

## Opportunity Cost, Excess Profit, and Counterfactual Conditionals

---

Carlo Alberto Magni<sup>\*</sup>

---

### Abstract

Counterfactual conditionals are cognitive tools that we incessantly use during our lives for judgments, evaluations, decisions. Counterfactuals are used for defining concepts as well; an instance of this is attested by the notions of opportunity cost and excess profit (residual income), two all-pervasive notions of economics: They are defined by undoing a given scenario and constructing a suitable counterfactual milieu. Focussing on the standard paradigm and Magni's (2000, 2005, 2009a,b) proposal this paper shows that the formal translation of the counterfactual state is not univocal and that Magni's model retains formal properties of symmetry, additive coherence, homeomorphism, which correspond to properties of frame-independence, time invariance, completeness. Two introductory studies are also presented to illustrate how people cope with these counterfactuals and ascertain whether either model is seen as more "natural". A brief discussion of the results obtained is also provided.

*Keywords:* Opportunity cost, excess profit, residual income, counterfactual, modelling

*JEL classification:* A12, B41, D40, G11, G12, G31, M41

---

<sup>\*</sup>Carlo Alberto Magni, Department of Economics, University of Modena and Reggio Emilia, Italy, e-mail: magni@unimo.it

## 1- Introduction

Counterfactual conditionals are used by individuals in almost every domain of their life. The literature on counterfactuals has considerably increased in the recent past in various disciplines, in particular psychology, in social psychology [Roese and Olson, 1995a; Roese, 1997]) as well as in cognitive psychology [Kahneman and Tversky, 1982; Kahneman and Miller, 1986; Harris, German and Mills, 1996; Byrne and Tasso, 1999; Byrne, 2002]. The role of language in shaping the counterfactual thought has also been investigated [Bloom, 1981, 1984; Au, 1983, 1984; Liu, 1985]; inquiries on the relationships between counterfactuals and causation have long since attracted the attention of philosophers of science [Chisholm, 1946; Will, 1947; Goodman, 1947, 1983; Popper, 1949; Kneale, 1950; Sosa, 1975]; also, the ability of constructing counterfactuals is investigated in medical researches [Hooker, Roese and Park, 2000]. Likewise, economists do make extensive use of counterfactual reasoning, mostly self-unconsciously though. For example, a vast literature is concerned with economic and financial decisions: If it is true that, normatively, von Neumann and Morgenstern's (1944) utility theory maintains its charm among decision theorists and economists, in a descriptive approach the role of counterfactuals in decision-making processes is highly recognized in regret theory [Bell, 1982, 1985; Loomes and Sugden, 1982, 1986], which is gaining ground among cognitive psychologists [Zeelenberg et al., 1996; Zeelenberg and Beattie 1997; Tsiros, 1998]; implications for marketing are also studied [McGill, 2000; Roese, 2000]; further, the way counterfactuals affect economic and financial decisions is investigated [Lundberg and Frost, 1992; McConnell et al., 2000; Tsiros and Mittal, 2000], as well as their impact on a personal domain [Landman and Manis, 1992]. Nevertheless, so far as I am aware, the role of counterfactual thought in the formulation of economic *concepts* has not yet been adequately recognized, let alone the epistemological implications on economic models [but see Sugden, 2000, for an interpretation of economic models as counterfactual worlds; see also Elster, 1978] and, in general, on decision-making processes. The purpose of this paper is to show that counterfactuals may explain the fundamental features of the concepts of opportunity cost and excess profit, two important concepts in economics. In particular, it aims at showing that the *counterfactualization* of a scenario is not univocal.<sup>1</sup> The paper presents both theoretical considerations and experimental results. The first part is theoretical and is devoted to showing that three interpretations of the above mentioned

---

<sup>1</sup> See also Chisholm (1975) and Rescher (1975) on counterfactuals' ambiguities.

concepts are possible: Two of them are *semicounterfactuals*, the third one is a *genuine counterfactual*. The latter is the sole to be represented by a symmetric and additively coherent operator, which is a multiplicative homeomorphism. Conceptually, this represents, respectively, independence from framing, invariance with respect to the unit of time selected, completeness of the counterfactualization. The second part of the paper is experimental: Two introductory studies have been conducted which seem to corroborate the thesis according to which the genuine counterfactual is more likely to be adopted by reasoners. Tentative explanations of these results are offered.

## 2 - Cost and profit

“You face a choice. You must now decide whether to read this [article], to read something else, to think silent thoughts, or perhaps to write a bit for yourself. The value that you place on the most attractive of these several alternatives is the *cost* you must pay if you choose to read this [article] now” (Buchanan, 1969, p.vii, italics supplied). When you calculate the benefit from reading this paper you must then take other available opportunities into account. The most valuable of these alternatives represents the cost of reading the paper. If you say that ‘it is not worth the cost’ you mean that alternatives are at your disposal which you prefer to reading this note. The idea of cost as an *opportunity cost* has been developed by Austrian economists (in particular Ludwig von Mises) as well as by economists of the London School of Economics such as Hayek, Coase, Thirlby, Shackle:

The person is faced with the possibility of taking one or other of (at least) two courses of action, but not both. He considers the relative significance to him of the two courses of action, and finds that one course is of higher significance than the other. He ‘prefers’ one course to the other. His prospective opportunity of taking the less-preferred course of action becomes the prospective cost of his taking the more preferred course. [Thirlby, 1946, pp.33-34]

The cost of doing anything consists of the receipts *which could have been obtained if that particular decision had not been taken*. When someone says that a particular course of action ‘is not worth the cost’, this merely

means that he prefers some other courses ... This particular concept of cost would seem to be the only one which is of use in the solution of business problems, since it concentrates attention to the alternative courses of action which are open to the businessman. Costs will only be covered if he chooses, out of the various courses of action which seem open to him, that one which maximizes profit. [Coase, 1938, p.123, as quoted in Buchanan, op.cit., italics supplied]

In this light, cost is not money outlay but an outcome that might occur (*ex ante* analysis) or that might have occurred (*ex post* analysis) if the decision maker selected or had selected a different course of action.<sup>2</sup> Cost is then “subjective, it exists in the mind of the decision-maker and nowhere else” [Buchanan, op.cit., p.43], for “outcomes cannot be matters of fact but are things imagined by the decision-maker. They exist in his imagination” [Shackle, 1961, pp.ix-x], outcomes “are figments of imagination ... figments of the individual mind (no matter of whether in some later actuality they shall be observed to have come true)” (ibidem, pp.9-10).

Thus, the concept of cost has a distinctive counterfactual characterization: It is the outcome the decision maker would receive (have received) if she undertook (had undertaken) a different course of action. One would be well advised to distinguish (opportunity) cost and (excess) profit in economics from cost and profit in accounting. In accountancy cost refers to expenditures and profit is given by the difference between revenues and costs. Suppose a firm undergoes an 80-euros expense obtaining, at the end of the period, total revenue receipts of 100 euros; the (accounting) cost is 80 and the (accounting) profit is  $100 - 80 = 20$ . In this sense, ‘profit’ is conceptually a synonym of ‘return’. Conversely, in economics, cost is generated by a counterfactual alternative the decision maker takes as a reference point. Excess profit is given by the difference of accounting profit and the value of the counterfactual course of action. In the specific case, assuming that 15 is such a value, the (excess) profit will be  $20 - 15 = 5$ . In other words, 20 is the benefit (measured in money value) of the course of action at hand, 15 is the value of the alternative course of action (the ‘lost opportunity’), the difference between these two gives the (excess) profit. Therefore, for an economist “to cover costs and to maximize profits are essentially two ways of expressing the

---

<sup>2</sup> For other notions of cost see Buchanan, op.cit.

same phenomenon” [Coase, *op.cit.*, p. 123]. Many synonyms have been coined to mean ‘excess profit’: ‘excess realizable profit’ [Edwards and Bell, 1961], ‘excess income’ [Kay, 1976], ‘abnormal earnings’ [Peasnell, 1981], ‘supernormal profit’ [Begg, Fischer, and Dornbusch, 1984, p.121], ‘residual income’ [Solomons, 1965] and others. The concept of ‘Goodwill’ [Preinreich, 1936] is also strictly related to that of excess profit, and the term ‘economic profit’ is widespread in finance [e.g. Damodaran, 2007, p.42].

I will henceforth use the term ‘excess profit’ or ‘residual income’ interchangeably, and keep the term ‘profit’ in an accounting sense.<sup>3</sup>

It is worthwhile noting that such a definition of excess profit makes the alternative course of action to act as a norm in the sense of Kahneman and Miller (1986). The expression ‘normal profit’ has been often used to mean opportunity cost [see Edey, 1957; Bodenhorn, 1964; Carsberg, 1966; Archer and D’Ambrosio, 1972; Begg, Fisher and Dornbusch, 1984].

