Risk in Emerging Stock Markets from Brazil and Mexico: Extreme Value Theory and Alternative Value at Risk Models

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Abstract

Conventional Value-at-risk (VaR) models tend to underestimate stock market losses, as they assume normality and fail to capture the frequency and severity of extreme fluctuations, Extreme value theory (EVT) overcomes this limitation by providing a framework in which to analyze the extreme behavior of stock-markets returns and by quantifying possible losses during financial turbulences. This study uses the *c*-quantile of a fat-tailed distribution for VaR analysis. An innovation in the present work is the application of EVT not only to the left tail of the returns distribution but also to its right tail, while assessing long and short positions. A generalized extreme value distribution (GEVD) is used to analyze the two largest stock markets from Latin America, Brazil and Mexico; a conditional VaR (CVaR) model is applied to determine risk exposure from investing in those markets, with daily index data for the period 1970-2004. The results confirm the presence of fat tails in both markets as a result of the excess of kurtosis; the empirical evidence shows that VaR and CVaR based on EVT yield more precise and robust information about financial risk than conventional parametric estimations.

Keywords: Extreme Values, Value-at-Risk, Risk Management, Emerging Capital Markets, Mexico, Brazil.
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1 – Introduction

Risk management has become one of the most important issues in contemporary finance. Financial and economic globalization has generated many investment opportunities in both mature and emerging markets and thus has generated greater diversification schemes aiming at optimizing portfolio returns. However, a very complex environment influences portfolio holdings in domestic and international variables as well as changes in expectations determine variations in asset prices. Potential losses might result from adverse trends in interest rates, exchange rates, and inflation rates, and high volatility of prices of goods and services, along with risk factors derived from local and international social and political variables.

In response to these challenges, risk management has developed important tools to measure and control risk to prevent potential losses. In practice, Value-at-Risk (VaR) has become one of the most important tools used to manage risk. Parametric and non parametric (simulation) VaR models have been developed in the financial literature. However, these models cannot always cope with large periods of strong financial instability such as the Mexican 1994-1995 peso crisis and the Brazilian crisis of 1998. Indeed large portfolio losses have also occurred from other financial crises affecting the international financial markets in recent decades.[3] The presence of atypical and unexpected movements limits the efficiency of conventional VaR models because of excess kurtosis present in the distributions of returns (Duffie and Pan, 1997). Similarly, GARCH models with normal innovations or t-distributions introduced by Jorion (1998), and Yang and Brorsen (1995) have proved insufficient to capture different degrees of leptokurtosis, as well as asymmetries of risk factors. Although non parametric VaR models yield more reliable results than the conventional delta-normal modeling, they are limited in adjusting for extreme returns, which are very important to measure risk adequately to insure the survival of financial institutions and the stability of financial systems (Hendricks, 1996; Jackson, Maude and Perraudin, 1997; Vlaar, 2000). In short, conventional VaR models fail to estimate extraordinary

[3] In the case of two-parameter distributions, to estimate VaR for long-horizons practitioners generally first calculate the daily VaR and then apply it to a longer investment horizon by using the Square Root Rule (SRR). Kaplanski and Levy (2010) show that SSR is theoretically incorrect inducing an overestimation for short periods and an underestimation for long periods; they propose a correct measure employing Tobin's multiperiod formulas to calculate the n-period mean and standard deviation of returns, which are used as inputs for the n-period VaR estimation.

losses that might arise in times of financial turbulences. This shortcoming has been overcome with a conditional Value-at-Risk model (CVaR), introduced by Artzner et al (1999), which measure the expected shortfall. These measures have been presented in the financial literature as coherent risk measures because they share the same properties when applied to continuous distributions.[4] CVaR estimates the tail risk in a conservative and efficient manner by incorporating both the frequency and the size of extreme events. Nevertheless, assuming normality, CVaR underestimates risk for high confidence levels because it fails to incorporate all the information from the tails of a returns distribution. In response to these limitations, extreme value theory (EVT) sets forth sound models to understand and model the statistical behavior of extreme values. Longin (1996, 2000) pioneered the application of EVT to VaR analysis and other researchers have followed. The generalized extreme value distribution (GEVD), first developed by Jenkinson (1969) and Longin (2001), has been applied to estimate daily VaR for the S&P stock index. In the case of emerging capital markets, the application of EVT has remained very limited.

