
	(2.776)	(2.953)	(2.035)	(2.633)
α_1	-0.066	-0.070	0.089	0.039
	(-2.347)	(-2.468)	(3.078)	(1.258)
β_o	$2.00*10^{-07}$	$2.14*10^{-07}$	$3.57*10^{-07}$	$4.64*10^{-07}$
	(2.728)	(2.912)	(2.491)	(2.778)
β_1	0.121	0.123	0.091	0.109
	(8.434)	(7.980)	(8.054)	(9.255)
β_2	0.879	0.877	0.908	0.891
	(67.464)	(62.415)	(88.997)	(79.752)
Diagnostics				
ARCH (1)	0.707	1.249	0.811	5.316
ARCH (10)	13.734	9.987	12.186	13.306
LB Q test (1)	0.580	0.749	0.478	0.757
LB Q test (10)	8.543	8.590	5.649	9.142
Log Likelihood	5695	5705	5039	4995
AIC	-11386	-11407	-10074	-9986
BIC	-11384	-11404	-10071	-9984

The table reports the estimation results of the GARCH model for the continuous returns of the spot and future prices of FTSE-100 and ATHEX-20. The estimation is performed by the method of quasi maximum likelihood BFGS using Winrats software package. The sample period is from January 3, 2005 to August 31, 2010 (1432 observations). The numbers in the parentheses are the t-statistics. The 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307. The AIC and BIC criteria are estimated as follows:
$AIC = -2\log(LLF) + 2m$, and $BIC = -2\log(LLF) + m\log(T)$, where m denotes the number of parameters and T is the number of observations.

Figure 1. The estimated conditional variance of the FTSE-100 and ATHEX-20 spot returns



The second model to be examined is the SK-GARCH model - Equation (7) - for the equity indices of spot and futures changes which are reported in Table 3. Once again, the mean equation is considered to be a AR process of order one, following the Akaike (AIC) and Schwarz Bayesian Information Criterion (SBIC). The ARCH and Ljung and Box diagnostic tests do not reveal any autocorrelation and ARCH effects in the standardised residuals. According to the value of Log Likelihood Function, Akaike (AIC) and Schwarz (SBIC), the GARCH-SK model outperforms the GARCH model for the in-sample period.

The coefficients of the lagged squared error, β_1, the lagged conditional variance, β_2, lagged cubic error terms, γ_1, the lagged conditional skewness, γ_2, lagged error terms to the power of four, δ_1, and the lagged conditional kurtosis, δ_2, are all significant in all models, suggesting the presence of time-varying moments and the ability of the model to capture their dynamics. The lowest coefficients of estimated skewness and kurtosis are reported for the equity indices of the futures continuous returns. Table 3 also presents the sum of coefficients of lagged error terms and the lagged variance of each model and indicates a moderate persistence level. However, this persistence level is lower than the one exhibited in the GARCH model, which suggests that, by incorporating the dynamics of the higher moments, the persistence of volatility is reduced. These results are in accordance with those displayed by Lamoureux and Lastrapes (1990), Campbell and Siddique (1999), Leon, *et al.* (2004) and Alizadeh and Gabrielsen (2010). Furthermore, the persistence of kurtosis tends to show a relatively higher degree of persistence vis-à-vis the persistence pertaining to skewness. These results are similar to those reached by Leon, *et al.* (2004) and Alizadeh and Gabrielsen (2010). The former study argues that periods of high (low) kurtosis are followed by periods of high (low) kurtosis, since the coefficients of the lagged kurtosis are positive and statistically significant.

Table 3: Estimation of the GARCH-SK model

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t, \quad \sigma_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1},$$

$$h_t = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 h_{t-1}, \quad k_t = \delta_0 + \delta_1 \eta_{t-1}^4 + \delta_2 k_{t-1}, \quad \varepsilon_t = \sqrt{\sigma} \eta_t$$

	FTSE-100		ATHEX-20	
	SPOT	FUTURES	SPOT	FUTURES
α_0	-0.004	-0.034	-0.015	-0.011
	(-222.431)	(-24.083)	(-612.657)	(-151.090)
α_1	-0.115	-0.063	-0.128	-0.025
	(-80.928)	(-6.205)	(-213.033)	(-8.552)

