Computational Efficiency and Accuracy in the Valuation of Basket Options

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Abstract

The complexity involved in the pricing of American style basket options requires careful consideration of both computational efficiency and accuracy. The conventional assumption of lognormal distribution for the value of a basket is the key for the trade-off. This paper examines the mispricing errors of Bermudan basket options based on the assumption. The mispricing error is measured by the price differences between the price resulting from the assumption of lognormal distribution and the ñtrueò option price. The ñtrueò option prices are obtained from simulation based on procedure described in Longstaff and Schwartz (2001). The effects on the maturities, the volatilities, the correlations, the dividend payments for the underlying assets, number of underlying assets in the basket and the ñmoneynessò on mispricing are addressed.

Key words: Basket option**,** Bermudan option, mispricing**,** lognormal, simulation
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1 - Introduction

Basket options are options written on a portfolio of risky assets. A stock index option is a typical example. A basket option can be used to hedge the risk exposures of an investment portfolio, an industry sector, a portfolio of different industries, and an entire market index. This kind of options has two distinct advantages. First, fund managers can employ multi-asset basket options to reduce burdens from monitoring individual assets in the portfolio. Second, basket options can be used to cut significantly transactions costs.

The complexity involved in the pricing of American style basket options requires careful consideration of both computational efficiency and accuracy. A lognormal distribution is a standard and convenient assumption for modelling the value of an underlying asset in option pricing.[2] Given this assumption, numerous theoretical and numerical solutions in option pricing are readily available. However the value of a basket in theory is not log-normally distributed unless some strict conditions are imposed on the basket weights, even if the individual assets in the basket are. Prior research suggests that the lognormal distribution remains a good approximation if (i) the maturity of an American basket option is short; (ii) the maturity of an Asian option is short and the volatility of returns on the underlying asset is small (Ju (2002), Levy (1992)).[3] An interesting question is that how ógoodô is this approximation and how does any error vary with the maturities, the volatilities and the ñdeepnessò of the options. In order to answer this question, we need a benchmark ñtrueò value of a basket option. In this paper, the Least-Squares Monte Carlo simulation (LSM) developed by Longstaff and Schwartz (2001) is employed to obtain this ñtrueò value.

While a closed form analytic valuation formula for an American basket option does not exist, the standard valuation approaches can be classified into two categories. First, we derive a Black-Scholes style partial differential equation based on an arbitrage argument, and then either solves it by a theoretical approximation or a numerical method. Probably the most

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[2] Recent study employs a simple jump process - the Bernoulli jump process to develop approximate basket option valuation formulas (Flamouris and Giamouridis (2007)).

[3] Levy (1992) states that ñfine accuracy is generally secured for $\sigma\sqrt{\tau} \le 0.20$ò, where σ is the volatility and τ is the time to maturity of the option.

popular theoretical approximation is based on the concept of reducing a high dimensional problem into a single variable one (Levy (1992), Wan (2002), Ju (2003), Dionne, et al (2006)). The commonly employed numerical methods include finite difference and lattice approaches, such as the use of binomial or trinomial trees. However these numerical methods become impractical when options have multiple underlying state variables. The difficulty caused by the dimension of the problem can be seen from considering the three-dimensional pyramid that results from two underlying assets as compared with a two-dimensional tree for a single underlying asset in Rubinstein (1994).

In the second approach, we apply the Monte Carlo simulation method although ñ(it) cannot easily handle situations where there are early exercise opportunitiesò (Hull (2003)), as is the case with American options. Nevertheless, a great deal of effort in recent years has been devoted to developing Monte Carlo simulations that deal with early exercise for pricing American-style options (Broadie and Glasserman (1996, 1997)). As a result when the dimension of the underlying state variables increases and the feature of early exercise emerges, Monte Carlo method becomes increasingly attractive compared to other numerical methods. Broadie and Glasserman summarize three techniques used when pricing American options using Monte Carlo simulation. First, we can parameterize the exercise boundary and use simulation to maximize the expected payoff with respect to the parameters. Second, we can characterize the upper and lower bounds for the option price and provide a confidence interval for the true value. Third, we can employ a dynamic programming technique to determine the optimal exercise policy in backward recursion. The last approach includes the Least-Squares Monte Carlo simulation method (Longstaff and Schwartz (2001)).

The main thrust of this paper is to discuss the computational efficiency and accuracy in the valuation of basket options. We can estimate the first few moments of value of the basket based on the distributions of returns on the individual underlying assets. The lognormal distribution is a convenient assumption here since it can be fully characterized by the first two moments: mean and variance. Given the assumption of a lognormal distribution for the value of the basket, we can apply a lattice approach to price an American basket option once the variance is known. This approach transfers the issue of a multi-dimensional American option to a standard one dimension American option (Wan (2002), Brigo et al (2003)). This method with the strong assumption of log-normality gives a computational efficient solution but is of unknown accuracy. I refer to this method as LN-approach in the following analysis. The value obtained from this approach is denoted by V_{ln} .

The Least-Squares Monte Carlo simulation (LSM) directly deals with the multi-dimensional problem with early exercise feature of American options. High dimensional underlying state variables can be incorporated though at the expense of thousands of simulated sample paths. The regression technique simplifies the sample path by relating the payoffs at time *t+1* to underlying asset values at time *t*. Comparing the intrinsic value and the conditional expectation at each sample period gives a complete specification of the optimal exercise strategy along each path. This methodology is heavy in computational requirements but can provide us with a solution of arbitrary accuracy. Therefore, the ñtrueò value, V_{mc}, for an American basket option can be computed by the LSM.

Comparing value V_{ln} to V_{mc}, we can investigate the accuracy and reliability of the assumption of lognormal distribution in the valuation of basket options. I examine how relative mispricing error $(V_{ln} - V_{mc})/ V_{mc}$ may change in the following five dimensions: (i) different option maturity (1-year, 2-year, and 3-year); (ii) increases in the volatilities of underlying assets in the basket given other parameters constant; or changes in the correlations among underlying assets given other parameters constant; (iii) the ñmoneynessò (ñat-the-moneyò, ñout-of-the-moneyò, and ñin-the-moneyò options) of the option; (iv) number of underlying assets in the basket (two-asset, three-asset, and five-asset); (v) the dividend payments of underlying assets. I discuss both symmetric scenarios and asymmetric scenarios regarding to the volatilities and the correlations of the underlying assets in the basket.