2 - Literature review

A large body of literature offers empirical evidence~~s~~ of the presence of excess kurtosis in financial series, the heavy or fat-tails effect, as well as different degrees of asymmetry. Following Mandelbrot (1963) and Fama (1965) who pioneered this line of research, several alternative distributions have been proposed to solve the problem of fat-tails in statistical estimations. A fat-tails distribution proposed to model extreme movements is the mixture of normal distributions suggested by Boness et al (1974). The application of this model for risk analysis using VaR was first described by Zangari (1996); Ventakaraman (1997) later improved this model by estimating its parameters using quasi-Bayesian maximum-likelihood estimation. More recently, a clear fat-tails model applied to VaR analysis was introduced by Hull and White (1998). Similarly, Huisman, Koedijk and Pownall (1998) and Heikkinen and

[4] However, Fabozzi and Tunaru (2006) show that while the theoretical superiority of expected shortfall is evident, it is possible to have estimators of expected shortfall that do not satisfy the sub-additivity condition. Similarly, responding to criticisms to conventional VaR models for not providing a coherent risk measure, Acerbi (2002) and Gianapoulos and Tunaru (2005) have presented and applied an entirely new family of risk known as "spectral measures of risk," proving that these new risk measures are both spectral and coherent.

Kanto (2002) proposed the Student's *t*-distribution to capture excess risk by combining the finite variance with the fat-tails distribution of returns. Andreev and Kanto (2005) developed a closed model to determine the impact of leptokurtosis in CVaR estimation. Clark, Giannopoulos and Tunaru (2005) show that by assuming a constant variance/covariance matrix over the holding period, the VaR limits can often be exceeded within the relevant horizon period. To minimize this risk they reformulate, the problem by taking into consideration the variability of risk on all assets eligible to be included in the portfolio, applying a CVaR model that minimizes the VaR at each point of the holding period. Finally, Topaloglu, Vlamidirou and Zenois (2002) develop an integrated simulation and optimization framework for multicurrency asset allocation problems; their simulation applies principal component analysis to generate scenarios depicting the discrete joint distributions of uncertain asset returns and exchange rates and then develop and implement models that optimize the conditional value-at-risk (CVaR); lastly, they also compare the performance of the CVaR model against that of a model that employs the mean absolute deviation (MAD) risk measure. Their evidence shows that in static tests the MAD and CVaR models often select portfolios that trace practically indistinguishable ex-ante risk–return efficient frontiers; in successive applications over several consecutive time periods the CVaR model attains superior ex-post results in terms of both higher returns and lower volatility. These alternatives to the normal distribution model solve some of the problems with heavy tails and yield better risk estimates; however they present several disadvantages, mainly the lack of closed form expressions; additionally, their models neglect adjustment in the distribution the asymmetry levels observed in financial returns.

Responding to inconsistencies from previous models, to capture the magnitude and probability of extreme values, the theory of extreme events provides a set of sound tools to assess and model risk, present in empirical distributions. Although EVT has a long standing in the scientific world, successfully having solved problems related to hydrology, weather changes, and insurance, it is only recently that it has begun to be applied in financial economics to model the impact of tail events on portfolio holdings. Empirical studies that apply EVT to examine the behavior of returns to changes in exchange rates include Danielsson and de Vries (1997), Hols and de Vries (1991) and Koedijk, Shanfgans and de Vries (1990). Interest in EVT for risk analysis and management to measure the behavior of financial markets has grown considerably during the last decade, particularly in the case of mature markets. Longin (1996, 2000) pioneered the application of EVT in VaR analysis, using the generalized extreme-values distribution, estimating VaR for daily returns of the S&P stock market index.[5] A more recent study, by Danielsson and de Vries (2000) and based on a hybrid model, estimated the tail index and quantified VaR for random portfolios. McNeil (1999) and McNeil and Frey (2000) used daily data to illustrate the potential of the generalized Pareto distribution (GPD) to measure tail risk related to the empirical distributions, in the case of the DAX and S&P stock indexes, and Loretan and Phillips (1994) focused on stock returns at a daily frequency. Finally, an important contribution is that from Giot and Laurent (2003), who model VaR using daily data for three international stock indexes and three US stocks of the Dow Jones index, using a collection of parametric models of the ARCH family based on the skewed Students t-distribution. Their evidence shows that models that rely on a symmetric density distribution for the error term underperform with respect to skewed density models when the left and right tails of the distribution of returns must be modeled. This approach to assess VaR for long and short positions is also used in this study.

The choice of right risk measures, including alternative VaR models, is particularly critical for the case of emerging markets which are characterized by high instability (Consigli, 2002). Important research has been accomplished by, among others, Jondeau and RocKinger (2003) and Susmel (2001); they used GPD to compare the asymptotic behavior from mature and emerging stock markets; Gencay and Selcuk (2004) estimated VaR using EVT for several emerging markets; da Silva and Mendes (2003), and Ho et al (2000), based on the construction of block maxima, examined potential losses in the Asian stock markets. Similarly, Bao, Lee and Saltoglu (2006), Fantazzini (2008) and Caillault and Guégan (2009) compared the predictive power of various classes of VaR models. Bao, Lee and Saltoglu (2006) examined the stock markets of five Asian economies that suffered from the 1997 financial crisis (Indonesia, Malaysia, Korea, Taiwan, Thailand); measuring the quantile loss, their study provided evidence that the Riskmetrics model performed reasonably well during tranquil periods and some EVT-based models did better in crisis periods. Caillault and Guégan (2009) apply several parametric and nonparametric models to compute VaR