Cost and excess profit are but derivations of counterfactual alternatives or norms subjectively constructed by the evaluator. Typically, the concept of excess profit is deduced from a comparison between the profit an economic agent achieves and the profit she could achieve (have achieved) if she undertook (had undertaken) a different course of action.

Let us consider an example. Consider firm Alpha and suppose its net worth is 100 and its prospective rate of return for next period is 10%. Its prospective profit is then

$$100(1.1) - 100 = 0.1 \cdot 100 = 10.$$

Suppose now that an alternative course of action is at its disposal at the return rate 9%. In the latter case, the firm would generate a profit of

$$100(1.09) - 100 = 0.09 \cdot 100 = 9.$$

The excess profit for Alpha is  $10 - 9 = 1$ . The notion of excess profit emphasizes “the role of distance from an ideal or paragon as a determinant of typicality” [Kahneman and Miller, 1986, p.143].

---

<sup>3</sup> I will use the term profit as excess profit only when the context makes it unambiguous.

This correspondence between opportunity cost and norm is also reflected in economic decisions concerning investments. In this case the norm is given by a (comparable<sup>4</sup>) alternative investment, which acts as a benchmark for acceptability. Suppose a decision maker faces the opportunity of undertaking a one-period project consisting of an initial outlay of 60 with a rate of return of 15%. In evaluating such an investment and in deciding whether to accept it or reject it the evaluator compares such a course of action with another course of action open to him. Suppose the investor may alternatively invest the same sum in a one-period project whose rate of return is 11%.<sup>5</sup> Then  $0.11 \cdot 60 = 6.6$  is the (opportunity) cost of undertaking the former project. The net payoff generated by the project at hand (the excess profit) is then given by  $0.15 \cdot 60 - 0.11 \cdot 60 = 9 - 6.6 = 2.4$ . In general, things may be framed in the following way: Let  $a_0$  be the initial outlay of the project under examination,  $x$  be the rate of return,  $i$  be the rate of return of the counterfactual alternative. The investment is profitable if its residual income is positive, namely if

$$xa_0 - ia_0 > 0. \quad (1)$$

Eq. (1) can be rewritten as

$$a_0(1+x) - a_0(1+i) > 0. \quad (2)$$

The left-hand member of eq. (2) is known, in financial economics, as Net Future Value (NFV) of the project ('excess return' is also used. See Damodaran, 2007, Young and O'Byrne, 2001). Dividing both members by  $(1+i)$  we have

$$-a_0 + \frac{a_0(1+x)}{(1+i)} > 0. \quad (3)$$

---

<sup>4</sup> 'Comparable' often means 'equivalent-risk'.

<sup>5</sup> The alternative rate of return is often called, in the financial literature, 'opportunity cost of capital', though this expression should be properly used to mean the *value* placed on the alternative course of action.

The left-hand side of eq. (3) is called the Net Present Value (NPV) of the project. The net-present-value rule states that a project is profitable if its NPV is positive. The idea of maximizing NPV is standard in financial economics [see Brealey and Myers, 2000; Rao, 1992; Finnerty, 1986; Copeland and Weston, 1988] and traces back to Fisher (1930), whose analysis is carried out under assumption of certainty [see MacMinn, 2005, for the Fisher model under uncertainty]. The idea of net present value maximization is commonplace in economic theory: “The firm attempts to maximize the present value of its net cash flow over an infinite horizon” [Abel, 1990, p.755]; “the net present value rule is also the basis for the neoclassical theory of investment ... Much of the theoretical and empirical literature on the economics of investment deals with issues of this sort” (Dixit and Pindyck, 1994, p.5). The NPV analysis is equivalent to that of Jorgensen (1963) and to the  $q$  theory of investment [Tobin, 1969]: “In all of this, the underlying principle is the basic net present value rule” [Dixit and Pindyck, 1994, p.5] and “the net present value rule is appropriate for decision making” [MacMinn, 2005, p.1]. The NPV of a project is then nothing but a mathematical transformation of the concept of excess profit. The term  $ia_0$  in eq. (1) is the cost of undertaking the project, that is, the profit the investor would attain (would have attained) if he undertook (had undertaken) the alternative course of action. The latter is a yardstick with which the project at hand must be compared for evaluation and decision purposes.

### 3 - Conditionals and the concepts of cost and (excess) profit

Counterfactual conditionals are interspersed in private reasoning during the lives of each individual, serving several functions [Roese and Olson, 1995b]. In the economic domain counterfactuals are one of the main ways economists adopt to explore the world [see Sugden, 2000].<sup>6</sup> In particular, counterfactuals (and norms) are used by economists in order to *define* concepts: The notions of costs and (excess) profit make sense only if any judgmental comparison is involved. We can say that economists “do not wish to explain an outcome per se but rather the divergence between an outcome and some default alternative” [Roese and Olson, 1995c, p.13]. But, whilst norm theory suggests that individuals’ “norms are computed after the

---

<sup>6</sup> Other important ones are regression and experiments. The latter tool, not frequently used in the past, is gaining increasing attention among scholars [see Hey, 1991].

event rather than in advance” [Kahneman and Miller, 1986, p.136], in economics a norm is evoked in order to formulate the very definition of an economic concept.

The state of affairs (being firm Alpha) or the course of action (undertaking a project) does not provide any information on cost and excess profit unless it is accompanied by an alternative state of affairs or course of action serving as a benchmark.<sup>7</sup> To this end, economists rely on both ordinary language and mathematics. Linguistically, cost is connected with the following counterfactual conditional:<sup>8</sup>

*If I (had) selected the alternative course of action, my return would be (have been) ... (4)*

This conditional statement needs to be converted in mathematical symbols. It seems that the construction of the above counterfactual should be an easy task: Opportunity cost is made to coincide with the profit corresponding to the alternative state, and residual income is just the difference between the profit released by the action selected (factual profit) and the profit released by the counterfactual alternative (counterfactual profit):

$$\text{EXCESS PROFIT} = \text{factual profit} - \text{counterfactual profit.} \quad (5)$$

Because profit is given by the rate of return times the capital invested, excess profit is given by

$$\begin{aligned} \text{EXCESS PROFIT} &= \text{factual (rate of return} \cdot \text{capital invested)} \\ &- \text{counterfactual (rate of return} \cdot \text{capital invested).} \end{aligned} \quad (6)$$

One just has now to substitute words with symbols (and then numbers) and the translation from ordinary language to mathematics is complete. The following section shows the way the factual situation can be undone is not unique.

---

<sup>7</sup> With no such alternative, excess profit turns into accounting profit and cost turns into expenditure.

<sup>8</sup> I do not distinguish among prefactual, nonfactual and counterfactual conditionals. I will always refer to them as counterfactuals; such a distinction is quite irrelevant to the analysis (counterfactual interpretation is possible even in indicative mood: see Dudman, 1988).

#### 4 - Three counterfactual interpretations

The paradigm accepted in the literature as the correct formal translation of the notion of excess profit comes from the financial and accounting literatures. The conventional approach has been forerun by Preinreich (1938), who states that “Capital value equals the book value, plus the discounted excess profits” (p. 240. See also his eq. (57)). This statement was then proved by Edwards and Bell (1961, chapter 2, Appendix B), who use the term ‘excess realizable profit’. The term ‘residual income’, widely used in corporate finance and management accounting, appears for the first time in Solomons (1965, p. 63). Important studies have been conducted by Peasnell (1981, 1982) and Ohlson (1989, 1995), who have triggered a renewed interest in this notion, which is now used for various purposes: Valuation, incentive compensation, capital budgeting [see Arnold and Davies, 2000; Martin and Petty, 2000].<sup>9</sup> Many metrics have been generated complying with this approach, and many others may be in principle constructed, but the individual differences among these measures are immaterial to our subject, depending on the choice of book values as opposed to market values, and the choice of an equity perspective as opposed to an entity one. They all share the same modelling approach. The most widely known of these models is Stewart’s (1991) Economic Value Added (EVA).<sup>10</sup> EVA is

the leading example of a new class of metrics that attempt to measure an underlying concept called residual income. Recognized by economists since the 1770s, residual income is based on the premise that, in order for a firm to create wealth for its owners, it must earn more on its total invested capital than the cost of the capital. Whereas traditional accounting net income measures “profits” ... residual income measures

---

<sup>9</sup> Independently, Peccati (1987, 1989, 1991) has developed a model of decomposition of a project's NPV, which is consistent with the standard approach.

<sup>10</sup> EVA has been attracting increasing attention among scholars and practitioners and is presented in any modern finance textbook [e.g., Damodaran, 1999; Brealey and Myers, 2000, Fernández, 2002]. The literature on it has increased exponentially in the past fifteen years. It is so common that even many dictionaries report it as an entry [see, for example, the *Longman Business English Dictionary*, 2000, Pearson Educational Limited, or the *Dictionary of Accounting Terms*, 2001, Barron’s Educational Series, 3rd edition].