My results show that in general the LN-approach is likely to underestimate the calls and overestimate the puts in all symmetric and asymmetric scenarios. In contrast to Levyôs (1992), which suggests that for a limited range of volatilities and option maturities, the distribution of an arithmetic average is well-approximated by the lognormal distribution when the underlying price process follows the conventional assumption of a geometric diffusion, my results show that the relative pricing errors seem to be reduced with the increases in the maturities of basket options.

While the LN-approach enlarges mispricing errors for the corresponding puts in the asymmetric scenarios, I find that the LN-approach reduces mispricing errors for the corresponding calls. In contrast to Brigo et al

(2001), which argue that the approximation based on the assumption of lognormal distribution gives a reasonable good accuracy with respect to the ñtrueò price only for the symmetric scenarios (i.e. when volatilities are roughly the same) and high correlations, the evidence here shows that the mispricing errors for the calls based on the LN-approach in the asymmetric scenarios are less than that in symmetric scenarios.

I also find that when the volatilities of underlying assets increase and other parameters keep constant, or the correlations of underlying assets increase and keep other parameters constant, the LN-approach reduces the mispricing errors. My results also indicate that ñout-of-the-moneyò puts have bigger mispricing errors than ñin-the-moneyò puts whilst ñin-the-moneyò calls have smaller mispricing errors than ñout-of-the-moneyò calls.

In addition, my analysis shows that the dividend payments of underlying assets have big effect on the mispricing errors for the LN-approach. In the symmetric scenarios when underlying stocks in the baskets pay no dividend, the LN-approach misprices the calls by less than 1% and the puts by less than 2% in my simulated sample. When the underlying stocks pay 5% dividends, the calls will be underestimated by 6.6% and the puts will be overestimated by about 6.1%. The dividends payments also significantly increase the magnitudes of mispricing errors in the asymmetric scenarios.

With respect to the number of underlying assets in the basket, the evidence generally supports that the magnitudes of underestimation of the calls decreases and the magnitudes of overestimation of the puts increases in the number of assets in the basket for the pooled sample.

Note from the payoff profiles of call and put options that the upside potential benefit is unlimited for a call option but less than the exercise price for a put option. My findings may suggest that the return of a basket is not behaved as ógoodô as a lognormal distribution. An assumption of a typical option pricing model is that the volatility of return of underlying asset is not influenced by an optionôs time to maturity, the underlying asset price, the exercise price, i.e., the volatility is a constant. The existence of smile effect and sloppy smile effect from option market is inconsistent with the constant volatility assumption. Asset-price dependent volatility in a basket option pricing may provide one of possible explanations for some puzzles summarized above.

The rest of the paper is organized as follows. In Section 2, I introduce LSM simulation and issues on dimension reduction based on lognormal distribution. In Section 3, I describe the simulation procedure and present my results. In Section 4, I conclude the paper.

2 - LSM simulation and dimension reduction based on lognormal distribution

2.1 Value of basket and underlying stock processes

Assume that there are *n* underlying assets in the basket and the price of each individual underlying asset follows a lognormal distribution. Specifically, in a risk neutral economy, we have

$$dS_t^i = (r - \delta_i) S_t^i dt + \sigma_i S_t^i dz_t^i, \ i = 1, 2, \ldots n.$$

where S_t^i is the price of asset *i* at time *t*, dz_t^i $(i=1,2,\ldots n)$ are standard Wiener processes, σ_i is the constant instantaneous volatility for asset *i*, and δ_i is the rate of continuous dividends payment on underlying asset *i*. The returns on different assets are assumed to be instantaneously correlated: $dz_t^i \, dz_t^j = \rho_{ij} dt$. Denote $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$.

The value of the basket at each time period *t* is given by:

$$B_t = \sum_{j=1}^{n} w_j S_t^j$$

where w_j is the percentage of asset *j* in the basket and hence $\sum_j w_j = 1$. The payoff of a call option on the basket is defined as $max(B_T - K, 0)$ and the payoff of a put option is defined as $max(K - B_T, 0)$, where *K* is the exercise price of the option.

By matching the first moment, we obtain

$$E[B_t] = \sum_{j=1}^{n} w_j E[S_t^j] = \sum_{j=1}^{n} w_j S_0^j \exp[(r - \delta_j)t] \equiv B_0 \exp[(r - \bar{\delta})t]$$

where

$$\bar{\delta} = -\frac{1}{t}\ln\left(\frac{1}{B_0}\sum_{j=1}^{n} w_j S_0^j \exp(-\delta_j t)\right)$$

By matching the variance, we have

$$\sigma^2 = \sum_{i,j=1}^{n} w_i w_j E[(S_t^i - E(S_t^i))(S_t^j - E(S_t^j))]$$

$$= \sum_{i,j=1}^{n} w_i w_j S_0^i S_0^j \exp[(2r - \delta_i - \delta_j)t][\exp(\rho_{ij}\sigma_i\sigma_j t) - 1]$$

$$\equiv B_0^2 \exp[2(r - \bar{\delta})t][\exp[(\bar{\sigma})^2 t] - 1]$$

where

$$(\bar{\sigma})^2 = \frac{1}{t}\ln\left(\frac{1}{B_0^2}\sum_{i,j=1}^{n} w_i w_j S_0^i S_0^j \exp[(2\bar{\delta} - \delta_i - \delta_j)t][\exp(\rho_{ij}\sigma_i\sigma_j t) - 1] + 1\right)$$

$$= \frac{1}{t}\ln\left(\frac{\sum_{i,j=1}^{n} w_i w_j S_0^i S_0^j \exp[(-\delta_i - \delta_j + \rho_{ij}\sigma_i\sigma_j)t]}{\sum_{i,j=1}^{n} w_i w_j S_0^i S_0^j \exp[-(\delta_i + \delta_j)t]}\right)$$

These are the continuous dividend yield and instantaneous variance for the basket obtained by Brigo et al (2001). This variance will be used in the lattice approach to pricing one dimension American (Bermudan) option later. Of course, if we assume that the basket value follows a lognormal distribution, these two parameters are sufficient to characterize the distribution of the value of underlying basket.

Nevertheless, the dynamics of basket value is:

$$dB_t = \sum_{j=1}^{n} w_j dS_t^j = rB_t dt - \sum_{j=1}^{n} w_j \delta_j S_t^j dt + \sum_{j=1}^{n} w_j \sigma_j S_t^j dz_t^j$$

Let $Y(t)=lnB(t)$ and $W_j(t) = w_j S_t^j / B_t$, the relative weights for individual asset *j (j=1,2,é n)* . Then itôôs Lemma gives

$$B_t = B_0 \exp\{rt - \int_0^t \Big(\sum_{j=1}^{n} \delta_j W_j(\tau) + \frac{1}{2}\sum_{i,j=1}^{n} \sigma_{ij} W_i(\tau) W_j(\tau)\Big) d\tau + \sum_{j=1}^{n} \sigma_j \int_0^t W_j(\tau)\, dz_\tau^j\}$$

It is clear that the value of the basket, in general, does not follow a lognormal distribution.[4] While a short maturity of a basket option and lower volatilities of the underlying assets may be a natural starting point to approximate the value of the basket option, no clear analytic expression leads to a conclusion that a lognormal distribution is a good approximation for the value of the basket option.

