Is a Necessary Good *Necessarily* an Inferior Good?
Slutsky Equation is Re-examined

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Abstract

In this short paper, we bring out the century-old celebrated Slutsky equation, and then ask the question: is a necessary good *necessarily* an inferior good? On the surface, the question appears quite strange, and yet further inquiry into this work reveals that the question, as it is posed, is very meaningful. The continued lack of an answer to this question, in spite of critical examination by microeconomic theorists and analysts including John R. Hicks and Paul A. Samuelson, is baffling. We present the question within the straitjacket of the Slutsky equation and the analytical vehicle we subsequently employed.

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Rational consumer behavior and optimum choices for the consumption basket have been explored for a long time in the economic literature.[i] But it was Slutsky [8]—just a century ago—who enunciated the optimum decision-making rule for a rational consumer. From the maximization of utility function,

$$U = U(C_1, C_2, ..., C_n), \tag{1}$$

subject to the budget constraint

$$\sum_{i=1}^{n} P_i C_i = M, \tag{2}$$

the following result is derived:

$$\frac{\partial C_i}{\partial P_i} = \lambda \frac{S_{ii}}{|S|} - C_i \frac{\partial C_i}{\partial M} \tag{3}$$

Here C_i is the consumption level, P_i the price of the $i-th$ good ($(1 \le i \le n)$, M the consumer's money income (expenditure), and λ the marginal utility of money income (Lagrange multiplier). This constrained maximization, of course, presupposes that the bordered Hessian matrix of the Lagrangean function is negative semi-definite, which means that $\lambda \frac{S_{ii}}{|S|} < 0$ where S_{ij} is the cofactor of the $i - j\ th$ element of the bordered Hessian matrix and $|S|$ is the determinant of the same Hessian matrix. Equation (3) is the celebrated century-old Slutsky result [1, 2, 3, 4, 5, 7, **8**, 9]. This result has been scrutinized by two outstanding economists, John R. Hicks [3, 4] and Paul A. Samuelson [6], for more than sixty years.

Note that without any difficulty, equation (3) can be rewritten in the following forms:

$$\frac{\partial C_i}{\partial P_i} = \left(\frac{\partial C_i}{\partial P_i}\right)_{dU=0} - C_i \frac{\partial C_i}{\partial M} \tag{4}$$

or alternatively,

$$\xi_i = \bar{\xi}_i - \theta_i \zeta_i \tag{4*}$$

where

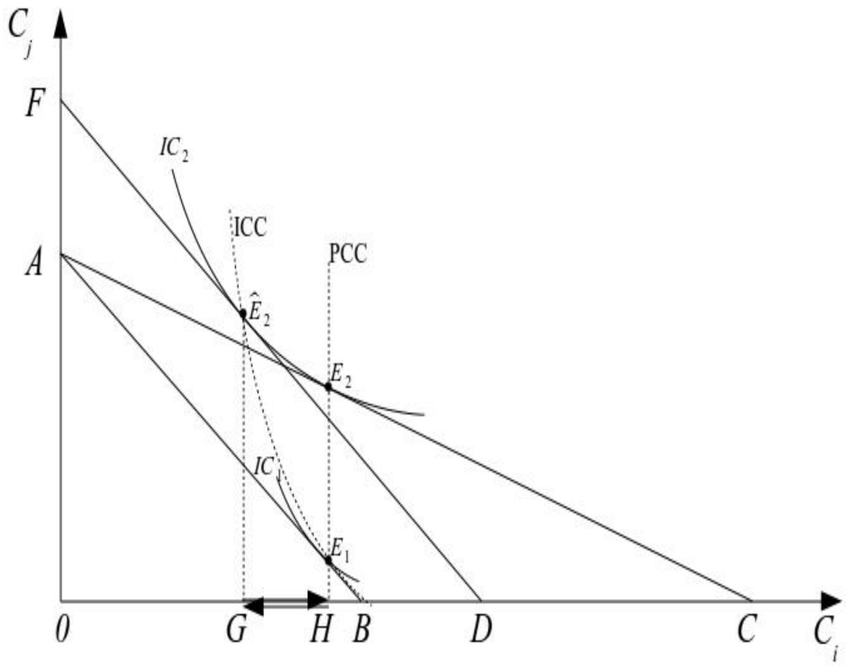
$$\xi_i \equiv \frac{P_i}{C_i}\frac{\partial C_i}{\partial P_i}, \quad \bar{\xi}_i \equiv \left(\frac{P_i}{C_i}\frac{\partial C_i}{\partial P_i}\right)_{dU=0}, \quad \theta_i \equiv \frac{P_i C_i}{M}, \text{ and } \zeta_i \equiv \frac{M}{C_i}\frac{\partial C_i}{\partial M}.$$