“profits” net of the ... cost of ... equity capital.<sup>11</sup> [Biddle, Bowen and Wallace, 1999].

Stewart’s EVA is currently used as a measure of projects’ periodic performance, as an index for evaluating firms, as a tool for forecasting asset prices, as a yardstick for rewarding managers. The hub of Stewart’s methodology is just that of calculating a periodic excess profit on the basis of eq. (6): A straightforward definition of EVA is

EVA = Net Operating Profit After Taxes – Cost of Capital \* Capital Invested.

The latter coincides with eq. (6), because ‘Cost of Capital’ is the rate of return of an alternative course of action (the reference to Taxes is not relevant to our subject). The acronym EVA is often used, in a general sense, to denote the very excess profit [see Rogerson, 1997; Ehrbar, 1998; Magni, 2001, 2005; Stoughton and Zechner, 2007].

In a vast array of papers, Magni (2000, 2001, 2003, 2004, 2005, 2006, 2009b) has shown that the standard paradigm of excess profit not the only way of interpreting the notion of excess profit; that is, EVA is only one possible way of using eq. (6). This claim is proved by providing an alternative paradigm, named Systemic Value Added (SVA) or lost-capital paradigm [Magni, 2009a, 2009b], which conforms to eq. (6) as well. As a matter of fact, it is here shown that a third interpretation is possible.

We will work on the following scenario and describe three alternative ways of coping with the corresponding counterfactual situation.

**Scenario 1.** Firm Alpha, operating in sector A, has a net worth of 100 yielding a 10% return each year. If firm Alpha operated in sector B its rate of return would be 8%.

*Question (a).* What profit will Alpha get in the third year?

*Question (b).* What profit would Alpha get in the third year if it operated in sector B?

---

<sup>11</sup> The cost of equity capital is just our counterfactual profit.

Note that the answers to (a) and (b) are just the two terms in eq. (6), namely the factual profit and the counterfactual profit.

#### 4.1 Answers: Type I

*Question* (a). Alpha's capital increases by an annual 10%, starting from 100. So, we have

capital at time 0	100
capital at time 1	$110=100(1+0.1)$
capital at time 2	$121=110(1+0.1)$ .

The profit in the third year is  $0.1 \cdot 121=12.1$ .

*Question* (b). If Alpha operated in sector B, it would earn 8% on its capital, so the third year's profit would be  $0.08 \cdot 121=9.68$ .

#### 4.2 Answers: Type II

*Question* (a). As in Type I

*Question* (b). If Alpha operated in sector B, then its capital would raise by an annual 8% rate of return, so that

capital at time 0	100
capital at time 1	$108=100(1+0.08)$
capital at time 2	$116.64=108(1+0.08)$

This means that the firm would earn, in the third year, its 10% on a capital of 116.64. The third year's profit would then be  $0.1 \cdot 116.64=11.664$ .

#### 4.3 Answers: Type III

*Question* (a). As in Type I

*Question* (b). If Alpha operated in sector B, it would earn an annual 8% on its capital. The value of the latter would thus increase by 8% so that it would equal 116.64 at the beginning of the third year. On 116.64 the firm would earn an 8% so that the third year's profit would be  $0.08 \cdot 116.64=9.33$ .

Type I is the standard way of formally translating eq. (6) (EVA). Type III is Magni’s lost-capital excess profit, Type II is another different way of interpreting eq. (6), not considered in the literature.

### 5 - Semicounterfactuals and genuine counterfactuals

To understand the cognitive implications of the three types of counterfactuals and their relationships with mathematics, let us have a closer glance to eq. (6). It says that the excess profit depends on the factual profit and on a counterfactual profit. Both factual profit and counterfactual profit depend on the relation existing between two elements: The rate and the capital. Denote with  $a$  and  $b$  such elements and with  $*$  the operator linking them (we may read  $a*b$  as “ $a$  is applied to”): In our case, a multiplication. We could rewrite eq. (6) as

$$F(a*b) - C(a*b)$$

where the operators  $F$  and  $C$  mean, respectively, ‘factual’ and ‘counterfactual’.

Let us focus on the counterfactual term  $C(a* b)$ . It says that we have to *counterfactualize* the situation; that is, we have to undo a scene whose characters are  $a$  and  $b$ . This can be accomplished in three ways. The first method (Type I) is to undo  $a$  and then relate it to the factual  $b$ . The second way (Type II) is to undo  $b$  and then relate it to the factual  $a$ . The third way (Type III) is to undo both  $a$  and  $b$  and then relate them. In symbols, we have

Type I	$C(a*b) = C(a) * F(b)$
Type II	$C(a*b) = F(a) * C(b)$
Type III	$C(a*b) = C(a) * C(b)$

Type I assumes that capital in the third period is the factual one, and is concerned with a change in the rate of return. Type II is interested in changing the capital while maintaining the factual rate of return. Type III undoes both rate of return and capital. Type I and Type II only partially change reality maintaining one of the two features in the factual mode, so they deserve to be named semicounterfactuals (the scenario is undone *ceteris paribus*); Type III

shifts from reality to a counterfactual realm where both elements are ruled out and replaced by their counterfactual counterparts, so it may be regarded as a genuine counterfactual (the scenario is undone *mutatis mutandis*). Note that the completeness of the counterfactualization in Type III may be mathematically interpreted by saying that operator C is a multiplicative homeomorphism in the same sense as the power operator is a homeomorphism with respect to the product operator: Letting  $N$  be the power operator we have  $N(ab)=N(a)N(b)$  (that is,  $(ab)^t = a^t b^t$  for any real  $t$ ).

The three ways of counterfactualizing the same scenario grow out of different interpretations of Question (b). The three (cognitive and) mathematical interpretations can be linguistically represented by the following statements:

#### Scenario 1

Type I (undoing rate). What profit would Alpha get in the third year if it operated in sector B *in that year* ?

Type II (undoing capital). What profit would Alpha get in the third year if it operated in sector B *up until the third year*?

Type III (undoing both rate and capital). What profit would Alpha get in the third year if it operated in sector B *each year*?

or, equivalently,

#### Scenario 1

Type I (undoing rate). What profit would Alpha get in the third year if *in that year its rate of return were that of a sector B's firm*?

Type II (undoing capital). What profit would Alpha get in the third year if *in that year its capital were that of a sector B's firm*?

Type III (undoing both rate and capital). What profit would Alpha get in the third year if it *earned each year* like a sector B's firm?

The existence of Type I, Type II, and Type III shows that it is not obvious how the scenario should be undone in constructing counterfactual alternatives. Type I is implicitly regarded, in the financial and accounting literature, as the only possible interpretation and thus the correct interpretation of eq. (6). In fact, Type I is only one of three possible interpretations of the counterfactual alternative and one may characterize it as a semicounterfactual way of looking at what might be (have been). Moving from ordinary language to mathematics, one can say that for the undoing of the scenario to be complete (genuine) one needs an operator which is a homeomorphism with respect to the operator linking  $a$  and  $b$  (i.e., the operator \*).

In the next sections it will be shown that, though legitimate, Type I and Type II reveal some oddities.<sup>12</sup>

---

<sup>12</sup> Scenario 1 is only a particular case of a more general pattern of situations where the counterfactual state of affairs is not unambiguous. The following scenario is an equivalent (noneconomic) example of how counterfactual situations may have multiple interpretations.

**Scenario 2.** Naima is a Swedish six-year-old child. She is 100 cm tall. For the next years (say, five), she is expected to grow in stature at a rate of 10% a year. If she were Italian, the increase rate would be 8% a year.

*Question (a).* What increase in stature will Naima have in her 9th year?

*Question (b).* What increase in stature would Naima have in her 9th year if she were Italian?

In Scenario 2 a child named Naima replaces firm Alpha of Scenario 1, and the notions of 'stature' and 'increase rate' replace the notions of 'capital' and 'rate of return', respectively, but the cognitive framework is identical. The corresponding linguistic representations are:

Type I (undoing increase rate). What increase in stature would Naima have in her 9th year if *in that year her increase rate were that of an Italian child*?

Type II (undoing stature). What increase in stature would Naima have in her 9th year if *in that year her stature were that of an Italian child*?

Type III (undoing both increase rate and stature). What increase in stature would Naima have in her 9th year *if she grew each year like an Italian child*?