2.2 The Cholesky decomposition

The first thing in a basket option pricing by simulation is to deal with the correlation with high dimensional assets. For the purpose of the present paper, I assume that the correlation matrix is positive definite. Therefore, the Cholesky decomposition applies.[5] Given a symmetric and positive definite correlation matrix Σ, the Cholesky decomposition is an upper triangular matrix U such that $\Sigma = U^T U$. Specifically, if $\Sigma = (\sigma_{ij})_{n\times n}$ and $U = (u_{ij})_{n\times n}$, then for any *i =1,2,é n* and *j = i+1, é n,*

$$u_{ii} = \sqrt{\sigma_{ii} - \sum_{k=1}^{i-1} u_{ik}^2}$$

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[4] One can easily show that there exist conditions, such that the value of the basket is log-normally distributed. For instance, if the relative weight $W_j(t)$ satisfies that $\sum_{j=1}^{n} \delta_j W_j(\tau) + \frac{1}{2}\sum_{i,j=1}^{n} \sigma_{ij} W_i(\tau) W_j(\tau) = 0$ and $\sum_{j=1}^{n} \sigma_j \int_0^t W_j(\tau)\, dz_\tau^j = \sum_{j=1}^{n} \sigma_j \overline{W_j} z_t^j$, then $B_t = B_0 \exp\{rt + \sum_{j=1}^{n} \sigma_j \overline{W_j} z_t^j\}$.

[5] Prior literature documents that the Cholesky decomposition is an efficient way to deal with correlated (random) variables although one can apply eigenvalue and eigenvector approach to more general symmetric matrix. It is well known that the Cholesky decomposition is used in linear least squares problems.

$$u_{ji} = \left(\sigma_{ji} - \sum_{k=1}^{i-1} u_{jk} u_{ik} \right) / u_{ii}$$

Because Σ is symmetric and positive definite, the expression under the square root is always positive, and all l_{ij} are real. In the following analysis, I use the Cholesky decomposition to simulate the correlated random asset prices. Since

$$\begin{pmatrix} \sigma_1 dz_t^1 \\ \cdots \\ \sigma_n dz_t^n \end{pmatrix} \begin{pmatrix} \sigma_1 dz_t^1 & \cdots & \sigma_n dz_t^n \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \cdots & \cdots & \cdots \\ \sigma_{1n} & \cdots & \sigma_n^2 \end{pmatrix} dt = U^T U d_t$$

We can let

$$\begin{pmatrix} dZ_t^1 \\ \cdots \\ dZ_t^n \end{pmatrix} = U^{-T} \begin{pmatrix} \sigma_1 dz_t^1 \\ \cdots \\ \sigma_n dz_t^n \end{pmatrix}$$

hence $dZ_t^i \, dZ_t^j = 0$ when $i \neq j$, dt when $i=j$.

In other words, we can efficiently generate n correlated Weiner processes dz_t^i from n independent Weiner processes dZ_t^i ($i=1,2,$**é** n) by applying the Cholesky decomposition. As a result, it is expected to improve the efficiency of the Monte Carlo simulation.

2.3 The simple Least-Squares regression and conditional expected option payoff

Assume that a Bermudan option can be exercised N times until its maturity and the exercise dates are $t_1 < t_2 <$**é** $..< t_N = T$, where T is the maturity of the option.[6] The risk free interest rate is r. We can simulate m sample paths for the value of the basket in a risk-neutral world. For instance, one realization for the value in sample path j $(j=1,2,$**é** $,m)$ at exercisable date $(t_1, t_2,$**é** $, t_N)$ is $(B_0, B_1^j, ..., B_N^j)$, where B_0 is the initial basket value.

[6] For example, a Down and Out Basket Bermudan Put may be exercised at the discretion of the holder at weekly intervals (Cao et al (2006)).

As usual, I employ a backward algorithm to pricing American options. At final expiration date t_N, the cash flows in sample path j are realized for the Bermudan option, that is, $max\{\Upsilon\,(B_N^j - K),0\}$, where Υ is a binary operator with 1 for a call option and -1 for a put option. It is either zero if the option is out-of-the- money or $\Upsilon\,(B_N^j$ -K) if the option is in-the-money for all sample paths j. Back to time t_{N-1}, if the option is in the money at time t_{N-1} for sample path j, then we have to decide whether to exercise the option immediately or keep the option alive until the final expiration date at time t_N.

Let $X = (B_{N-1,N}^1, B_{N-1,N}^2, ..., B_{N-1,N}^m)$ be the value of the basket at time t_{N-1} for all m paths, where $B_{N-1,N}^j = B_{N-1}^j$ if the option is in-the-money, otherwise 0. Let $Y = (\Upsilon\,(B_{N,N}^1 - K)\, e^{(t_N - t_{N-1})r}$, $\Upsilon\,(B_{N,N}^2 - K)\, e^{(t_N - t_{N-1})r}$, é , $\Upsilon\,(B_{N,N}^m - K)\, e^{(t_N - t_{N-1})r})$ be the corresponding discounted cash flows received at time t_N, where $B_{N,N}^j = B_N^j$ if $B_{N-1,N}^j \neq 0$, otherwise corresponding element in Y is 0 for sample path j. By doing so, the computational time will be saved because paths leading to zero payoffs contribute nothing to the determination of the value of the option.

Following Longstaff and Schwartz (2001), in order to make the analysis and program tractable, I use weighted parabolas as basis functions in the simple ordinary lease square regression. That is,

$$L_0(X) = 1, L_1(X) = X, L_2(X) = X^2$$

which are argued to have significantly improved in computational speed and efficiency. Regressing Y on the above basis functions by OLS approach, we have

$$E\{Y \mid X\} = \beta_0 L_0(X) + \beta_1 L_1(X) + \beta_2 L_2(X)$$

which is called the conditional expected present value of next periodôs cash flows.

We can decide whether it is optimal to exercise the option at time t_{N-1} for the paths obtained above by comparing the value of immediate exercise at time t_{N-1} with the value from the continuation. The value of immediate exercise equals the intrinsic value $\Upsilon\,(B_{N-1}^j - K)$ for the path j if the option is in-the-money, whereas the value from continuation is given by substituting X into the conditional expectation function formula to match the first moment.