[5] Alentorn and Markose (2008) developed fine computational methods and analytical models applying the generalized extreme value distributions.

and expected shortfall, to measure risk associated to a two-dimensional portfolio composed of three Asian composite indexes (the Thai SET index, the Malaysian MKLC index, and the Indonesian JCI); data sets start from July 2, 1987 and finish December 17, 2002. Their evidence shows that the bivariate distribution of the portfolios is not stable along the period under analysis. For this reason they compute value-at-risk and expected shortfall in a dynamic way, applying the Riskmetrics approach, the multivariate GARCH models, the multivariate Makov switching models, the empirical histogram, and the dynamic copulas. The GARCH model seems more appropriate, except duing the Asian crisis; the copula approach seem to be more appropriate for banks because it allows them to take into account all crises. Seymur and Polakow (2003) estimated VaR applying historical simulation, using EVT and a model that included both EVT and volatility updating via GARCH-type modeling. The latter model provided significantly better results than the previous ones, which was consistent with Bali and Theodossiou (2007) who proposed a skewed generalized t distribution based on GARCH models to incorporate mean and volatility estimates in the estimation of VaR and expected shortfall measures.[6] Considering the Latin American emerging markets, Mendes, Ferreira and Duarte (2000) applied EVT to estimate VaR for the stock markets from this region; Fernandez (2003) presented empirical evidence for the Chilean stock market; Hotta, Lucas and Palaro (2008) estimated VaR for a two-asset portfolio composed of the stock indices from the stock exchanges from Brazil and Argentina. In short, the literature related to extreme returns associated with booms and crash of emerging Latin American stock markets is scarce, even though financial authorities and investors have a growing need to assess risks concerning their institutions and markets.[7]

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[6] It is worth mentioning that Bali and Theodessiou (2008) also found that the skewed generalized distribution performed as well as the more specialized generalized extreme value distributions in modeling tail behavior in portfolio returns. Fantazzini (2008) proposes dynamic copula and marginal functions to model the joint distribution of risk factor returns affecting portfolios profit and loss over a specified holding period. With this methodology is possible to take into account marginal functions capable of considering higher moments. Using Monte-Carlo simulation for portfolios made of the S&P stock index, the DAX Index, and the Nikkei225 Index, their empirical analysis shows that in the true marginal distributions of financial assets are usually present fat-tails and strong skewness.

[7] Research on emerging markets is often limited due to the lack of data. Neuroppos et al (2006) considered VaR analysis a creative approach to overcome this problem. Their risk analysis

3 - Data and extreme values modeling of risk

3.1 Data and sample

The dynamics of extreme movements in high-frequency financial series, which allows estimating a distribution's tail risk, are analyzed in this work using daily data from Brazilian and Mexican emerging stock markets. Daily data were gathered for the period January 2, 1970 through December 31, 2004. A total of 8772 observations for the Mexican Stock Market Index (IPC) and for the Brazilian Stock Market Index (BOVESPA) were obtained from the Reuters Data Bank, complemented with local information from the Central Banks from these countries. A total of 35 years of daily data constituted a fine sample to carry out a thorough analysis concerning extreme movements and risk in Brazil and Mexico. The series include, for instance, financial crises related to the international debt crisis of the 1980's, the 1987 worldwide stock markets crash, the 1994-1995 Mexican currency crisis, the 1997 Asian financial crisis, the 1998 Russian crisis, the 1999 Brazilian currency crisis, and the 2001 Argentinean currency crisis.

3.2 Methodology

Risk analysis and management has evolved to the point that investors are now concerned with the impact of low-probability events in their portfolios. In particular, massive losses might be caused by extreme movements. In this context, conventional Value-at-risk (VaR) estimates are inappropriate, as they are limited to model extreme movements captured on the tails of the returns distributions. Extreme value theory overcomes such shortcomings. This theory studies the asymptotic behavior from extreme observations in order to develop rational procedures that serve as a basis to estimate with greater precision extreme quantiles of a distribution without a need to know the full distribution.

Univariate extreme value theory

Let a set of random variables $R_1, R_2, \ldots, R_n$ represent the daily observations on the returns of a stock market index, and $M_n = \mathrm{Max}\{R_1, R_2, \ldots, R_n\}$ the

applied to the Cyprus and Athens stock markets performed successfully, using enriched historical simulation to supplement local data with data from other markets.

maximum return that a financial asset can attain during n days of trade. If returns are independent and identically distributed and belong to an unknown distribution F_R, then the distribution for maximum return, M_n can be expressed as follows:[8]

$$P\{M_n \le r\} = P\{R_1 \le r, R_2 \le r, \ldots, R_n \le r\} = \prod_{j=1}^{n} P\{R_j \le r\} = F_R^n(r) = F_{M_n}(r). \quad (1)$$