## 6 - Summing excess profits

If one assumes that a semicounterfactual interpretation is acceptable, one encounters some oddities I now focus on. But first, suppose we were to compute the whole factual profit earned by firm Alpha in the span of three periods. We can tackle the task in two ways: One way is to directly compute the whole profit as the difference between Alpha's net worth after three years and its net worth at the outset. At the outset, net worth is 100; at the end of the period it is  $100(1.1)^3=133.1$  (the initial net worth plus all annual profits), so the whole profit is  $133.1-100=33.1$ . The second way is to explicitly sum the annual profits: They are, respectively,

$$\begin{aligned} \text{first year's profit} &= 0.1 * 100 = 10 \\ \text{second year's profit} &= 0.1 * 110 = 11 \\ \text{third year's profit} &= 0.1 * 121 = 12.1. \end{aligned}$$

Summing the three shares, one finds back  $10+11+12.1=33.1$ . Let us now focus on the counterfactual case and suppose we want to calculate the whole counterfactual profit (i.e., the whole profit generated in the span of three periods by the counterfactual scenario). I now show that the coincidence just found for the factual profit keeps valid for the counterfactual profit only if Type III is adopted. Indeed, if one computes the whole counterfactual profit directly, one obtains  $100(1.08)^3-100=25.9712$ , irrespective of which interpretation of excess profit we rest on. We would expect that the latter be found as a sum of the periodic counterfactual profits as well. Strangely enough, this is not the case for Type I and Type II. We have

year	Type I $C(a)*F(b)$	Type II $F(a)*C(b)$	Type III $C(a)*C(b)$
1 <sup>st</sup>	$0.08*100=8$	$0.1*100=10$	$0.08*100=8$
2 <sup>nd</sup>	$0.08*110=8.8$	$0.1*108=10.8$	$0.08*108=8.64$
3 <sup>rd</sup>	$0.08*121=9.68$	$0.1*116.64=11.664$	$0.08*116.64=9.3312$

If one sums the three shares one finds

$$8+8.8+9.68 = 26.48 \neq 25.9712$$

for Type I,

$$10+10.8+11.664 = 32.464 \neq 25.9712$$

for Type II, and

$$8+8.64+9.3312 = 25.9712$$

for Type III. The whimsical results of Type I and Type II tell us something about the way the scenario is counterfactualized. In particular, let  $P$  be the whole profit (factual or counterfactual) and  $P_s$  the  $s$ -th year profit (factual or counterfactual); then  $P = \sum_{s=1}^3 P_s$  is the whole profit. Denoting with  $F(P)$  the whole factual profit and with  $C(P)$  the whole counterfactual profit, one finds that the operator  $F$  is additively coherent:

$$F(P) = \sum_{s=1}^3 F(P_s)$$

whereas the operator  $C$  is (additively) noncoherent for Type I and II and coherent for Type III. That is, only for the latter we have:

$$C(P) = \sum_{s=1}^3 C(P_s).$$

The noncoherence means that there is a cognitive discrepancy between the way in which the scenario is undone for the whole length of time and the way in which the scenario is undone period by period. To put it differently, consider our firm whose capital is 100 at time 0 and assume that the capital becomes 125.9712 after one period, if the counterfactual course of action is employed. The excess profit is evidently 25.9712. Now, take the third part of this period as unit of time and consider an 8% per period return. Type III is invariant with respect to this change, that is calculating the three excess profits and summing one gets to 25.9712. In other terms, it is irrelevant which unit of time one uses for computing excess profit. Unlike Type III, Type I and Type II are not invariant with respect to the unit of time selected, since the residual income calculated with the new unit of time differs from the previous one.

Additive coherence for a residual income measure is equivalent to the so-called property of earnings aggregation in accounting [see Penman, 1992, for the importance of such a property. See also Ohlson, 1995]. In Type III, the total abnormal earnings aggregated lead to the Net Future Value. In terms of forecasting errors, Type III enables to neutralize the role of timing in forecasting: it is not necessary to know when residual incomes are generated.

This induces a simple heuristic for valuation: rather than forecasting each and every residual income one may rest on an average residual income [see Magni, 2009b].

## 7 - Symmetry

In the previous example normality was predetermined, that is the reference point was explicitly stated. We will now turn to a decision-making process where the reference point depends on the way a scenario is depicted and will apply the three approaches to it.

Consider the two following ways of describing the same scenario:

**Scenario 3.1.** Roberta faces the opportunity of investing 100 in project A, whose annual rate of return is 10%. Alternatively, she can invest the same amount in project B yielding an annual 8%.

*Question (a).* Suppose Roberta chooses A: What is the profit in the third period?

*Question (b).* What would the profit be in the same period, should Roberta choose B?

**Scenario 3.2.** Roberta faces the opportunity of investing 100 in project B, whose annual rate of return is 8%. Alternatively, she can invest the same amount in project A yielding an annual 10%.

*Question (a).* Suppose Roberta chooses B: What is the profit in the third period?

*Question (b).* What would the profit be in the same period, should Roberta choose A?

Table 1 collects the answers for each Type of counterfactual.

Table 1: Two frames for Scenario 3

Scenario	Question	Type I	Type II	Type III
3.1	(a)	(0.10)(121.00)	(0.10)(121.00)	(0.10)(121.00)
	(b)	(0.08)(121.00)	(0.10)(116.64)	(0.08)(116.64)
3.2	(a)	(0.08)(116.64)	(0.08)(116.64)	(0.08)(116.64)
	(b)	(0.10)(116.64)	(0.08)(121.00)	(0.10)(121.00)

Note that Scenario 3.1 and Scenario 3.2 are just the same scenario but the framing is changed. In the former description B acts as the reference point and A is the project under consideration, in the latter the role is reversed. So the factual (counterfactual) course of action in Scenario 3.1 turns to be the counterfactual (factual) alternative in Scenario 3.2. Therefore, questions (a) and (b) in 3.1 are, respectively, questions (b) and (a) in 3.2. The symmetry of the situation is well reflected in Type III, where the factual (counterfactual) profit of 3.1 coincides with the counterfactual (factual) profit of 3.2. Conversely, Types I and II are asymmetric and as Table 1 shows, there is a sort of *duality* between either Type: Type I's counterfactual alternative of 3.1 (3.2) is just Type II's counterfactual alternative of 3.2 (3.1). So Types I and II are whimsical ways of undoing scenarios since, if adopted, they lead to different evaluations of the same situation. In terms of excess profit, this means that while Type III keeps the same residual income (in absolute value) regardless of the way the decision process is represented (look at the symmetry of the last column of the table), each of the other ones leads to two excess profits differing in absolute value. Adopting Types I and II our Roberta would compute different excess profits depending on how she describes the decision process.

This awkward result seems to suggest that advocates of Type I and Type II should suffer from a frame-dependent cognitive illusion [Tversky and Kahneman, 1981; Kahneman and Tversky, 1984]. Types I and II seem

therefore to be examples of counterfactuals which “leave people subject to biases and to errors of judgement” [Sherman and McConnell, 1995, p. 203].<sup>13</sup>

## 8 - Study 1

The purpose of this study was to determine which one among Types I, II and III is the more natural way of thinking of cost and (excess) profit and therefore to determine whether or not people adhere to the standard definition as given in the financial literature.

**Participants** The sample was composed by 104 Italian second-year students of the Faculty of Economics of the University of Modena and Reggio Emilia. Forty-three (41%) students were female and 61 (59%) were male, with a mean age of 21, ranging from 20 to 28. They all had previously attended a first course in economics and in accounting. The subjects participated in partial fulfilment of financial calculus course requirements. Forty-four out of the 104 students had been exposed, during the course, to the standard paradigm, that is to Type I interpretation. The remaining 60 had had no exposure to Type I interpretation. No one had had any exposure to Type III interpretation (Magni’s lost-capital paradigm) nor to Type II.

**Procedure** Each subject received Scenario 1 in my presence. The task was administered in the classroom. Participants have been given 25 minutes to complete the task but all of them finished within 15 minutes. Any answer was assigned to one of the three types on the basis of the solutions shown in Section 3. Any other different answer (included incomplete or inconsistent solutions) has been classified as undefined.<sup>14</sup>

**Results** Results are collected in Table 2 (percentages have been rounded). I expected that exposed students would show a greater inclination to engage in a Type I interpretation. Contrary to expectations, the percentage of subjects giving a Type III interpretation is very high for both nonexposed and exposed students. In particular, the latter show a very high percentage of

---

<sup>13</sup> Another relevant situation where the point of reference is not predetermined regards two firms or two business units, whose performance is measured by comparison with the other one’s performance.

<sup>14</sup> Two students previously exposed to Stewart’s model provided both Type I and Type III solutions: Their answers have been classified as undefined.

genuine-counterfactual interpretation (84%). It seems that individuals do not find the standard notions of cost and excess profit as the natural ones. They seem to find Type III approach more appropriate as a definition of excess profit (so doing, they prevent themselves to be hidden in the cognitive trap we have seen in section 7). Note that Type II has been completely neglected by students.