Specifically, in the path *j* at time t_i, the cash flow $CF(t_i)$ is either $E\{Y|X\}$ or the intrinsic value (ITV). If $E\{Y|X\}$ is larger than ITV, then let $CF(t_i) = 0$, and we should keep option alive. Of course, the corresponding path *j* value at time $t_i +1$, $CF(t_i+1) \neq 0$; otherwise exercise it at time $t_i -1$ and let $CF(t_i+1) = 0$, and $CF(t_i) \neq 0$ since an option can only be exercised once. When we proceed at time $t_i -1$, if ITV at time $t_i -1$ for path *j* is bigger than the continuation value at time t_i ï *1*, then we have to set all future cash flows zero. That is, each path has only one time period non-zero value, i.e. option can only be exercised once.

Repeating the above procedure, we can examine whether the option should be exercised at time $t_{N-2}, t_{N-3}, t_2, t_1$ for each path *j* (j=1,é , *m*). A $m \times N$ stopping rule matrix will be generated. After discounting cash flows according to the stopping time matrix, the simulated option value will be obtained by averaging overall sample paths.

3 - Simulation procedure and results

In the following analysis, I set some common parameters in my simulation programs on different dimensions. A Bermudan basket option can have one-year, two-year or three-year maturity in my simulations. If the maturity $T = 1$, I assume that the option can be possibly exercised 4 times until its maturity (say, at the end of each quarter); if $T = 2$, the option can be exercised 8 times until its maturity; and if $T = 3$, the option can be exercised 12 times until its maturity. The basket options can have number of two, three or five underlying assets for the tractability of presentation. For a 2-asset basket option, two assets have weights (0.45, 0.55) in the basket. For a 3-asset option, three assets have weights (0.15, 0.35, 0.50) in the basket. For a 5-asset option, five assets have weights (0.05, 0.15, 0.20, 0.25, 0.35) in the basket. The weights above are set for the purpose of parsimony. Assume that each underlying stock has an initial price of 100 (and hence a basket has initial value of 100) and each option has one of exercise prices (80, 90, 100, 110, 120), which is around initial value of the basket. Thus, there are 15 combinations between the number of assets and exercise prices. Risk-free interest rate is assumed to be 5%.

Based on all possible different volatilities and correlations of the underlying assets in the basket, I differentiate basket options and discuss the mispricing errors in the symmetric and asymmetric scenarios. In the symmetric scenarios, the volatilities of all underlying assets are the same and the correlations among assets are also the same. The pooled simulated sample option prices are generated from 270 basket options. Each of above 15 combinations in the number of assets and the exercise prices can then be incorporated with the following six mixtures of all underlying stocksô volatilities, the correlation coefficients and the rates of dividend yields. Specifically, I discuss (σ, ρ, δ) = (0.2, 0.5, 0), (0.2, 0.5, 0.05), (0.5, 0.5, 0), (0.5, 0.5, 0.05), (0.2, 0.9, 0) and (0.2, 0.9, 0.05) respectively. They include cases in which the underlying stocks paying or without paying dividends, increases in the volatilities and changes in the correlation coefficients among underlying stocks. Finally each option can have one-year, two-year or three-year maturity.

In the asymmetric scenarios, the pooled simulated sample option prices are generated from 150 Bermudan basket options. For a two-asset basket option with one of the exercise prices (80, 90, 100, 110, 120), consider the following combinations of two underlying stocksô volatilities, the correlation coefficient and the rates of dividend yields, $(\sigma_1, \sigma_2, \rho, \delta)$ = (0.2, 0.5, 0.5, 0) and (0.2, 0.5, 0.5, 0.05). For three-asset options, consider four combinations: $(\sigma_1, \sigma_2, \sigma_3, \rho, \delta)$ = (0.2,0.5,0.9,0.5, 0) and (0.2,0.5,0.9, 0.5, 0.05); $(\sigma, \rho_{12}, \rho_{13}, \rho_{23}, \delta)$ = (0.2, 0.7, 0.9, 0.9, 0) and (0.2, 0.7, 0.9, 0.9, 0.05). For five-asset options, also consider four combinations: $(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \rho, \delta)$ = (0.3,0.3,0.3,0.5,0.5, 0.5, 0) and (0.3,0.3,0.3,0.5,0.5, 0.5, 0.05); $(\sigma, \rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{23}, \rho_{24}, \rho_{25}, \rho_{34}, \rho_{35}, \rho_{45}, \delta)$ = (0.2, 0.6, 0.8, 0.8, 0.6, 0.6,0.6, 0.6, 0.6, 0.9, 0.6, 0) and (0.2, 0.6, 0.8, 0.8, 0.6, 0.6,0.6, 0.6, 0.6, 0.9, 0.6, 0.05). Again, calibration of these parameters is for the purpose of parsimony. They also include underlying stocks paying or without paying the dividends, the increases in the volatilities and changes in the correlation coefficients among underlying stocks. However there are either $\sigma_i \neq \sigma_j$ or $\rho_{ij} \neq \rho_{ik}$ in the combinations. Each option can also have one-year, two-year or three-year maturity.

I focus on the relative pricing errors by estimating basket option prices with and without lognormal distribution for the underlying basket values. The relative pricing errors are measured by $(C_{ln}$ ï $C_{mc})/C_{mc}$ for the calls and $(P_{ln}$ ï $P_{mc})/P_{mc}$ for the puts, where C_{ln} and P_{ln} are the Bermudan call price and put price obtained by applying the binomial tree approach under the assumption that the value of the basket follows a geometric Brownian motion, or the LN-approach, C_{mc} and P_{mc} are the Bermudan call price and put price obtained by applying the LSM approach.

I discuss the changes of pricing errors (i) in the maturities of basket options since prior studies show a short maturity has small pricing errors; (ii) in the volatilities (and the correlations) for the underlying assets since prior studies argue that small volatilities for the underlying assets have small pricing errors; (iii) in the exercise prices since early research claims that pricing errors differ for out-of-the-money and in-the-money options (Levy (1992)); (iv) in the number of underlying assets in the baskets to examine whether there is any ñsmooth effectò on the distribution of basket value and diversification effect; (v) in the dividend payments since theory tells us that option prices change in the underlying stocksôdividend payments.

3.1 Pricing errors change in maturities

Assume that the Bermudan basket option can be exercised at the end of each quarter.
Table 1 below shows the pricing errors based on the simulated pooled sample, which consists of 90 basket options with one-year, two-year or three-year maturity in the symmetric cases.