β_o	$1.19*10^{-06}$	$6.25*10^{-06}$	$4.61*10^{-07}$	$1.18*10^{-06}$
	(65.644)	(2.170)	(13.522)	(7.512)
β_1	0.048	0.002	0.019	0.004
	(123.878)	(3.191)	(102.645)	(5.322)
β_2	0.855	0.809	0.875	0.751
	(110.597)	(23.540)	(912.131)	(22.822)
γ_0	0.034	0.637	0.076	0.213
	(0.442)	(0.292)	(3.917)	(12.943)
γ_1	0.029	0.011	0.046	0.011
	(17.677)	(29.156)	(26.670)	(31.857)
γ_2	0.756	0.602	0.725	0.684
	(12.617)	(12.687)	(138.163)	(51.986)
δ_0	0.737	1.228	1.778	2.296
	(4.270)	(12.905)	(60.069)	(18.244)
δ_1	0.052	0.068	0.033	0.010
	(52.120)	(34.874)	(31.283)	(64.114)
δ_2	0.812	0.722	0.714	0.592
	(101.832)	(17.528)	(143.655)	(84.327)
Diagnostics				
ARCH (1)	1.084	3.584	1.153	0.532
ARCH (10)	16.4982	17.902	18.462	18.707
LB Q test (1)	0.018	2.709	0.014	3.579
LB Q test (10)	14.293	20.476	15.183	18.924
Log Likelihood	6829	6792	7291	6784
AIC	-13636	-13562	-14560	-13546
BIC	-13652	-13578	-14576	-13562

The table reports the estimation results of the GARCH-SK model for the continuous returns of the spot and future prices of FTSE-100 and ATHEX-20. The estimation is

performed by the method of quasi maximum likelihood BFGS using Winrats software package. The sample period is from January 3, 2005 to August 31, 2010 (1432 observations). The numbers in the parentheses are the t-statistics.
The 1% and 5% critical values for Engle's ARCH/GARCH test and Ljung-Box Q-test are for $\chi^2(1)$ 6.634 and 3.841, $\chi^2(10)$ 23.209 and 18.307. The AIC and BIC criteria are estimated as follows:
$AIC = -2\log(LLF) + 2m$, and $BIC = -2\log(LLF) + m\log(T)$, where m denotes the number of parameters and T is the number of observations.

The in-sample conditional skewness and kurtosis of the equity indices for spot and futures changes are presented in Figures, 2, 3, 4 and 5, respectively, and a consistent pattern of the higher-moments is observed. Time-varying skewness and kurtosis seem to fluctuate more and exhibit large spikes for the ATHEX-20 spot and futures indices, compared to those for the FTSE-100 spot and futures indices. Furthermore, time-varying skewness and kurtosis for the ATHEX-20 markets seem to spike at two particular periods. The first is during the beginning of the financial crisis and its transition from the US to the Greek banking system. The second is observed during the middle of 2010 and coincides with the period of Greece's bailout and austerity measures to reduce the budget deficit (Mahmud, 2010). Therefore, the GARCH-SK model is capable of capturing the different periods of financial crisis illustrated in the Greek economy, and this is depicted in the dynamics of higher-moments. Finally, time-varying skewness and kurtosis fluctuate more for the futures indices than the spot indices, illustrating higher levels of uncertainty regarding the future market movements.