*Table 2: Cognitive interpretation of Scenario 1*

Sample	Response rate (number)			
	Type I	Type II	Type III	Undefined
Exposed students	5% (2)	0% (0)	84% (37)	11% (5)
Nonexposed students	2% (1)	0% (0)	93% (56)	5% (3)

## 9 - Study 2

This experiment deals with a situation where an economic agent undertakes an investment consisting of withdrawing 100 from a current account and reinvesting two cash flows at time 1 and 2 respectively in the same account. The scenario is just a little more complex than Scenario 1 and the norm is given by the current account. The purpose is that of determining whether raising the degree of difficulty of the task encourages a change in the way individuals cognize the concepts of cost and profit.

The scenario is the following:

**Scenario 4.** A decision maker’s wealth amounts to 100, which is invested in a current account X yielding an annual rate of return of  $i=10\%$ . She faces the opportunity of undertaking the following investment: Withdrawal of 40 from account X and

investment of that sum on a current account Y yielding an annual rate of return of 15%; from Y she will withdraw, after one year and two years, the sums 26 and 23 respectively. The sums will be immediately reinvested in account X.

Suppose now the decision maker has undertaken investment Y.

*Question (a).* What is the profit in the second year?

*Question (b).* What would the profit have been in the second year if the investment had not been undertaken?

The solutions are given in the Appendix.

**Participants** The sample was composed by 112 Italian second-year students of the Faculty of Economics of the University of Modena and Reggio Emilia. Fifty-three (47%) students were female and 59 (53%) were male, with a mean age of 21, ranging from 19 to 23. They all had previously attended a first course in economics and in accounting. They participated in partial fulfilment of financial calculus course requirements. Eighty-one students out of the 112 had been taught the standard theory (Type I). The remaining 31 had had no exposure to Type I interpretation. No one of the students had had exposure to Type III interpretation (Magni's model) nor to Type II. The participants to this experiments have not participated to Experiment 1.

**Procedure** Each subject received Scenario 4 in my presence. The task was administered in the classroom. Subjects have been given 70 minutes to complete the task but most of them finished within 50 minutes. Any answer was assigned to one of the types on the basis of the solutions given in the Appendix. Any other solution (included incomplete or inconsistent solutions) has been classified as undefined.

**Results** Results are shown in Table 3 (percentages have been rounded). They mirror the results found for Scenario 1. A more complex milieu does not change the way cost and (excess) profit are cognized by individuals. A higher percentage of undefined answers is to be ascribed to a high number of incomplete or incorrect solutions, owing to the higher degree of difficulty of the task. Even exposed students keep on undoing scenarios in a genuinely counterfactual way (73%): They cleave to Type III even though the knowledge of the standard theory should encourage a standard interpretation.

Overall, ruling out the undefined answers, we see that 94% of exposed students (95% in Scenario 1) cleave to Type III interpretation. Type II is totally neglected, as it was in Scenario 1.

**Table 3: Cognitive interpretation of Scenario 4**

Sample	Response rate (number)			
	Type I	Type II	Type II	Undefined
Exposed students	5% (4)	0% (0)	73% (59)	22% (18)
Nonexposed students	0% (0)	0% (0)	68% (21)	32% (10)

## 10 - Discussion

The way Scenarios 1 and 4 have been undone by the subjects tested is radically different from the way financial economists are accustomed to undo the same scenarios in order to provide the notions of cost and excess profit. As we have seen, there are three possible ways of altering the scenarios. Financial economists focus on the rate of return and render the capital an immutable component. According to this view (Type I), the information conveyed relates to what might occur (have occurred) if the rate of return were (had been) different from the factual one while maintaining the capital in the factual mode. Financial economists iterate this reasoning for each period, that is they focus on a single period forgetting the events of the preceding periods (viz., forgetting that if the firm invested in the alternative business/sector, in the preceding years the rate of return would be different and thus the value of the capital would be different). Conversely, the subjects tested seem to focus on the entire wealth of the economic agent and its evolution through time. They take into account the whole story: If the firm invested in the alternative business, its capital would be different at each time, so the features to be mutated are both the rate of return and the capital.

It is not easy to ascertain the reasons why financial economists prefer Type I interpretation. They seem to construct a cognitive representation where different courses of action are associated to different rates of return but not to different capitals. At each period the capital is maintained in the factual mode and the rate is altered to the counterfactual mode. When interpreting the question

*What profit would Alpha get in the third year if it operated in sector B?*

reasoners have to retrieve the following cognitive representation:

RATE\*CAPITAL.

Financial economists focus on the first factor, whereas other people seem to focus on both factors. Linguistically, this means that the antecedent

*if it operated in sector B*

is intended as referred only to one year (the third year in our case) by financial economists, to the entire span by individuals. In this sense, we could say that financial economists focus their attention on the period rather than on the whole duration of the course of action. The reason might lie in the higher degree of mutability of a controllable event [Giroto et al., 1991]. Financial economists seem to find it easier to think of an alteration of the rate rather than a change in capital, which is perhaps regarded as an uncontrollable feature of the scenario, and, as a result, less mutable than the rate of return. But why then is capital regarded as an uncontrollable feature? It might be that financial economists, contrary to what is currently thought, are not concerned with what the decision maker's profit would be if she undertook a different course of action, but with what the profit *of a different agent* would be if she held the same (factual) capital of the decision maker. If so, they are not comparing the decision maker's profit to the profit that very decision maker would earn if she undertook another course of action; instead, they are comparing the decision maker's profit to the profit of some other agent owning the same capital as the decision maker's. In other terms, their decision maker does not seem to translate a counterfactual conditional such as "If I

selected the counterfactual course of action, then my return would be ...”, but, rather, a counterfactual such as “If an agent different from myself held my capital and selected a course of action corresponding to my counterfactual course of action, then her return would be...”. At any rate, it seems that people are reluctant to engage in such interpretation. They seem to have a broader perspective, they defocus the situation while undoing the scenario, and mutate both features, rate and capital. People do associate alternative courses of action with alternative rates of return, but they also take into consideration that the choice of a specific alternative made at the outset determines a specific evolution of capital. At time 0 the same capital can be invested at either rate of return, but once decision has been made the value of capital will evolve accordingly, so that at each period the counterfactual rate of return must be applied to a counterfactual capital in order to accomplish the counterfactualization. If the firm invested in the alternative sector, it would not only invest net worth at the annual 8%, but also that rate would be applied to a capital which would increase by an annual 8% (not an annual 10%). In terms of mental model theory [Johnson-Laird, 1983], this amounts to saying that financial economists’ mental model of the counterfactual state may be represented as

COUNTERFACTUAL RATE  
FACTUAL CAPITAL<sup>15</sup>

whereas our subjects’ mental model retrieve a pure counterfactual state:

COUNTERFACTUAL RATE  
COUNTERFACTUAL CAPITAL.

No one seems to be interested in Type II interpretation, though it is specular to Type I. As for financial economists, the reason lies in the fact that capital is seen as immutable since it bears no relation to alternative courses of action; as for ordinary people, they have a broader view, as we have seen, and the modification of one single element alone is unacceptable: The whole story must be changed.

---

<sup>15</sup> A genuine-counterfactual reasoner would say that that of economists “is an attempt to pull counterfactual rabbits out of actual hats” [Elster, 1978, p.203].

It might be that linguistic shaping plays a role in the way individuals undo such kinds of scenarios, and further researches could be addressed to gradually manipulating the linguistic description of a particular situation, so as to analyze to what extent the cognitive interpretation changes from one Type to another. As noted, economists have both language and mathematics to express notions and concepts. Could language (and the way language is used) affect thought in some specific cases? Could it be that a particular mental representation is elicited by a specific linguistic representation of a situation? The experiments conducted suggest that people cognize the notion of opportunity cost so that all elements are changed. But to become a financial economist implies to master a new discipline. So, whenever ordinary people learn to become financial economists, they have to change their own cognitive schemas in order to adopt the conventional one accepted in finance. In all this, language could play a role:

Learning a new discipline largely consists in coming to understand and use appropriately the individual labelled schemas that constitute its fundamental vocabulary. Labelled schemas ... play active roles in the way we categorize the ... world and hence in the way we construct our attitudes to it.  
[Bloom, 1981, p. 72]

It would be interesting to test scholars and see whether different linguistic labels and different representations of the same scenarios generate different notions of cost and excess profit.

## **11 - Concluding remarks**

This paper deals with modelling cost and excess profit, also known as residual income. The aims of this paper are: (i) To show that these fundamental economic concepts are based on implicit counterfactual reasoning; (ii) to show that the formalization of these counterfactual conditionals is not univocal; (iii) to show that ordinary people may interpret such counterfactuals in a different way from academics, who have created those implicit counterfactuals.