Table 1 : **Pricing Errors Changes in Maturities**

Table 1 shows the changes in the pricing errors inthe maturities of basket options. Relative pricing errors are measured by (C_{ln} ï C_{mc})/C_{mc} for calls and (P_{ln} ï P_{mc})/P_{mc} for put, where C_{ln} and P_{ln} are the Bermudan call price and put price under the assumption that the value of the basket follows a geometric Brownian motion, C_{mc} and P_{mc} are the Bermudan call price and put price obtained by applying the LSM approach. Options here can have 1-year, 2-year or 3-year maturity and numbers of 2, 3 or 5 underlying assets. The weights in each basket are respectively (0.45, 0.55), (0.15, 0.35, 0.50) and (0.05, 0.15, 0.20, 0.25, 0.35) for 2-asset, 3-asset and 5-asset basket options. Each underlying stock has an initial price of 100 and each option has one of the exercise prices (80, 90, 100, 110, 120). Interest rate is 5%. The pooled simulated sample option prices are generated from 90 basket options in the symmetric scenarios and 50 basket options in the asymmetric scenarios. They include cases in which the underlying stocks paying or without paying the dividends, increase in the volatilities and changes in the correlation coefficients among the underlying stocks.

Panel A: Symmetric and asymmetric cases						
	Symmetric cases					
	T=1		T=2		T=3	
	Call	Put	Call	Put	Call	Put
Mean(%)	-3.99	5.87	-3.6	3.44	-3.39	2.74
Stdev	9.2	8.37	4.47	3.78	3.58	3.02
t-ratio	-4.17	6.61	-7.61	8.59	-8.94	8.54
	Asymmetric cases					
	T=1		T=2		T=3	
	Call	Put	Call	Put	Call	Put
Mean(%)	-1.52	7.8	-1.45	5.41	-0.62	5.12
Stdev	8.81	9.09	4.38	4.91	4.85	4.07
t-ratio	-1.21	6.01	-2.32	7.71	-0.9	8.79
Panel B: Pooled sample						
	T=1		T=2		T=3	
	Call	Put	Call	Put	Call	Put
Mean(%)	-3.12	6.56	-2.84	4.15	-2.4	3.59
Stdev	8.99	8.65	4.54	4.31	4.28	3.61
t-ratio	-4.07	8.93	-7.36	11.35	-6.62	11.72

If the time to maturity of the basket option *T=1*, then Table 1 shows that the LN-approach is likely to underestimate the calls by nearly 4% and overestimate the puts by about 5.9%. If the time to maturity of the basket option *T=2*, the LN-approach will underestimate the calls by 3.6% and overestimate the puts by about 3.4%. If the time to maturity of the basket option *T=3*, the LN-approach will underestimate the calls by 3.4% and overestimate the puts by about 2.7%. All differences are statistically significant. In contrast to Levyùs (1992) findings that the LN-approach produces good approximation for ñtruthò Asian option price for an option with relative short maturity and small volatilities for the underlying assets, the relative pricing errors here seem to be reduced with the increase in maturity of basket options.[7] This may suggest that the effect of time to maturity differs for a path-dependent option and a typical American basket option.

[7] My results, not reported in this paper, show that the mispricing errors for relative short maturity options and small volatilities for the underlying assets based in the LN-approach in a

The reported pricing errors in the asymmetric scenarios are based on the simulated pooled sample with 50 basket options with one-year, two-year or three-year maturity. Whilst the general trend is the same as that in the symmetric scenarios: the relative pricing errors seem to be reduced with the increase in maturity of basket options, asymmetric scenarios enlarge the mispricing errors for the corresponding puts and reduce the mispricing errors for the corresponding calls. Specifically, the LN-approach underestimates the calls by nearly 1.5% and overestimates the puts by about 7.8% if the options have one-year maturity. If the options have two-year maturity, the LN-approach underestimates the calls by about 1.5% and overestimates the puts by about 5.4%. For $T=3$, the LN-approach underestimates the calls by 0.6% and overestimates the puts by about 5.1% if the options have three-year maturity. Similar to the symmetric cases, all differences are statistically significant. The evidence on the call options here does not fully support Brigo et alÕs (2001) findings on the symmetric feature.[8] The results on the total pooled sample are generally consistent with that in the segment samples, i.e., the relative pricing errors seem to be reduced with the increase in the maturity of basket options.

3.2 Pricing errors change in volatilities and correlations of underlying assets in the basket

Table 2 below shows the pricing errors based on the simulated pooled sample, which consists of 90 basket options for each volatility-correlation coefficient pair (σ, ρ) = (0.2, 0.5), (0.5, 0.5) and (0.2, 0.9) for the symmetric scenarios. For example, for a three-asset basket option, it can have possible exercise prices (80, 90, 100, 110,120), possible maturity one-year, two-year or three-year, and with or without the dividend payments for the underlying assets.

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pooled sample are not significantly different from that for relative long maturity options and relative big volatilities for the underlying assets. I have considered option maturities: T=0.08333 (1-month), 0.25 (1-quater), 0.5 (6-month), 0.75(9-month) with exercise prices (80, 90, 100, 110, 120) and various combinations of volatilities and correlations (σ, ρ) with and without dividend payments for the underlying assets.

[8] Results shown in Brigo et al (2001) suggest that the valuation approximations based on assumption of lognormal distribution gives a reasonable good accuracy with respect to the ÔtrueÕ price only for symmetric scenarios (i.e. when volatilities are roughly the same) and high correlations.

Table 2 : **Pricing Errors Change in Volatilities and Correlations**

Table 2 shows the changes in the pricing errors in the volatilities and the correlations of underlying assets in the basket options. Relative pricing errors are measured by $(C_{ln} ï C_{mc})/C_{mc}$ for calls and $(P_{ln} ï P_{mc})/P_{mc}$ for puts, where C_{ln} and P_{ln} are the Bermudan call price and put price under the assumption that the value of the basket follows a geometric Brownian motion, C_{mc} and P_{mc} are the Bermudan call price and put price obtained by applying the LSM approach. Options here can have 1-year, 2-year or 3-year maturity and numbers of 2, 3 or 5 underlying assets. The weights in each basket are respectively (0.45, 0.55), (0.15, 0.35, 0.50) and (0.05, 0.15, 0.20, 0.25, 0.35) for 2-asset, 3-asset and 5-asset basket options. Each underlying stock has an initial price of 100 and each option has one of exercise prices (80, 90, 100, 110, 120). Interest rate is 5%. The pooled simulated sample option prices are generated from 90 basket options in the symmetric scenarios and 150 basket options in the asymmetric scenarios. They include cases in which the underlying stocks paying or without paying the dividends, increase in the volatilities and changes in the correlation coefficients among the underlying stocks.