Since $\frac{\partial C_i}{\partial P_i}\Big|(dU = 0) = \lambda \frac{S_{ii}}{|S|} < 0,$ it is pretty obvious that $\frac{\partial C_i}{\partial P_i} < 0$ (or alternatively, $\xi_i \equiv \frac{P_i}{C_i}\frac{\partial C_i}{\partial P_i} < 0)$ if $\frac{\partial C_i}{\partial M}. > 0$ $(or \;\; \zeta_i \equiv \frac{M}{C_i}\frac{\partial C_i}{\partial M} > 0).$ It must, however, be noted that $\frac{\partial C_i}{\partial P_i}$ can still be negative even if $\frac{\partial C_i}{\partial M}. < 0$, provided the second term on the right-hand side of equation (3) is less dominant (that is, if $\left|C_i \frac{\partial C_i}{\partial M}\right| < \left|\lambda \frac{S_{ii}}{|S|}\right|$).

Now, note that the $i-th$ good is an *inferior* good to a consumer if $\frac{\partial C_i}{\partial M}. < 0$ $(or \;\; \zeta_i \equiv \frac{M}{C_i}\frac{\partial C_i}{\partial M} < 0).$ One may find then that even if the $i-th$ good is *inferior*, the price effect of the $i-th$ good may still be normal. So far, so good!

It is obvious now that if $\frac{\partial C_i}{\partial M} < 0,$ but $C_i\left|\frac{\partial C_i}{\partial M}\right| = \left|\lambda \frac{S_{ii}}{|S|}\right|,$ then $\frac{\partial C_i}{\partial P_i} = 0$ $(i.e., \; \xi_i = 0).$ When $\frac{\partial C_i}{\partial P_i} = 0$ $(i;e., \; \xi_i = 0),$ $i-th$ good is recognized as an absolutely *necessary* good, as its price elasticity of demand is zero. Does this then mean that a perfectly demand-inelastic good (with respect to its own price) which is an absolutely *necessary* good is necessarily an *inferior* good? The usual interpretation of price elasticity of demand seems to suggest that the higher the degree of inelasticity, the higher the degree of necessity of the good, and perfect inelasticity ($\xi_i = 0$ i) signifies that the good concerned is an absolute necessity. But it is pretty hard to accept that an absolutely necessary good (e.g., insulin to a diabetic) is an inferior good. Let us visualize the situation through the following diagram (Figure 1):