In general, there seems to be no awareness that counterfactuals crowd economic notions (their use is mainly implicit). There are exceptions, as is the case of historical economics, where the notion of counterfactual has been

self-consciously used [see Fogel, 1964; Fogel and Engerman, 1974; Thomas, 1965]. Theory of games is sometimes viewed as an inquiry into counterfactuals [Selten and Leopold, 1982; Stalnaker, 1996]. However, the most noticeable attempt to view economic laws as counterfactual statements is found in Hülsmann (2003): “a whole class of economic laws are counterfactual laws. They concern the relationship between what human beings actually do ... and what they could have done instead” (p. 57). So, “self-aware or not, economists will go on speaking counterfactually about noncooperative games, macroeconomic policy, and the retrospective welfare calculations of historical economics” [McCloskey, 1987]. Nor do scholars focus their attention on the relationships between such counterfactuals and their mathematical translation. This paper aims at eliciting interest in investigating the circumstances in which scholars make use of counterfactuals for defining concepts, identifying the implications for theoretical models (and for decision-making as well), and, in contrast, how ordinary people cognize the same situation and which mental model they are prone to activate.<sup>16</sup> Using counterfactuals is a major strategy for defining concepts but there is no unique way to undo a scenario, both in linguistic and in formal terms. In particular, we have counterfactual statements lending themselves to different cognitive representations and thus different mathematical formalizations. The three representations we have seen are but three different ways of defining the concepts of opportunity cost and excess profit, and provide different information. Type I is the standard way of formally translating the concept of cost, Type III is a model introduced by Magni (2000, 2001, 2004, 2005, 2006), Type II has never been considered in the literature. We have seen that only Type III interpretation presupposes that the counterfactual operator is a multiplicative homeomorphism and have the properties of symmetry and additive coherence. These properties correspond to three cognitive categories: The former is the mathematical correspondent of what one could name genuine counterfactual, a counterfactual where the reasoner alters all of the features, as opposed to the semicounterfactual undoing, which involves a partial mutation of the scenario. Symmetry has to do with framing: A definition of excess profit should be symmetric, that is, taking either alternative as a reference point should not result in a change of (the absolute value of) profit (Type I and Type II are asymmetric: The reasoner cognizes

---

<sup>16</sup> Byrne and Tasso (1999) and Thompson and Byrne (2002) show that individual differences lead to differences in interpretations of subjunctives. In our case the individual differences would refer not to preconstructed counterfactuals but to the very way a counterfactual is constructed by the reasoner.

the situation so that excess profit changes depending on which of the alternatives is taken as the benchmark). Additive coherence guarantees coherence in the calculation of the whole counterfactual profit: It can be deduced from a difference between counterfactual terminal net worth and initial net worth, as well as from the sum of all periodic counterfactual profits. Type I-reasoners are trapped in a paradox: The sum of the parts do not equal the whole, which is tantamount to saying that their counterfactual is not invariant with respect to changes in the unit of time.

Experimentally, two introductory studies have been conducted among Italian students of the University of Modena and Reggio Emilia. The studies seem to corroborate the tentative thesis according to which nonacademic reasoners lean toward a complete counterfactualization of the scenario. Individuals seem prefer to genuinely undo scenarios than focus on the alteration of the rate of return alone. Fruitful experiments may be conducted to pinpoint the relations among language, mathematics and reasoners' mental models. Moreover, it is worth studying the differences, if any, between a concept of cost generated by a prefactual (decision has not been taken) or by a nonfactual (decision has already been taken), as well as understanding whether any significant divergence originates from an *ex ante* analysis (action has not yet begun) as opposed to an *ex post* analysis (action is over). Furthermore, it would theoretically and practically helpful to single out situations where either model is more appropriate. For example, if a methodology for rewarding managers is to be modelled on the basis of excess profit, the lost-capital paradigm may be more suitable in a situation where a business unit's performance is set against another business unit's performance, given that this model fulfils the property of symmetry. The property of additive coherence makes the lost-capital paradigm more appealing in those where uncertainty lies in forecasting residual-income timing rather than quantifying residual income. The standard paradigm, of which Stewart's EVA is an instantiation, may be more appropriate in a situation where managers of a firm often change, possibly period by period: In this case the new manager should be considered as starting at the outset with a determined capital equal to the capital left by the preceding period's manager. Given that the two models may even lead to different signs [Ghiselli Ricci and Magni, 2006], the choice of the model may also depend on several other aspects (i.e., the piece of information one is willing to retrieve from the notion of excess profit, the degree at which a model motivates managers, the degree at which either model affects managers' compensation and therefore outflows from the firm etc.).

It is worth noting that the present work suggests a well-determined epistemological perspective: If the notion of opportunity cost is subjective, if multiple interpretations are possible, and if either model may be more suitable depending on the situation and on several qualitative and quantitative considerations, then a conventionalist view might be inferred, in the sense of Poincaré (1902). In this sense, opportunity cost is just a matter of convention, and the widespread adoption of the standard model in the financial literature is just an agreed upon convention (this aspect pertains to the sociological side of the issue as well as the epistemological side).

## Appendix

Solutions to Scenario 4.

### Type I

*Question* (a). The value of account Y after one year has increased at a rate of 15% but has decreased by the sum 26, so the value is  $20=40(1+0.15)-26$ . Accordingly, the profit in the second year is  $0.15 \cdot 20=3$ .

*Question* (b). The counterfactual is here interpreted as

What would the profit have been in the second year *if in that year* the investment had not been undertaken?

In this case, the value of 20 just mentioned would have been placed in account X generating a 10% return. As a result, the profit would have been  $0.1 \cdot 20=2$ .

### Type II

*Question* (a). As in Type I

*Question* (b). The counterfactual is here interpreted as

What would the profit have been in the second year *if up until then* the investment had not been undertaken?

In this situation account X's value after one year would have been different: In fact, the sum 40 would have remained in account X, yielding a 10% return, which means a total value of  $44=40(1+0.1)$ ; also, no 26 would have been reinvested in account X. So, after one year account X's value would have been greater (with respect to the factual case) by 44 and smaller by 26, which

results in a surplus of  $44-26=18$ . But if the investor had had a surplus of 18 in account X he would have invested it in account Y at the beginning of the second year, earning a 15% return. Therefore, the profit in the second year would have been  $0.15 \cdot 18=2.7$ .

### Type III

*Question* (a). As in Type I

*Question* (b). The counterfactual is here interpreted as

What would the profit have been if the investment had *never* been undertaken?

In this case, account X's value after one year would have been different: In fact, the sum 40 would have remained in account X, yielding a 10% return, which means a total value of  $44=40(1+0.1)$ ; also, no 26 would have been reinvested in account X. So, after one year account X's value would have been greater (with respect to the factual case) by 44 and smaller by 26, which results in a surplus of  $44-26=18$ . Therefore, in the second year our decision maker would have earned a 10% return on that 18: Hence, the counterfactual profit equals  $0.1 \cdot 18=1.8$ .

An alternative solution is possible for Type III:

*Question* (a). After one year, account Y's value is  $20=40(1+0.15)-26$ , while account X's value is  $92=60(1+0.1)+26$ . The second year's profit on X is  $0.1 \cdot 92=9.2$ , the second year's profit on Y is  $0.15 \cdot 20=3$ , so  $9.2+3=12.2$  is the overall profit generated by the two accounts in the second year.

*Question* (b). Account X's value after one year would have been  $110=100(1+0.1)$ , so the profit in the second year would have been  $0.1 \cdot 110=11$ .

In terms of excess profit, this solution is equivalent to the preceding one. In fact, note that we have calculated three factors:

$$\begin{aligned} &0.15 \cdot 20 \\ &0.1 \cdot 92 \\ &0.1 \cdot 110 \end{aligned}$$

whence the excess profit is

$$(0.15 \cdot 20 + 0.1 \cdot 92) - 0.1 \cdot 110 = 1.2.$$

The two last terms in the left-hand side can be grouped:

$$0.15 \cdot 20 - 0.1(110 - 92) = 0.15 \cdot 20 - 0.1 \cdot 18 = 3 - 1.8 = 1.2.$$

Note that 3 and 1.8 are just the answers to *Question (a)* and *Question (b)* previously seen. So the line of argument we have here described is equivalent.<sup>17</sup>

## References

- Abel, A.B., 1990. Consumption and investment. In B.M. Friedman and F.H. Hahn (Eds.), *Handbook of Monetary Economics*. Amsterdam: North-Holland.
- Archer, S.H., D'Ambrosio, A.A., 1972. *Business Finance. Theory and Management*, second edition. New York: MacMillan Company
- Arnold, G. and M. Davies (Eds.), 2000. *Value-based Management: Context and Application*. Chichester, UK: John Wiley & Sons.
- Au, T.K., 1983. Chinese and English counterfactuals: The Sapir-Whorf hypothesis revisited. *Cognition*, 15, 155–187.
- Au, T.K., 1984. Counterfactuals: In reply to Alfred Bloom. *Cognition*, 17, 289–302.
- Begg, D., S. Fischer and R. Dornbusch, 1984. *Economics*, Mc-Graw Hill, British Edition.
- Bell, D.E., 1982. Regret in decision making under certainty. *Operations Research*, 30, 961–981.
- Bell, D.E., 1985. Disappointment in decision making under certainty. *Operations Research*, 33, 1–27.
- Biddle, G.C., R.M. Bowen and J.S. Wallace, 1999. Evidence on EVA. *Journal of Applied Corporate Finance*, 12, 69–79.
- Bloom, A.H., 1981. *The Linguistic Shaping of Thought: A Study in the Impact of Language on Thinking in China and the West*. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.
- Bloom, A.H., 1984. Caution—the words you use may affect what you say: A response to Au. *Cognition*, 17, 275–287.