	Symmetric cases (σ, ρ)					
	(0.2, 0.5)		(0.5,0.5)		(0.2,0.9)	
	Call	Put	Call	Put	Call	Put
Mean(%)	-5.02	5.68	-1.33	2.65	-4.64	3.73
Stdev	6.25	5.84	4.03	4.08	7.12	6.59
t-ratio	-7.58	9.16	-3.11	6.12	-6.14	5.34
	Asymmetric cases			Pooled sample		
	Call	Put		Call	Put	
Mean(%)	-1.2	6.11		-2.78	4.76	
Stdev	6.31	6.48		6.31	6.08	
t-ratio	-2.32	11.51		-9.02	16.04	

If the volatilities of all underlying assets are equal to 0.2 and the correlation coefficients of all underlying assets are equal to 0.5, then Table 2 shows that the LN-approach is likely to underestimate the calls by nearly 5% and overestimate the puts by about 5.7%. If volatilities of all underlying assets are increased to 0.5 from 0.2 and keeping the same correlation coefficients 0.5, then the LN-approach is likely to underestimate the calls by 1.3% and overestimate the puts by about 2.7%. If the correlation coefficients of all underlying assets are increased to 0.9 from 0.5 and the volatilities of all underlying assets remain to be 0.2, then the LN-approach is to underestimate the calls by about 4.6% and overestimate the puts by about 3.7%. Therefore, both increasing in the volatilities and the correlations of the underlying assets reduce mispricing errors.

There are total 150 basket options on the simulated pooled sample in the asymmetric scenarios. The options can have five different exercise prices and three maturity dates. They include (σ_1, σ_2) = (0.2, 0.5) with and without the dividend payments for two underlying assets; $(\sigma_1, \sigma_2, \sigma_3)$ = (0.2, 0.5, 0.9), $(\rho_{12}, \rho_{13}, \rho_{23})$ = (0.7, 0.9, 0.9) with and without the dividend payments for three underlying assets; $(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5)$ = (0.3,0.3,0.3,0.5,0.5), $(\rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{23}, \rho_{24}, \rho_{25}, \rho_{34}, \rho_{35}, \rho_{45})$ = (0.6, 0.8, 0.8, 0.6, 0.6,0.6, 0.6, 0.6, 0.9, 0.6) with and without the dividends payments for five underlying assets. The LN-approach underestimates the calls by nearly 1.2% and overestimates the puts by about 6.1%. It seems that the asymmetric scenarios reduce mispricing errors for corresponding calls and enlarge mispricing errors for corresponding puts. Again, the evidence on the call options here does not fully support Brigo et alôs (2001) findings on the symmetric feature. The results in the total pooled sample are consistent with that in the segment samples, i.e., the LN-approach underestimates the calls and overestimates the puts.

3.3 Pricing errors change in exercise prices

Table 3 below shows the pricing errors based on the simulated pooled sample, which consists of 54 basket options for each of exercise prices (80, 90,100,110,120) in the symmetric scenarios. For each volatility-correlation pair (σ, ρ) = (0.2, 0.5), (0.5, 0.5) and (0.2, 0.9) with and without the dividend payments for the underlying assets, basket options may have one-year, two-year or three-year maturity and two-asset, three-asset or five-asset in the basket.

There are 30 basket options for each of exercise prices (80, 90, 100, 110, 120) for the simulated pooled sample in the asymmetric scenarios. They include (σ_1, σ_2) = (0.2, 0.5) with and without the dividend payments for two underlying assets; $(\sigma_1, \sigma_2, \sigma_3)$ = (0.2, 0.5, 0.9), $(\rho_{12}, \rho_{13}, \rho_{23})$ = (0.7, 0.9, 0.9) with and without the dividend payments for three underlying assets; $(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5)$ = (0.3,0.3,0.3,0.5,0.5), $(\rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{23}, \rho_{24}, \rho_{25}, \rho_{34}, \rho_{35}, \rho_{45})$ = (0.6, 0.8, 0.8, 0.6, 0.6,0.6, 0.6, 0.6, 0.9, 0.6) with and without the dividends payments for five underlying assets. Options may have one-year, two-year or three-year maturity.

Table 3 : **Pricing Errors Change in Exercise Prices**

Table 3 shows the changes in the pricing errors in the exercise prices of basket options. Relative pricing errors are measured by (C_{ln} ï C_{mc})/C_{mc} for calls and (P_{ln} ï P_{mc})/P_{mc} for puts, where C_{ln} and P_{ln} are the Bermudan call price and put price under the assumption that the value of the basket follows a geometric Brownian motion, C_{mc} and P_{mc} are the Bermudan call price and put price obtained by applying the LSM approach. Options here can have 1-year, 2-year or 3-year maturity and numbers of 2, 3 or 5 underlying assets. The weights in each basket are respectively (0.45, 0.55), (0.15, 0.35, 0.50) and (0.05, 0.15, 0.20, 0.25, 0.35) for 2-asset, 3-asset and 5-asset basket options. Each underlying stock has an initial price of 100. Interest rate is 5%. The pooled simulated sample option prices are generated from 54 basket options in the symmetric scenarios and 30 basket options in the asymmetric scenarios for each of exercise prices (80, 90, 100, 110, 120). They include cases in which the underlying stocks paying or without paying the dividends, increase in the volatilities and changes in the correlation coefficients among the underlying stocks.