Figure 2. The estimated conditional skewness of FTSE-100 spot and future returns

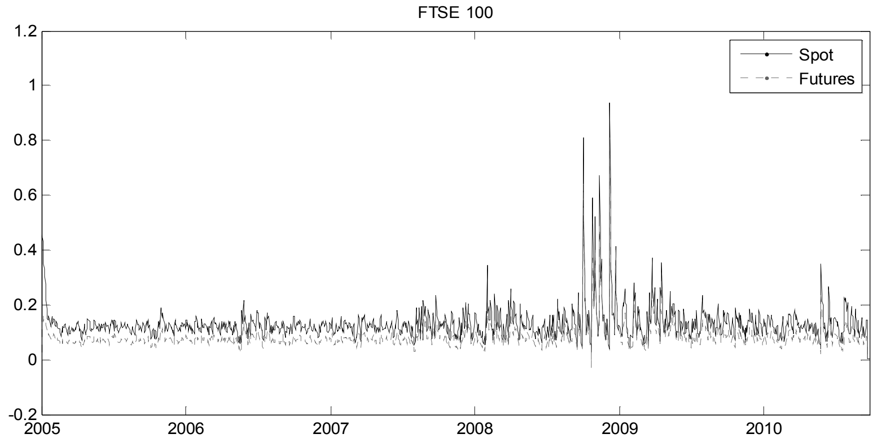


Figure 3. The estimated conditional skewness of ATHEX-20 spot and future returns

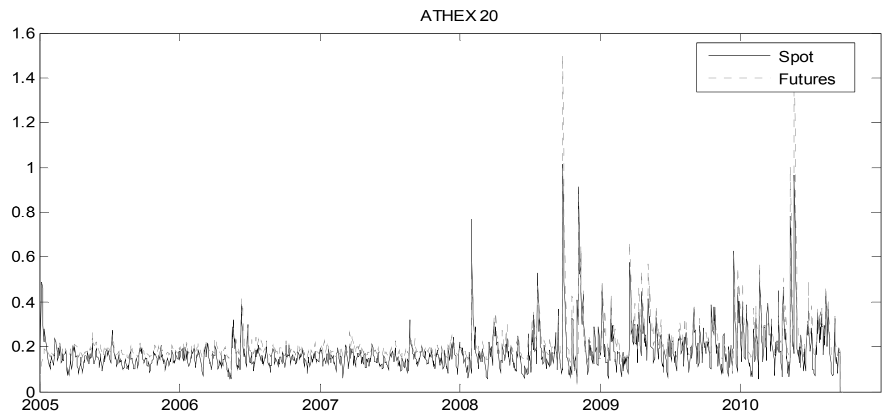


Figure 4. The estimated conditional kurtosis of FTSE-100 spot and future returns

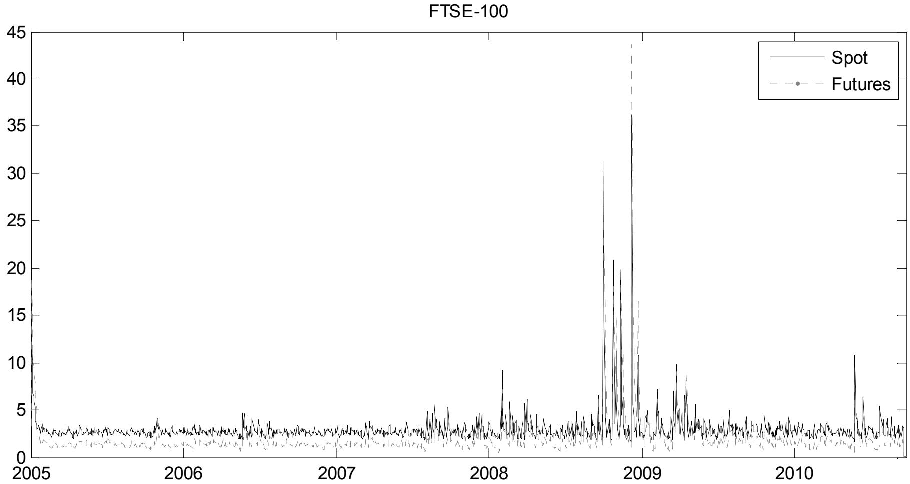
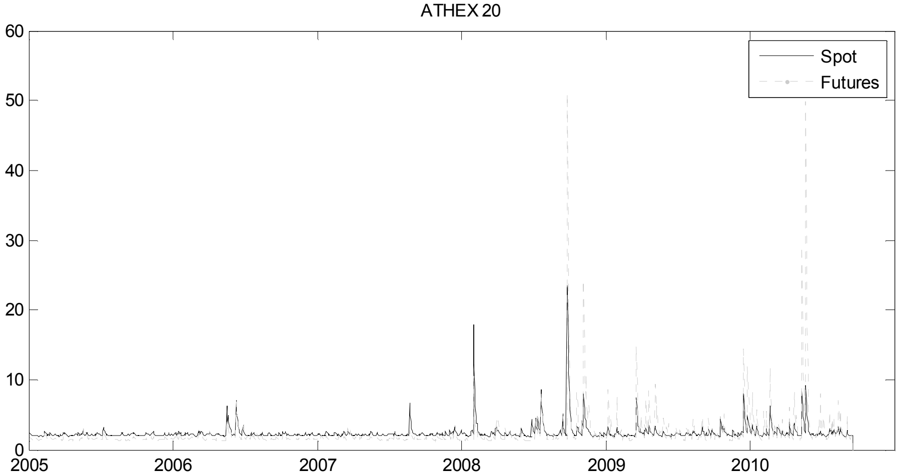