---

<sup>17</sup> The equivalence shown deserves more profound investigation, but it is omitted for reasons of space (see Magni, 2004, 2005).

- Bodenhorn, D. 1964. A Cash-flow concept of profit. *Journal of Finance* (March), 19(1), 16–31.
- Brealey, R.A. and S.C. Myers, 2000. *Principles of Corporate Finance*, 6th edition. New York: Irwin McGraw-Hill.
- Buchanan, J. 1969. *Cost and Choice. An Inquiry in Economic Theory*. Chicago: Markham. Republished as Midway reprint, Chicago: University of Chicago Press, 1977.
- Buchanan, J. 1987. Opportunity cost. In *The New Palgrave: A Dictionary of Economics*, 4 vols. London and Basingstoke: The MacMillan Press Limited.
- Byrne R.M.J. 2002. Mental models and counterfactual thoughts about what might have been. *Trends in Cognitive Science*, 6 (10), 426–431.
- Byrne R.M.J. and A. Tasso, 1999. Deductive reasoning with factual, possible, and counterfactual conditionals. *Memory and Cognition*, 27 (4), 726–740.
- Carsberg, B.V., 1966. The contribution of P.D. Leake to the theory of Goodwill valuation. *Journal of Accounting Research*, 4(1), 1–15, Spring.
- Chisholm, R.M., 1946. The contrary-to-fact conditional. *Mind*, 55, 289–307, October.
- Chisholm, R.M., 1975. Law Statements and Counterfactual Inference. In *Causation and Conditionals*. See Sosa (1975).
- Coase, R.H., 1938. Business Organization and the Accountant, *The Accountant*, October-December. Reprinted in Solomons, D. (Ed.), *Studies in Costing*. London: Sweet and Maxwell, 1952.
- Copeland, T.E. and J.F. Weston, 1988. *Financial Theory and Corporate Finance*. Addison-Wesley Publishing Company, Inc.
- Damodaran, A., 1999. *Applied Corporate Finance. A User's Manual*. New York: John Wiley & Sons, Inc.
- Damodaran, A., 2007. *Valuation Approaches and Metrics: A Survey of the Theory and Evidence*, Delft, NL: now Publishers Inc.
- Dixit, A. and R. Pindyck, 1994. *Investment under Uncertainty*. Princeton NJ: Princeton University Press.
- Dudman, V.H., 1988. Indicative and subjunctive. *Analysis*, 48, 113–122.
- Edey, H.C., 1957. Business valuation, goodwill and the super-profit method. *Accountancy*, January/February. Reprinted in W.T. Baxter & S. Davidson (1962) (Eds.), *Studies in Accounting Theory*. London: Sweet & Maxwell, pp. 201–217.
- Edwards, E. and P. Bell., 1961. *The Theory and Measurement of Business Income*. Berkeley: University of California Press.

- Elster, J., 1978, *Logic and Society: Contradictions and Possible Worlds*. Chichester, UK: John Wiley & Sons, Ltd.
- Fernández, P., 2002. *Valuation Methods and Shareholders Value Creation*. San Diego: Academic Press.
- Finnerty, J.D., 1986. *Corporate Financial Analysis*. New York: McGraw-Hill.
- Fisher, I., 1930. *The Theory of Interest*. New York: The Macmillan Company. Reprint by Augustum M. Kelley Publishers, Clifton, NJ, 1974.
- Fogel, R.W., 1964. *Railroads and American Economic Growth: Essays in Economic History*, Baltimore: John Hopkins Press.
- Fogel, R.W. and S.L. Engerman, 1974. *Time on the Cross: The Economics of American Negro Slavery*. Boston: Little, Brown.
- Friedman, L.A. and B.R. Neumann, 1980. The effects of opportunity costs on project investment decisions: A replication and extension. *Journal of Accounting Research*, 18 (2), 407–419.
- Ghiselli Ricci, R. and C.A. Magni, 2006. Economic Value Added and Systemic Value Added: Symmetry, additive coherence and differences in performance. *Applied Financial Economics Letters*, 3, 151–154.
- Giroto, V., P. Legrenzi and A. Rizzo 1991. Event Controllability in Counterfactual Thinking. *Acta Psychologica*, 78, 111–133.
- Goodman, N. 1947. The problem of counterfactual conditionals. *Journal of Philosophy*, 44, 113–28.
- Goodman, N., 1983. *Fact, fiction, and forecast*. Cambridge MA: Harvard University Press, 4<sup>th</sup> edition.
- Johnson-Laird, P.N., 1983. *Mental models*. Cambridge, MA: Cambridge University Press.
- Jorgensen, D., 1963. Capital theory and investment behavior. *American Economic Review*. 53, 247–259, May.
- Harris, P.H., T. German and P. Mills, 1996. Children’s use of counterfactual thinking in causal reasoning. *Cognition*, 61, 233–259.
- Hey, J. 1991. *Experiments in Economics*. Oxford, UK.: Basil Blackwell Ltd.
- Hooker, C., N.J. Roese and S. Park, 2000. Impoverished counterfactual thinking associated with schizophrenia. *Psychiatry*, 63, 326–335.
- Hülsmann, J.G., 2003,. Facts and counterfactuals in economic laws. *Journal of Libertarian Studies*, 17(1), 57–102.
- Kahneman, D. and A. Tversky, 1982. The simulation heuristic. In Kahneman, D., Slovic, P. and Tversky (Eds.), *Judgement under uncertainty: Heuristics and biases*. New York: Cambridge University Press.

- Kahneman, D. and D.T. Miller, 1986. Norm theory: Comparing reality to its alternatives, *Psychological Review*, 93, 136–53.
- Kahneman, D. and A. Tversky, 1984. Choices, values and frames, *American Psychologist*, 39, 341–50.
- Kay, J., 1976. Accountants, too, could be happy in a Golden Age: The accountant's rate of profit and the internal rate of return. *Oxford Economic Papers*, 28, 447–460.
- Kneale, W., 1950. Natural Laws and Contrary to Fact Conditionals. *Analysis*, 10, 121–125.
- Landman, J. and J.D. Manis, 1992. What might have been: Counterfactual thought concerning personal decisions. *British Journal of Psychology*, 83, 473–477.
- Legrenzi, P., V. Girotto and P.N. Johnson-Laird 1993. Focussing in reasoning and decision making. *Cognition*, 49, 37–66.
- Liu, L.G., 1985. Reasoning counterfactually in Chinese: Are there any obstacles? *Cognition*, 21, 239–270.
- Loomes, G. and R. Sugden, 1982. Regret theory: An alternative theory of rational choice under uncertainty. *Economic Journal*, 92, 805–824.
- Loomes, G. and R. Sugden, 1986. Disappointment and dynamic inconsistency in choice under uncertainty. *Review of Economic Studies*, 53, 271–282.
- Lundberg, C.G. and D.E. Frost, 1992. Counterfactuals in financial decision making, *Acta Psychologica*, 79, 227–44.
- MacMinn, R.D., 2005. *The Fisher Model and Financial Markets*. Singapore: World Scientific Publishing.
- Magni, C.A., 2000a. Decomposition of a certain cash flow stream: Differential systemic value and net final value, *Proceedings of the XXIV Conference of AMASES* (Association for Mathematics Applied to Economic and Social Sciences), Padenghe, Italy, September. Available at SSRN: <<http://ssrn.com/abstract=1096213>>.
- Magni, C.A., 2000b. Valore aggiunto sistemico: un'alternativa all'EVA quale indice di sovraprofitto periodale [Systemic Value Added: an alternative to EVA as a residual income model, *Budget*, 25(1), 63–71. Available in bilingual version at SSRN: <<http://ssrn.com/abstract=1103492>>.
- Magni, C.A., 2001. Scomposizione di sovraprofitto: Economic Value Added e Valore Aggiunto Sistemico [Excess-profit decomposition: Economic Value Added and Systemic Value Added], *Finanza, Marketing e Produzione*, 19(4), 94–119, December. Available at SSRN: <<http://ssrn.com/abstract=1140163>>.