This expression stresses the viability of obtaining the distribution function for the maximum for a finite simple when the distribution F_R is known. However, empirically the use of equation (1) is limited. Based on empirical existing evidence, it can be inferred that neither the distribution of the returns, nor the distribution for the maximum (cf. Longin and Solnik, 2001) is well known. Therefore, the distribution F_R^n is degenerated as $n \to \infty$. To solve this problem the random variable M_n must be transformed so that the asymptotic distribution of the new random variable becomes a non-degenerated distribution. This can be obtained with a simple normalization operation, finding out two sequences of positive numbers $\{\beta_n\}$ and $\{\alpha_n\}$, and $\alpha_n > 0$ that stabilize or adjust the location and scale parameters for $M_n^* = \frac{M_n - \beta_n}{\alpha_n}$, so that it converges in distribution to some non-degenerated random variable as $n \to \infty$. This is one of the fundamental principles from EVT established by Fisher and Tippett (1928)[9] and finally demonstrated by Gnedenko (1943).

Suppose that $R_1, R_2, \ldots, R_n$ are random variables that are independent and identically distributed with a distribution function $F_R(r)$,

[8] Risk management also requires models to describe the behavior of minimum returns due to its importance for VaR estimation. However the remaining of this section only presents the theoretical results for the maximum because results for the minimum can be deduced from the maximum by only changing the sign, that is,

$$\text{Max}\{R_1, R_2, \ldots, R_n\} = -\text{Min}\{-R_1, -R_2, \ldots, -R_n\}$$

[9] This results from the Central Limit Theorem which studies the asymptotic behavior of the maximum of a set of random variables in lieu of the asymptotic behavior of the partial sum.

and let $R_{n1}, R_{n2}, \ldots, R_{nm}$ be its order statistics. If there are the normalization successions $\{\beta_n\}$ and $\{\alpha_n\}$, with $\alpha_n > 0$, so that

$$P\left\{\frac{(M_n - \beta_n)}{\alpha_n} \leq r\right\} = F^n(\alpha_n r + \beta_n) \xrightarrow{d} H(r)$$

where H is a non-degenerated distribution, then H belongs to one of the following families.[10]

Type I: $H(r) = \exp\{-\exp(-r)\}, \qquad -\infty < r < \infty;$

Type II: $$H(r) = \begin{cases} \exp\{-(r^{-\tau})\}, & r > 0, \quad \tau > 0 \\ 0, & r \leq 0 \end{cases}$$

Type III: $$H(r) = \begin{cases} \exp\{-(-r)^{\tau}\}, & r < 0, \quad \tau < 0 \\ 1, & r > 0. \end{cases}$$

Therefore, the distribution F_R is in the maximum domain of attraction of the limiting distribution H. These families of distributions are known as the distributions of standard extreme value. Type I is known as the Gumbel (1958) distribution or else as distribution of light tails, because all its moments are finite and because the behavior of the tails of the distribution decays exponentially. Type II is defined as the Fréchet (1928) distribution or distribution of heavy tails. Gnedenko (1943) showed that if the tails from

[10] The notation "$\xrightarrow{d}$" means convergence in distribution. See: Mittnik and Rachev (1993).

$F_R(r)$ decay slowly, then the distribution represents the maximum domain of attraction of the Fréchet distribution. Finally, Type III is known as the Weibull (1951) distribution or distribution of short or bounded tails. This distribution is characterized by the fact that has a finite right endpoint. The form parameter τ plays a key role in the identification on the shape of the limiting distribution, that is, it represents the weight of the tail of the distribution $H(r)$. Nevertheless, one of the main problems in the analysis of extreme values is the determination of the form parameter τ. This problem can be easily solved reformulating the models of the Fisher and Tippett (1928) theorem reparametrizing $\xi = -1/\tau$ known as the tail index. For estimation purposes it is a very useful alternative representation for the three types of standard extreme value distributions, which can be expressed as follows:

$$H_{\xi}(r) = \begin{cases} \exp\left[-(1-\xi r)^{1/\xi}\right] & \textbf{if} \quad \xi \neq 0 \\ \exp\left[-\exp(r)\right] & \textbf{if} \quad \xi = 0, \end{cases} \qquad \textbf{(2)}$$

for $r < 1/\xi$ if $\xi < 0$ and for $r > 1/\xi$ if $\xi > 0$. This family of distribution functions depending on the tail index ξ, commonly known as the generalized extreme value distribution (GEVD) was first introduced independently by Mises (1936) and Jenkinson (1955) and was reparametrized to a more general context by Maritz and Munro (1967).