Panel A: Symmetric and asymmetric cases

	Exercise price	Calls			Puts		
		Mean	st.dev	t-ratio	mean	st.dev	t-ratio
Symmetric	80	-1.95	2.71	-5.24	3.2	7.39	3.15
	90	-1.6	2.83	-4.12	8.68	7.51	8.41
	100	-6.08	3.67	-12.06	1.3	3.54	2.7
	110	-0.72	4.31	-1.22	3.81	2.89	9.61
	120	-7.96	10.19	-5.69	3.08	1.75	12.85
Asymmetric	80	0.58	4.11	0.76	10.59	10.33	5.52
	90	0.48	3.62	0.72	10.24	5.36	10.28
	100	-4.12	4.33	-5.11	2.27	2.54	4.82
	110	0.76	3.67	1.12	4.31	2.24	10.33

120	-3.71	10.8	-1.85	3.13	1.66	10.2

Panel B: Pooled sample

Exercise price	Calls			Puts		
	mean	st.dev	t-ratio	mean	st.dev	t-ratio
80	-1.04	3.47	-2.74	5.84	9.21	5.78
90	-0.86	3.27	-2.39	9.23	6.83	12.32
100	-5.37	4	-12.22	1.65	3.23	4.66
110	-0.19	4.14	-0.42	3.99	2.67	13.6
120	-6.44	10.55	-5.57	3.1	1.7	16.57

Consistent with the findings above, in general, the LN-approach underestimates the calls and overestimates the puts. Specifically, in the symmetric scenarios, the LN-approach underestimates calls by nearly 6% and 8% for ñat-the-moneyò calls and deep ñout-of-the-moneyò calls respectively. In the asymmetric scenarios, the LN-approach underestimates the calls by about 4.1% and 3.7% for ñat-the-moneyò calls and deep ñout-of-the-moneyò calls respectively. In contrast, the mispricing error, 1.3%, is the smallest for ñat-the-moneyò puts. Note that ñout-of-the-moneyò puts have bigger mispricing errors than ññin-the-moneyò puts while ññin-the-moneyò calls have smaller mispricing errors than ñout-of-the-moneyò calls. Comparing to the symmetric scenarios, the LN-approach has smaller mispricing errors for the corresponding call options and larger mispricing errors for the corresponding put options in the asymmetric scenarios.

The pooled sample generally supports the above findings. The LN-approach underestimates the calls and overestimates the puts. The mispricing errors for ññin-the-moneyò calls, 1% for the options with exercise price 80 and 0.9% for the options with exercise price 90, are less than that for ñout-of-the-moneyò puts, 5.8% for the options with exercise price 80 and 9.2% for the options with exercise price 90. The mispricing error for deep ñout-of-the-moneyò calls, 6.4%, is more than twice that of deep ññin-the-moneyò puts.

3.4 Pricing errors change in dividend payments for the underlying assets in the basket

The calculation of the pricing errors in Table 4 is based on the simulated pooled sample, which consists of 135 basket options for dividend payments, 0% or 5%, of all underlying assets in the basket options for the symmetric scenarios. For example, for a three-asset basket option, it can have possible exercise prices (80, 90, 100, 110,120), possible maturities one-year, two-year or three-year, and volatility-correlation pair (σ, ρ) = (0.2, 0.5), (0.5, 0.5) and (0.2, 0.9) for the underlying assets.

There are 75 basket options for dividend payments, 0% and 5%, of underlying assets on the simulated pooled sample in the asymmetric scenarios. The options can have five different exercise prices and three maturity dates. They include (σ_1, σ_2) = (0.2, 0.5) for two underlying assets; $(\sigma_1, \sigma_2, \sigma_3)$ = (0.2, 0.5, 0.9), $(\rho_{12}, \rho_{13}, \rho_{23})$ = (0.7, 0.9, 0.9) for three underlying assets; $(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5)$ = (0.3,0.3,0.3,0.5,0.5), $(\rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{23}, \rho_{24}, \rho_{25}, \rho_{34}, \rho_{35}, \rho_{45})$= (0.6, 0.8, 0.8, 0.6, 0.6,0.6, 0.6, 0.6, 0.9, 0.6) for five underlying assets.

Table 4 : **Pricing Errors Change in Dividends**

Table 4 shows the changes in the pricing errors in dividend payments of the underlying assets in the basket options. Relative pricing errors are measured by $(C_{ln} ï C_{mc})/C_{mc}$ for calls and $(P_{ln} ï P_{mc})/P_{mc}$ for put, where C_{ln} and P_{ln} are the Bermudan call price and put price under the assumption that the value of the basket follows a geometric Brownian motion, C_{mc} and P_{mc} are the Bermudan call price and put price obtained by applying the LSM approach. Options here can have 1-year, 2-year or 3-year maturity and numbers of 2, 3 or 5 underlying assets. The weights in each basket are respectively (0.45, 0.55), (0.15, 0.35, 0.50) and (0.05, 0.15, 0.20, 0.25, 0.35) for 2-asset, 3-asset and 5-asset basket options. Each underlying stock has an initial price of 100 and each option has one of exercise prices (80, 90, 100, 110, 120). Interest rate is 5%. The pooled simulated sample option prices are generated from 135 basket options in the symmetric scenarios and 75 basket options in the asymmetric scenarios for dividend payments of 0% and 5% for all underlying stocks. They include cases in which the volatilities increase and changes in the correlation coefficients among the underlying stocks.

	Symmetric cases				Asymmetric cases			
dividend yield	0%		5%		0%		5%	
	Call	Put	Call	Put	Call	Put	Call	Put
Mean(%)	-0.72	1.96	-6.6	6.07	1.41	4.86	-3.8	7.36
Stdev	4.38	5.05	6.28	5.62	4.89	6.79	6.51	5.94
t-ratio	-1.91	4.48	-12.2	12.5	2.49	6.16	-5	10.65
	Pooled sample							
dividend yield	0%		5%					
	Call	Put	Call	Put				
Mean(%)	0.04	2.99	-5.6	6.53				
Stdev	4.67	5.89	6.48	5.76				
t-ratio	0.12	7.35	-12.5	16.4				

Table 4 shows that dividend payments of underlying assets have big effect on the mispricing errors for the LN-approach. In the symmetric scenarios when underlying stocks in the baskets pay no dividend, the LN-approach mis-prices the calls by less than 1% and the puts by less than 2%. When the underlying stocks pay 5% dividends, the calls will be underestimated by 6.6% and the puts will be overestimated by about 6.1%.

In the asymmetric scenarios, while the LN-approach overestimates the calls by 1.4% for non-dividend paying underlying stocks, it underestimates the calls by 3.8% if underlying stocks pay 5% dividends. The dividends payments also increase the magnitudes of overestimating errors on the put options from about 4.9% to 7.4%.
While the mispricing errors on both the calls and the puts are small for no-dividend paying underlying assets in the pooled sample, the LN-approach underestimates the calls by 5.6% and overestimates the puts by about 6.5% if the stocks in the basket pay dividends.

3.5 Pricing errors change in number of assets in the basket

Table 5 below shows the pricing errors based on the simulated pooled sample, which consists of 90 basket options for each of two-asset, three-asset or five-asset basket options in the symmetric scenarios. For each volatility-correlation pair $(\sigma, \rho) = (0.2, 0.5), (0.5, 0.5)$ and $(0.2, 0.9)$ with and without dividend payments for the underlying assets, basket options may have one-year, two-year or three-year maturity and one of exercise prices (80, 90,100,110,120).