- Magni, C.A., 2003. Decomposition of net final values: Systemic Value Added and residual income, *Bulletin of Economic Research*, 55(2), 149–176.
- Magni, C.A., 2004. Modelling excess profit. *Economic Modelling*, 21, 595–617.
- Magni, C.A., 2005. On decomposing Net Final Values: EVA, SVA and shadow project. *Theory and Decision*, 59, 51–95.
- Magni, C.A., 2006. Zelig and the art of measuring excess profit. *Frontiers in Finance and Economics*, 3(1), 103–129, June.
- Magni, C.A., 2009a. Residual income and value creation: An investigation into the lost-capital paradigm. *European Journal of Operational Research*, forthcoming. Available at SSRN: [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1029680](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1029680).
- Magni, C.A., 2009b. Splitting up value: A critical review of residual income theories. *European Journal of Operational Research*. Available online 25 September 2008. DOI:10.1016/j.ejor.2008.09.018.
- Martin, J.D. and J.W. Petty 2000. *Value Based Management. The Corporate Response to Shareholder Revolution*. Boston, MA: Harvard Business School.
- McCloskey, D.N., 1987. Counterfactuals. In *The New Palgrave: A Dictionary of Economics*. See Buchanan (1987).
- McConnell A.R., K.E. Niedermeyer, J.J. Leibold, A.G. El-Alayli, P.G. Chin and N.M. Kuiper, 2000. What if I find it cheaper someplace else?: Role in prefactual thinking and anticipated regret in consumer behaviour. *Psychology and Marketing*, 17, 281–298.
- McGill A.L., 2000. Counterfactual reasoning in causal judgments: Implications for marketing. *Psychology and Marketing*, 17, 281–298.
- Ohlson, J.A., 1989. Accounting earnings, book values, and dividends: The theory of the clean surplus equation in equity valuation. Reprinted in R. Brief and K.V. Peasnell (Eds.) (1996), *A Link Between Accounting and Finance*. New York and London: Garland Publishing.
- Ohlson, J.A., 1995. Earnings, book values, and dividends in equity valuation. *Contemporary Accounting Research*, 11(2) 661–687, Spring.
- Peasnell, K.V., 1981. On capital budgeting and income measurement. *Abacus*, 17(1), 52–67, June.
- Peasnell, K.V., 1982. Some formal connections between economic values and yields and accounting numbers. *Journal of Business Finance & Accounting*, 9(3), 361–381.
- Peccati, L., 1987. DCF e risultati di periodo [DCF and period margins] . In *Proceedings of XI Convegno A.M.A.S.E.S.* Aosta, Italy: AMASES.

- Peccati, L. 1989. Multiperiod analysis of a levered portfolio. *Decisions in Economics and Finance*, 12(1) (March), 157–166. Reprinted in J. Spronk & B. Matarazzo (Eds.), 1992, *Modelling for Financial Decisions*. Berlin: Springer-Verlag.
- Peccati, L., 1991. Valutazione analitica e sintetica di attività finanziarie. *Rivista Milanese di Economia*, serie Quaderni n. 21. Milano: Cariplo-Laterza.
- Penman, S.H., 1992. Return to fundamentals. *Journal of Accounting Auditing and Finance*, Reprinted in R. Brief and K. V. Peasnell (Eds.) (1996). *Clean Surplus: A Link Between Accounting and Finance*. New York and London: Garland Publishing.
- Poincaré, J.-H., 1902. *La Science et L'Hypothèse*. Paris: Flammarion.
- Popper, K.R., 1949. A note on natural laws and so-called “contrary-to-fact conditionals”. *Mind*, 49, 62–66.
- Preinreich, G., 1936. The Law of Goodwill. *The Accounting Review*, XI (4), 317–329, December.
- Preinreich, G., 1938. Annual survey of economic theory: The Theory of Depreciation. *Econometrica*, 6(1), 219–241, January.
- Rao, R.K.S., 1992. *Financial Management*. MacMillan Publishing Company.
- Rescher, N., 1975. Belief-contravening suppositions and the problem of contrary-to-fact conditionals. In *Causation and Conditionals*. See Sosa (1975).
- Roese N.J., 1997. Counterfactual thinking. *Psychological Bulletin*, 121, 133–148.
- Roese N.J., 2000. Counterfactual thinking and marketing: Introduction to the special issue. *Psychology and Marketing*, 17, 277–280.
- Roese, N.J. and J.M. Olson (Eds.), 1995a. *What Might Have Been. The Social Psychology of Counterfactual Thinking*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.
- Roese, N.J. and J.M. Olson, 1995b. Functions of counterfactual thinking. In *What Might Have Been*. See Roese and Olson (1995a).
- Roese, N.J. and J.M. Olson, 1995c. Counterfactual thinking: A critical overview. In *What Might Have Been*. See Roese and Olson (1995a).
- Rogerson, W.P., 1997. Intertemporal cost allocation and managerial investment incentives: A theory explaining the use of Economic Value Added as a performance measure. *Journal of Political Economy*, 105(4), 770–795.
- Selten, R. and U. Leopold, 1982. Subjective conditionals in decision and game theory. In *Studies in Contemporary Economics*. Berlin: Springer-Verlag.

- Shackle, G.L.S., 1961. *Decision, Order and Time in Human Affairs*. Cambridge, UK: Cambridge University Press.
- Sherman, S.J. and A.R. McConnell, 1995. Dysfunctional Implications of Counterfactual Thinking: When Alternatives to Reality Fail Us. In *What Might Have Been*. See Roese and Olson (1995a).
- Solomons, D., 1965. *Divisional Performance: Measurement and Control*. Homewood, IL: Richard D. Irwin.
- Sosa, E. (Ed.), 1975. *Causation and Conditionals*. Oxford, UK: Oxford University Press.
- Stalnaker, R., 1996. Knowledge, belief, and counterfactual reasoning in games. *Economics and philosophy*, 12(2), 133–163.
- Stewart, G.B., 1991. *The Quest for Value: The EVA<sup>TM</sup> Management Guide*. New York: HarperCollins Publishers Inc.
- Stoughton, N.M. and J. Zechner, 2007. Optimal capital allocation using RAROC<sup>TM</sup> and EVA<sup>TM</sup>. *Journal of Financial Intermediation*, 16(3), 312–342.
- Sugden, R., 2000. Credible worlds: The status of theoretical models in economics. *Journal of Economic Methodology*, 7(1), 1–31.
- Thirlby, G.F., 1946. The subjective theory of value and accounting Cost. *Economica*, XIII, 32–49, February.
- Thomas, R.P., 1965. A quantitative approach to the study of the effects of British imperial policy upon colonial welfare. *Journal of Economic History*, 25, 615–38.
- Thompson, V.A. and R.M.J. Byrne, 2002. Reasoning counterfactually: Making inferences about things that didn't happen. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 28, 1154–1170.
- Tobin, J., 1969. A general equilibrium approach to monetary theory. *Journal of Money, Credit and Banking*, 1, 15–29, February.
- Tsiros M., 1998. Effect on regret on post-choice evaluation: The case of more than two alternatives. *Organizational Behavior and Human Decision Processes*, 76, 48–69.
- Tsiros, M. and V. Mittal, 2000. Regrets: A model of its antecedents and consequences in consumer decision making. *Journal of Consumer Research*, 26, 401–417.
- Tversky, A. and D. Kahneman, 1981. The framing of decisions and the psychology of choice. *Science*, 211, 453–458.
- von Neumann, J. and O. Morgenstern, 1944. *Theory of Games and Economic Behavior*. Princeton, NJ: Princeton University Press.

- Wells, G.L., B.R. Taylor, and J.W. Turtle, 1987. The undoing of scenarios. *Journal of Personality and Social Psychology*, 53(3), 421–430.
- Will, F.L., 1947. The contrary-to-fact conditional. *Mind*, 56, 236–249.
- Young, S.D. and S.F. O’Byrne, 2001. *EVA and Value-Based Management*. McGraw-Hill.
- Zeelenberg M., J. Beattie, J. van der Pligt, and N.K. de Vries, 1996. Consequences of regret aversion: Effects on expected feedback on risky decision making. *Organizational Behavior and Human Decision Processes*, 65, 148–158.
- Zeelenberg M. and J. Beattie, 1997. Consequences of regret aversion: 2. Additional evidence for effects of feedback on decisions. *Organizational Behavior and Human Decision Processes*, 72, 63–78.