Figure 5. The estimated conditional kurtosis of ATHEX-20 spot and future returns



Following the estimation of the empirical discussion of the time-varying higher moments, the next step is to evaluate the performance of the hedges computed by the various models. Such models are the OLS model, the Vector Error Correction Model (VECM) of spot and future returns (Johansen, 1988) and the MVGARCH model, by evaluating the following expression:

$$Var\left(\Delta S_t - \gamma^* \Delta F_t\right)$$

where Var denotes the variance of hedge ratio, ΔS_t refers to spot price changes, ΔF_t futures price changes, γ^* are the estimated hedge ratios of the different specifications and is applied for both in- and out-of sample periods. Moreover, a benchmark naive hedge is considered by taking a futures position, which offsets the spot position exactly (i.e., $\gamma^* = 1$). Additionally, both the in- and out-of-sample hedging effectiveness of the models will be evaluated. Although the in-sample performance of the different strategies gives an indication of their historical performance, it is the out-of-sample performance that is of greater interest to investors, since it evaluates the effectiveness of the conditional hedge ratios. The in-sample analysis is performed for the period of 3rd January 2006 to 31st August 2010. For the out-of-sample analysis 65 pairs of observations for each market are withheld for the period of September 1, 2010 to November 30, 2010. The models are estimated using only the data up to August 31, 2010 to forecast hedge ratios for the out-of-sample period.

The portfolio variances as well as the percentage variance improvement over the GARCH-SK model for both portfolios and sample periods are reported in Table 4 and a number of stylized facts are observed. All models offer a significant reduction in the portfolio variance compared with the naive hedge, with the exceptions of OLS and VECM for the out-of-sample period for the FTSE 100 whose percentage variance reduction over the GARCH-SK is 11% and 22%, respectively. For both the in-sample and out-of-sample periods, the GARCH-SK outperforms the OLS, VECM, GARCH and MVGARCH specifications, in terms of in-sample and out-of-sample portfolio variance reduction. Furthermore, it is indicative that the performance of the GARCH-SK model is more pronounced for the out-of-sample for the ATHEX 20 index, during the period in which the market experienced a number of extreme movements. Finally, the performance of the GARCH-SK model may not be surprising as it is capable of capturing the dynamics and properties of the time series.

Table 4: Estimation of hedging effectiveness

	Naive	OLS	VECM	GARCH	GARCH-SK	BEKK
			In-Sample			
Variance						
FTSE-100	0.00829	0.00824	0.00824	0.00815	0.00811	0.00817
ATHEX-20	0.11962	0.11363	0.11150	0.11122	0.11120	0.11869
Variance Improvement over GARCH-SK						
FTSE-100	2.218%	1.525%	1.593%	0.443%	-	0.697%
ATHEX-20	7.572%	2.184%	0.267%	0.019%	-	6.736%
			Out-of-Sample			
Variance						
FTSE-100	0.00386	0.00403	0.00440	0.00360	0.00360	0.00369
ATHEX-20	0.13371	0.12882	0.12740	0.12737	0.12698	0.13132
Variance Improvement over GARCH-SK						
FTSE-100	7.286%	11.994%	22.343%	0.083%	-	2.640%
ATHEX-20	5.299%	1.445%	0.328%	0.303%	-	3.420%

This table reports the portfolio variances estimated as $Var\left(\Delta S_t - \gamma^* \Delta F_t\right)$ where γ^* are the estimated hedge ratios of the different specifications, as well as the percentage variance improvement over the GARCH-SK model for both portfolios and sample periods. The in-sample period is from January 3, 2005 to August 31, 2010 (1432 observations), while the out-of-sample is from September 1, 2010 to November 30, 2010 (65 observations).

5- Conclusions

This paper examined the performance of hedge ratios estimated through the GARCH-SK model, which is based on the Gram-Charlier expansion and it is capable of capturing the time-varying dynamics of skewness and kurtosis of financial time series. The hedging effectiveness of the GARCH-SK model was investigated on the FTSE 100 and ATHEX 20 markets. The selection of the ATHEX 20 market was primarily based on the prolonged extreme movements observed during the credit crisis period, which would put any model under a real stress. It is, therefore, an ideal market for the investigation of the dynamics of higher moments and hedging effectiveness during the credit-crisis period.

The in-sample performance, measured by the Log Likelihood values, the Akaike and the Schwarz criteria, illustrated that the GARCH-SK specification outperformed the GARCH model and was able to capture the characteristics and dynamics of the higher moments and better fit the empirical distribution of the data. Additionally, the GARCH-SK model was more powerful to capture the two different periods of financial crisis presented in the Greek economy, as highlighted by the time-varying dynamics of the higher-moments.

The hedge ratios, estimated through the GARCH-SK model, outperformed the GARCH, OLS, VECM and Naive hedges in the FTSE 100 and ATHEX 20 markets, according to a set of in- and out-of-sample tests. Moreover, the superior hedging effectiveness of the GARCH-SK model was also illustrated in the ATHEX 20 market for the out-of-sample period, highlighting the model's ability in capturing the extreme movements in the index. These extreme movements were depicted in the higher-moments of the underlying market. Future research in this field would include the application of the GARCH-SK model in financial series that exhibit sudden or extreme moments, such as those related to the emerging economies or to commodities.

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