There are 60 basket options for each of 3-asset and 5-asset basket and 30 basket options for each of 2-asset basket for the simulated pooled sample in the asymmetric scenarios. Options can have one of exercise prices (80, 90, 100, 110, 120) and one-year, two-year or three-year maturity. For the two-asset options, I have $(\sigma_1, \sigma_2) = (0.2, 0.5)$ with and without the dividend payments. For the three or five underlying assets options, I have $(\sigma_1, \sigma_2, \sigma_3) = (0.2, 0.5, 0.9)$, $(\rho_{12}, \rho_{13}, \rho_{23}) = (0.7, 0.9, 0.9)$, $(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5) = (0.3,0.3,0.3,0.5,0.5)$, $(\rho_{12}, \rho_{13}, \rho_{14}, \rho_{15}, \rho_{23}, \rho_{24}, \rho_{25}, \rho_{34}, \rho_{35}, \rho_{45}) = (0.6, 0.8, 0.8, 0.6, 0.6,0.6, 0.6, 0.6, 0.9, 0.6)$ with and without the dividends payments.

Table 5 : **Pricing Errors Change in Number of Assets**

Table 5 shows the changes in the pricing errors in the number of underlying assets in the basket options. Relative pricing errors are measured by (C_{ln} ï C_{mc})/C_{mc} for calls and (P_{ln} ï P_{mc})/P_{mc} for put, where C_{ln} and P_{ln} are the Bermudan call price and put price under the assumption that the value of the basket follows a geometric Brownian motion, C_{mc} and P_{mc} are the Bermudan call price and put price obtained by applying the LSM approach. Options here can have 1-year, 2-year or 3-year maturity and number of 2, 3 or 5 underlying assets. The weights in each basket are respectively (0.45, 0.55), (0.15, 0.35, 0.50) and (0.05, 0.15, 0.20, 0.25, 0.35) for 2-asset, 3-asset and 5-asset basket options. Each underlying stock has an initial price of 100 and each option has one of exercise prices (80, 90, 100, 110, 120). Interest rate is 5%. The pooled simulated sample option prices are generated from 90 basket options in the symmetric scenarios. There are 30 basket options written on 2-asset basket, 60 basket options written on 3- and 5-asset basket in the asymmetric scenarios. They include cases in which underlying assets paying or without paying dividends, the volatilities increase and changes in the correlation coefficients among the underlying stocks.

Panel A: Symmetric and asymmetric cases

Symmetric cases

	2-asset		3-asset		5-asset	
	Call	Put	Call	Put	Call	Put

Mean(%)	-4.5	2.79	-3.51	4.18	-3.05	5.08
Stdev	6.02	5.67	6.34	5.47	6.05	5.79
t-ratio	-7.13	4.69	-5.23	7.21	-4.76	8.28
Asymmetric cases						
	2-asset		3-asset		5-asset	
	Call	Put	Call	Put	Call	Put
	-1.42	7.64	-0.38	5.7	-1.9	5.76
	3.89	8.54	7.35	5.9	6.14	5.82
	-1.97	4.81	-0.4	7.42	-2.38	7.59
Panel B: Pooled sample						
	2-asset		3-asset		5-asset	
	Call	Put	Call	Put	Call	Put
Mean(%)	-3.74	3.98	-2.25	4.79	-2.59	5.35
Stdev	5.72	6.79	6.91	5.68	6.09	5.79
t-ratio	-7.2	6.45	-3.99	10.3	-5.19	11.28

Consistent with early findings, the symmetric scenarios in Panel A of Table 5 show that the LN-approach underestimates the calls and overestimates the puts. It is interesting to note that the magnitudes of underestimation of the calls decreases and the magnitudes of overestimation of the puts increases in the number of assets in the basket. They are respective 4.5%, 3.5% and 3.1% for the two-asset, three-asset and five-asset call options, and 2.8%, 4.2% and 5.1% for the two-asset, three-asset and five-asset put options. All differences are statistically significant. This may reflect some kind of ósmoothô effect in a portfolio analysis. The pooled sample in Panel B generally supports this finding. Though the asymmetric scenarios support the underestimation of the calls and overestimation of the puts by the LN-approach, the changes in magnitudes of underestimation and overestimation in the number of assets in the basket produce mixed results in the asymmetric scenarios.

4 - Conclusion

An analytic pricing model on a standard option is typically built on the assumption that the underlying asset follows a lognormal distribution. With this conventional assumption on the value of a basket, pricing a Bermudan basket option becomes a simple matter and we can ignore that the complexity of multi-dimensionality of the underlying state variables. The difficulty on valuation of a basket option arises when the value of the basket does not follow a lognormal distribution although the individual underlying assets in the basket do. Indeed, there is no simple relationship in the underlying parameters of the option to result in the validity of this assumption. Without this assumption, most numerical methods become impractical when an option is written on a number of assets. Nevertheless the numerical exercises in the prior studies suggest that a lognormal distribution is a good approximation for the value of a basket.

In this paper, I address the accuracy and the computational efficiency in pricing a Bermudan basket option. I calculate the ñtrueò value of a Bermudan basket option by employing the least-squares Monte Carlo simulation developed by Longstaff and Schwartz (2001). My results show that in general the LN-approach is likely to underestimate the calls and overestimate the puts. This may result from the payoff profiles of call and put options and non-normality of value of a basket. The relative pricing errors seem to be reduced with the increases in the maturities of basket options. While the LN-approach enlarges mispricing errors for the corresponding puts in the asymmetric scenarios, I find that the LN-approach reduces mispricing errors for the corresponding calls. When the volatilities of underlying assets increase and other parameters keep constant, or the correlations of underlying assets increase and keep other parameters constant, the LN-approach reduces the mispricing errors. My results indicate that ñout-of-the-moneyò puts have bigger mispricing errors than ñin-the-moneyò puts whilst ñin-the-moneyò calls have smaller mispricing errors than ñout-of-the-moneyò calls.

The evidence shows that the dividend payments of underlying assets have considerable effect on the mispricing errors for the LN-approach. In general, the approximation is significantly improved if the underlying assets in the basket pay no dividends. My simulation results also suggest that the magnitudes of underestimation of call options decrease and the magnitudes of overestimation of the puts increase in the number of assets in the basket for the pooled sample.

The findings may suggest that the return of a basket is not behaved as ógoodô as a lognormal distribution. Asset-price dependent volatility in a basket option pricing may be responsible for some puzzles found in this paper. It would be interesting to examine whether there is a smile effect or a sloppy smile effect in the pricing of basket options. I leave it as a future research topic.

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