This way, the value of the tail index plays a key role for discriminating standard extreme-value distributions. If $\xi_n < 0$ the distribution function $F_R(r)$ is at the maximum domain of attraction of the Fréchet distribution, which is generally valid to model financial series. There are special cases of distributions that have tails that decay in a polynomial manner, which include the $\alpha-$stable, Cauchy, Student's t-distributions and the mixture of normal distributions, are among the most important. When $\xi_n > 0$ it is said that the distribution function $F_R(r)$ belongs to the maximum domain of attraction of the Weibull distribution, which is not very efficient to explain the behavior of financial return series. Some examples of this type of distributions are the uniform and beta distributions. Finally, if $\xi_n = 0$, the distribution function $F_R(r)$ is at the maximum domain of attraction of the Gumbel distribution, which includes the normal, exponential, gamma distributions and lognormal distribution, which has much heavier tails. Similarly if the location and scale parameters β_n y $\alpha_n > 0$ are considered, it is possible to extend the family of distributions such that $H_\xi(r) = H_{\xi_n,\beta_n,\alpha_n}\left((r-\beta_n)/\alpha_n\right)$, an important result for the analysis of extreme values supported by the Fisher and Tippet Theorem (1928), which states that if $F_R \in \mathrm{MDA}(H)$, then H is of the same type of H_ξ for some ξ.

The generalized extreme-value distribution depends on the behavior of a family of parameters. First, the location parameter β_n is defined. This parameter indicates where the extreme values are located on the average. Second is the definition of the scale parameter α_n, which determines the dispersion of the extreme observations. Last to be defined is the tail index ξ_n, which describes the behavior of the tail of the limiting distribution, generally denoted by $\tau = -1/\xi_n$.

Maximum likelihood method for extreme returns

One of the crucial challenges faced in measuring tail risk, presented in a number of empirical financial applications refers to the values of estimated parameters of the generalized extreme-value distribution, particularly the tail index. To estimate the three relevant parameters there are several standard statistical models such as probability-weighted moment estimation, maximum-likelihood estimation, and regression analysis. In this paper the estimation of the parameters of location β_n, scale α_n and tail index ξ_n is carried out using the maximum-likelihood model. Longin (1996, 2000), suggested dividing the total sample into several subsamples. Assume a sample of N returns $r_1, r_2, \ldots, r_N$, is observed that is a realization of random variables $R_1, R_2, \ldots, R_N$, which are independent and identically distributed. Dividing the total sample by m which represents the number of subsamples with n observations each one. In other words, if $N = mn$, the *ith* subsample of the returns series is defined by

$\{r_1, r_2, \ldots, r_n | r_{n+1}, r_{n+2}, \ldots, r_{2n} | \cdots | r_{(m-1)n+1}, r_{(m-1)n+2}, \ldots, r_{mn}\}$. Supposing that $r_{n,i}$ represents the maximum value of the ith subsample, then a set of maximum subsamples can be obtained from the sample of size m. Assuming independence, the log-likelihood function for the collection of maximum subsamples can be expressed as follows,

$$l\{\xi_n, \beta_n, \alpha_n; r_{n1}, r_{n2}, \ldots, r_{mn}\} = \prod_{i=1}^{m} \log h_{\xi_n, \beta_n, \alpha_n}(r_{n,i})$$

$$= -m \log \alpha_n + \left(\frac{1}{\xi_n} - 1\right) \sum_{i=1}^{m} \log\left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right) - \sum_{i=1}^{m}\left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{\frac{1}{\xi_n}}, \quad (3)$$

where $h_{\xi_n, \beta_n, \alpha_n}(r_{n,i})$ is defined as the probability density function of the generalized extreme-value distribution. The maximum-likelihood parameter estimation for generalized extreme value-distribution requires the computation of the first partial derivatives of the log-likelihood function. The partial derivatives with respect to the parameters are:

$$\frac{\partial l}{\partial \beta_n} = \left(\frac{\mathbf{1} - \xi_n}{\alpha_n}\right) \sum_{i=1}^{m} \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{-\mathbf{1}} - \left(\frac{1}{\alpha_n}\right) \sum_{i=1}^{m} \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{\frac{1}{\xi_n} - 1} + = \mathbf{0}$$

$$\frac{\partial l}{\partial \alpha_n} = -\frac{m}{\alpha_n} + \left(\frac{\mathbf{1} - \xi_n}{\alpha_n^2}\right) \sum_{i=1}^{m} \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{-\mathbf{1}} \left(r_{n,i} - \beta_n\right) - \left(\frac{1}{\alpha_n^2}\right) \sum_{i=1}^{m} \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{\frac{1}{\xi_n} - 1} \left(r_{n,i} - \beta_n\right) = \mathbf{0}$$

$$\frac{\partial l}{\partial \xi_n} = -\left(\frac{\mathbf{1}}{\xi_n^2}\right) \sum_{i=1}^{m} \log \cdot \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right) - \left(\frac{1}{\alpha_n}\right)\left(\frac{\mathbf{1}}{\xi_n} - 1\right) \sum_{i=1}^{m} \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{-1} \left(r_{n,i} - \beta_n\right)$$

$$+ \left(\frac{\mathbf{1}}{\xi_n^2}\right) \sum_{i=1}^{m} \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{\frac{1}{\xi_n}} \log \cdot \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right) + \left(\frac{1}{\alpha_n}\right)\left(\frac{\mathbf{1}}{\xi_n}\right) \sum_{i=1}^{m} \left(1 - \xi_n\left(\frac{r_{n,i} - \beta_n}{\alpha_n}\right)\right)^{\frac{1}{\xi_n} - 1} \left(r_{n,i} - \beta_n\right) = 0 \qquad (4)$$