Figure 1A

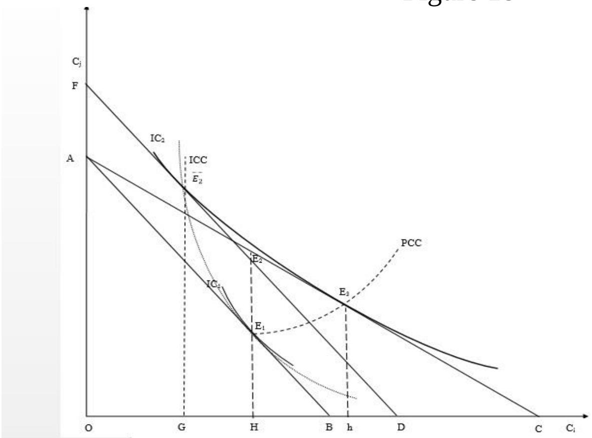


Here, in Figure 1A, *AB* is the original budget line, and the tangency point of the budget line with indifference curve IC_1 at E_1 defines the optimum purchase (OH units of C_i good). Next, the budget line changes from *AB* to *AC*, implying a drop in the price of C_i good alone, and thereby triggering the price effect to move equilibrium to E_2 (the tangency point of *AC* and IC_2). Here E_1E_2 defines the price consumption curve (*PCC*). When price effect is decomposed into income effect, and substitution effect, we move from E_1 to $\hat{E}_2$ (income effect) and from $\hat{E}_2$ to E_2 (substitution effect). In this scenario, $i-th$ good is obviously an inferior good. The income effect, consequent upon the price effect, moves the budget line to *FD* that becomes tangent with IC_2 (at point $\hat{E}_2$), and it triggers the drop in consumption of the $i-th$

good from OH to OG. Here $i-th$ good is indeed an inferior good. Substitution effect along the same indifference curve (IC_2) – measuring the move from $\hat{E}_2$ to E_2 – takes the consumer to E_2. It is a situation where the drop in the price of the $i-th$ good has not induced the consumer to change his consumption of the $i-th$ good at all. Price elasticity of demand is zero, and it is an absolutely necessary good, which is insensitive to price change. It is an inferior good and a necessary good.[ii]

One may now go a step further and argue that a good does not have to be an absolute necessity (as an insulin to a diabetic) to be construed as a necessary good. As long as the absolute value of the price elasticity of demand is less than unity (that is, $\xi_i \equiv \frac{P_i}{C_i} . \frac{\partial C_i}{\partial P_i} < 1$, the *i-th* good is a necessary good. Figure 1B here makes that point. If, in this diagram, the absolute value of $\frac{Hh}{BC}\frac{OB}{OH} < 1$, *i-th* good is a necessary good and concurrently an inferior good. The Slutsky equation has not answered the curious question: is a necessary good *necessarily* an inferior good?

Figure 1b



Despite the well-articulated definitions to the contrary in the existing literature (cited in the references), some people, of course, may consider a good as an absolute necessity if its income elasticity of demand is zero. It then transpires that the *compensated* demand condition
only explains the (absolute) necessity characteristic of a good. More importantly, one should bear in mind the fact that $\zeta_i = 0$ means that the $i-th$ good is a borderline case of inferiority and superiority. Here also the same problem haunts the rational and pragmatic mind: how can a necessary good be strictly non-superior?
If we attempt to reconcile these two definitions of absolute necessity so that they perfectly agree with each other, then we have a case where both $\zeta_i = 0$ and $\bar{\zeta}_i = 0,$ and ξ_i is zero as well. This may occur in the situation of kinked indifference curves (Figure 2).[iii] In fact, one can draw the same scenario with non-intersecting, convex-to-the-origin, downward-sloping indifference curves (Figure 3).