In this case numerical methods and computer algorithms must be used to obtain the maximum-likelihood estimates because the log-likelihood function becomes a multidimensional non-linear optimization problem. If the form parameter $\xi_n > -0.5$, the family of the generalized extreme-value distribution satisfies all conditions of regularity as shown by Smith (1985). That is, the maximum-likelihood estimators are consistent, efficient and asymptotic normal, even if the data are not independent and identically distributed. Similarly, this model helps in calculating standard errors, constructing confidence intervals, and carrying out other inferences related to the GEV parameters $(\xi_n, \beta_n, \alpha_n)$ which follow immediately from the approximate of maximum-likelihood estimator. Nevertheless the statistical procedure of block maxima presents a trade-off problem between bias and variance. A sufficiently large block not only leads to a more precise model estimation, but also yields estimators with low bias and high variance because extreme values are rare by definition. With a small block the number of observations in the sample increases which reduces the variance; there remains the possibility that the asymptotic properties of the model estimation is violated.

Tests of goodness-of-fit

Once the parameters of a family of distributions have been estimated it is important to quantify the uncertainty derived from the estimations made. A goodness-of-fit test on the probability distributions, that is, a statistical validation of how well the model describes or explains the behavior of the available data must be carried out.

Several formal tests have been presented in the literature, and the great majorities are based on a comparison of the estimated distribution with the observed distribution. In this case, Sherman (1957) proposed a test of goodness-of-fit based on a series of ordered data denoted by $(r_{n,i})_{i=1,\ldots m}$. The calculation of the statistic is determined in the following way:

$$\Omega_m = \frac{1}{2}\left|\sum_{i=0}^{m} H_{\xi_n,\beta_n,\alpha_n}(r_{n,i+1}) - H_{\xi_n,\beta_n,\alpha_n}(r_{n,i}) - \frac{1}{m+1}\right|, \tag{5}$$

The statistic Ω_m is distributed asymptotically like a normal variable with mean $(m/(m+1))^{m+1}$ and variance $(2e-5)/me^2$. A low Ω_m statistic indicates that the discrepancy between the estimated distribution and the observed distribution is small; in other words, extreme value theory describes well the behavior of extreme returns.

Another common statistic that helps test whether the maximum-likelihood estimates are statistically significant is the test of the likelihood ratio defined by

$$\lambda = 2\left(l\left(\hat{\theta}\right) - l\left(\hat{\theta}^*\right)\right), \qquad (6)$$

where $l\left(\hat{\theta}\right)$ is the maximum value of the log-likelihood function for the generalized extreme-value function, or non-restricted model, and $l\left(\hat{\theta}^*\right)$ is the maximum value of the log-likelihood function for the Gumbel distribution, or restricted model. Testing the validity of the non restricted model represented by GEVD suffices to verify whether $\lambda > C_\alpha$, where C_α is the $(1-\alpha)\%$ percentile of the χ^2 distribution with one degree of freedom because it only has one restriction.

VaR and CVaR and models based on GEVD

VaR measures the maximum probable losses for a portfolio holding, for a determined time horizon, and given a confidence level. VaR is determined by the $c-$quantile of the returns distribution F with a negative sign:

$$\mathrm{VaR}_c = -F^{-1}(c)$$

Parametric and nonparametric (simulation) VaR models have been developed in the financial literature.[11] VaR measures, or $c-$quantile of the normal distribution of returns are a linear function of the variance and its implementation is an easy task. However, risk estimations are incorrect, as they do not take into account extreme values captured at the tails of the return distributions. On the contrary using $c-$quantile from a heavy-tailed distribution yield better information about financial risks and therefore reduce modeling risks.

[11] Dowd (2002) presents an excellent review of value-at-risk models and expected tail loss.

Assuming that extreme observations follow a generalized extreme-value distribution, with a distribution function $H_{\xi_n,\beta_n,\alpha_n}$, then VaR as a risk measure is represented by the quantile from the GEVD, which is obtained inverting such distribution and substituting the estimates of maximum likelihood from the family of parameters $(\xi_n, \beta_n, \alpha_n)$. Therefore, value at risk for maximum returns corresponding to a short position can be expressed as

$$\mathbf{VaR}_c(X) = \hat{\beta}_n + \frac{\hat{\alpha}_n}{\hat{\xi}_n}\left[1 - n(-\ln c)^{\hat{\xi}_n}\right] \quad \textbf{(7)}$$

Here, $\hat{\alpha}_n$, $\hat{\beta}_n$ and $\hat{\xi}_n$ represent estimators of maximum likelihood for the series of maximum returns; c is the probability that the maximum return exceed the VaR level, and n represents the size of the blocks from which the maximum observed returns are obtained during a determined number of days of operations. Choosing this parameter plays an important role in VaR estimation based on classic EVT. For a long position following similar procedures as for a short position, the value-at-risk for maximum returns can be expressed as follows.

$$\mathbf{VaR}_c(X) = \hat{\beta}_n - \frac{\hat{\alpha}_n}{\hat{\xi}_n}\left[1 - n(-\ln c)^{\hat{\xi}_n}\right], \quad \textbf{(8)}$$

where $\hat{\alpha}_n$, $\hat{\beta}_n$ and $\hat{\xi}_n$ represent maximum likelihood estimates; c is the probability the minimum return does not exceeds the VaR level, and n represent the size of the subsamples from which are obtained the minimum observed returns for a determined number of days of operations.

A more refined alternative that measures excess losses above VaR has been developed by Artzner et al (1997; 1999); Rockafellar and Uryasev (2000) named this model conditional Value-at-Risk (CVaR). This model has been proposed because conventional VaR does not take into account the statistical properties from extreme losses and additionally does not fulfill the sub-additivity principle; that is, conventional VaR does not satisfy the properties of coherent risk measures. Taking into account these properties, CVaR can be defined as

$$\mathrm{CVaR}_c(X) = -\mathrm{E}\left[X \mid X \leq \mathrm{VaR}_c(X)\right] \quad (9)$$

In the context of EVT this risk measure can be estimated as a simulation of a function of VaR for a specific returns distribution. This means that CVaR also depends on the returns distribution F and on the probability level c. The asymptotic relationship between these two risk measures can be obtained analytically; taking advantage of the maximum domain of attraction a heavy-tailed Pareto distribution, with location parameter equal to zero, parameter of scale equal to one, and a characteristic exponent greater than one $(\alpha > 1)$[12], that is, $F_X(x) = 1 - kx^{-\alpha}$,[13] $x \geq 0$ can be constructed where k is a function that changes slowly and α is a positive parameter known as the tail index of the distribution F.

Considering the inequality $x > u$ from the tail of distribution F can produce the CVaR for a heavy-tailed Pareto distribution located at the maximum domain of attraction of the generalized extreme-value distribution equal to

$$\mathrm{CVAR}_c(X) = \left(\frac{\alpha}{\alpha - 1}\right)\mathrm{VAR}_c(X) \tag{10}$$

Therefore, the CVaR is always greater than VaR when a distribution of heavy tails is assumed, as the difference between both measures does not converge to zero when the values from the tails of the returns distribution are taken into account. This result depends to a greater extend on the degree of excess of probabilistic mass that is captured at the tails of the distribution, which is measured by the tail index parameter. This means that a more negative tail index indicates a greater CVaR because the tail of the returns distributions is heavier. The empirical evidence shows that the characteristic exponent generally takes values between $1 < \alpha < 2$ for the insurance sector while taking values between $1.5 < \alpha < 5$ for financial applications.[14]

Evaluation of VaR Estimates

After the estimation of VaR's for all competing models, we assess the acuteness of value at risk models based on the likelihood ratio test of

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[12] An important fact that must be pointed out is that the tail index does not have any relation with the scale parameter of the GEVD.

[13] All distributions with heavy tails were defined in this manner by Gnedenko (1943).

[14] For more details of CVaR measure based on extreme value theory, see Embrechts et al (1998).

unconditional coverage or backtesting. The backtesting technique consists of comparing the VaR estimates with the actual losses observed on the next day using historical daily return series. In this analysis we employ the likelihood ratio test developed by Kupiec (1995) which examines whether the failure rate is statistically equal to the expected failure rate (i. e., $\alpha = 1 - c$, where c is the confidence level for the estimation). If we assume the confidence level $1-\alpha$, sample size T, and days of failure N, then the frequency of failure can be regarded as $f = N/T$, which follows a binomial distribution with probability of occurrence equaling α. Thus, the alternative hypothesis are: $H_0 : N/T = \alpha$ and $H_a : N/T \neq \alpha$, respectively.

The likelihood ratio test statistic is given by

$$LR = 2\ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] - 2\ln\left[(1-\alpha)^{T-N}(\alpha)^{N}\right] \tag{11}$$

where α is the desired significance level, which is equal to one minus the VaR confidence level. Under the null hypothesis of correct unconditional coverage, the LR statistic has a Chi-squared distribution with one degree freedom. Therefore, if LR is larger than the corresponding critical value, then the null hypothesis should be rejected, in other words, the VaR model is not adequate to estimate the risk.

4 - Empirical results

4.1 Basic statistical characteristics and tail Diagnosis

The Mexican and Brazilian stock markets are no exception to the presence of heavy tails in their time series. In the long-run, during the period under analysis, the stock market indexes from Brazil and Mexico experienced significant growth, particularly in the case of Brazil. The boom experienced by these two emerging markets was partly induced by liberalization and deregulation policies undertaken by local authorities especially during the last two decades in the 20th Century aiming to improve their efficiency and liquidity, as well as to attract international portfolio investments (Cabello, de Jesús and Ortiz, 2006).