Figure 2

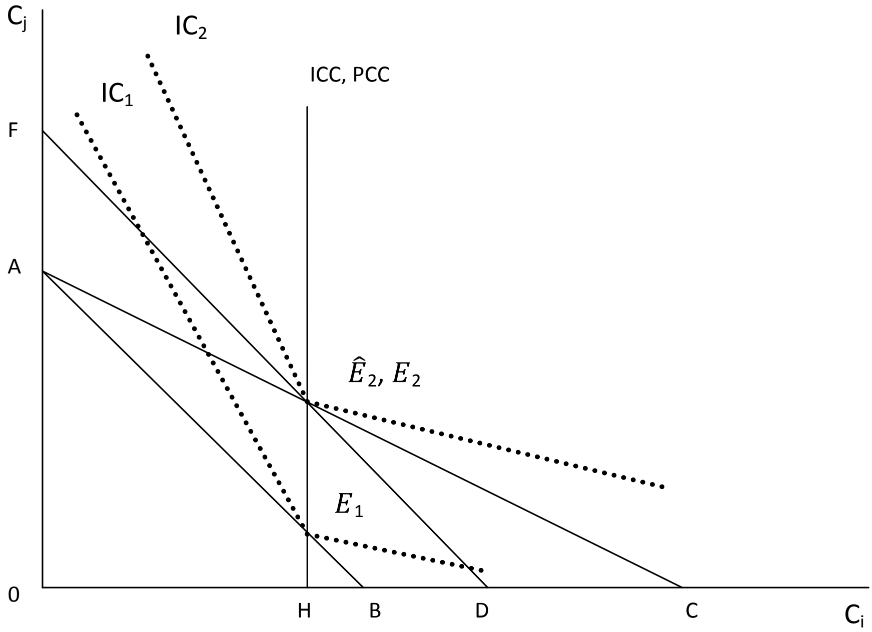
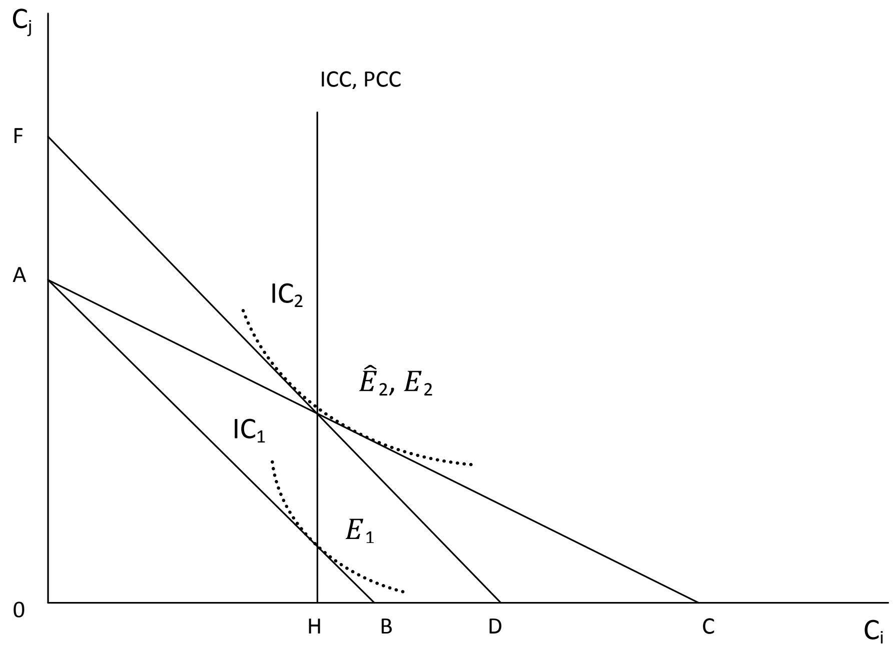


Figure 3



These diagrams (Figure 2 and Figure 3) are self-explanatory in view of the exposition of Figure 1. Kinked indifference curves, as shown in Figure 2, or smooth indifference curves, as shown in Figure 3, will have zero income effect and substitution effect. Normally, according to the Slutsky equation, the price effect should be zero, too.

How general are these cases? We must be wondering how the Slutsky equation has escaped our scrutiny (including that of Paul Samuelson, John Hicks, and every scholar of demand theory and consumer behavior). Does the Slutsky equation or its interpretation explain the anomaly that has baffled us all for such a long time? It is a serious question. The original question as to how a necessary good can be accepted as an inferior (or at best a non-superior) good is a very puzzling issue, and it still seeks a good answer.

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[i] Please see the Postscript on Consumer Behavior in Ghosh [1]. The literature, following Adam Smith's *value-in-use* to further developments in the hands of Say, Lloyd, Senior, Jennings, Dupuit, Gossen, Jevons, Menger, Walras, Edgeworth, Antonelli, and Irving Fisher, can be traced in this Postscript.

[ii] A highly-priced inelastic good (a case where ξ_i is less than unity) can also be an inferior good. One does not in fact have to consider the case of perfect inelasticity. See some well-recognized textbooks—for instance, Mas-Colell, Whinston, and Green [5], Henderson and Quandt [2], Shone [7], Ghosh [1], and Varian [9]—on the definition and characterization of goods.

[iii] Is such a creation a viable option? This is another intriguing issue in consumer behavior. However, it must be noted that in the Slutsky analysis, utility function is continuously twice differentiable. Kinked indifference curves do not satisfy the differentiable space.