The presence of heavy tails can be diagnosed by examining the basic statistical characteristics of the return series. To work with stationary series distributed identically and independently, the empirical analysis must be based on continuous returns: $R_t = \ln(P_t) - \ln(P_{t-1})$, where $R_t =$ return for day t and $P_t =$ closing price for day t. The daily logarithm of emerging market returns is computed in the local currency of the stock market indices. Table 1 summarizes the basic statistical characteristics of the return series from the stock markets under analysis. Both series present positive mean returns and high volatility. The Mexican stock market's mean return was of 0.131 percent and its standard deviation averaged 1.83 percent.[15] The Brazilian market experienced higher return and volatility, which were 0.361 percent and 3.10 percent, respectively. High returns and high volatility in these markets can be attributed not only to liberalization and deregulation, but especially to high inflation rates experienced in these countries; nevertheless it is worth pointing out that the stock markets from these countries constituted a hedge against inflation (Cabello, de Jesús and Ortiz, 2006). Maximum and minimum returns confirm the volatility of the Brazilian and Mexican stock markets; the maximum and minimum returns for Mexico were 23.58 percent and -20.24 percent, respectively; Brazil shows a wider differential: 30.79 to -25.20 percent. At any rate the statistical characteristics of both markets are very similar. Two facts must be underlined; the lowest return for the Mexican Stock Market took place November 16, 1987, almost one month later than the world stock market crash initiated with the fall from the S&P which took place October 19, 1987; in the case of Brazil the minimum return occurred on March 23, 1990, when the nation was experiencing inflation rates above 100 percent in annual terms.

Both series also present an asymmetrical behavior. As shown in Table 1, this implies that the tails from the return series present different characteristics. Both markets exhibit positive skewness, with values of 0.27 for Brazil and 0.099 for Mexico; this means that the right tail of the empirical returns distributions is slightly heavier than the left tail, particularly for the case of the Brazilian stock exchange, i.e. extreme positive returns usually take place more frequently than extreme negative returns. This results from the

[15] This analysis is based on nominal returns; inflation adjusted returns cannot be used due to the lack of high frequency (daily) inflation information. For an in depth coverage of EVT and its applications to emerging stock markets using nominal returns, see Gencay and Selcuk (2004), da Silva and Mendes (2003) and Ho et al (2000).

explosive growth experienced by both markets during the last two decades. In this respect, evidence from negative skewness presented earlier by Duffie and Pan (1997) cannot be confirmed for the stock markets from Brazil and Mexico.

Observing the fourth-moment estimates, both financial series show an extremely high and statistically significant kurtosis. The highest kurtosis corresponded to the Mexican market, 25.16 points, whereas the Brazilian stock market had a kurtosis of 8.71 points; at any rate these coefficients mean that tails from the return distributions from both markets are heavier than the normal distribution because they are characterized by a major probabilistic density. In the financial literature the presence of leptokurtosis has been significantly stronger among high-frequency returns than for low-frequency returns, or than for long-period returns.[16] Absence of normality is also confirmed by the Jarque-Bera (1980) statistic which follows a chi-square distribution with two degrees of freedom.

Table1: Basic Statistics for Daily Returns from the Stock Markets from Brazil and Mexico

	Brazil	Mexico
Mean	0.361	0.131
Maximum	30.792	23.583
Minimum	-25.199	-20.242
Std. Deviation	3.104	1.834
Skewness	0.265	0.099
Kurtosis	8.716	25.159
Jarque-Bera	12005*	177977*
Probability	0	0
Observations	8745	8698

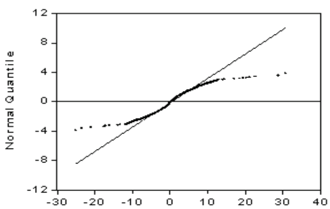
Note: The Basic statistics for daily returns from the stock markets from Brazil and Mexico are expressed as percent points. An * indicates statistical significance at the 1% confidence level.

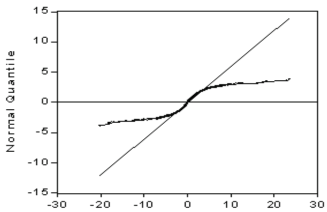
The evidence rejecting the assumption of normality for the daily return series is notorious, particularly for the Mexican stock market, for the

[16] Empirical evidence proving that the returns on shares present excess kurtosis include Fama (1965); Praetz (1972), Blattberg y Gonedes (1974); Kon (1984); Tucker (1992); Kim y Kon (1994). For evidence on exchange rates see: Boothe y Glassman (1987); Tucker y Pond (1988).

Jarque-Bera statistic is extremely high (177977) and statistically significant.[17] Another procedure to identify the main structure of a financial market and determine if its distribution exhibits heavy tails is the QQ-plot (quantile-quantile plot). Basically the QQ plot compares the quantiles from the empirical distribution with the quantiles from a given distribution, that is, the quantile is the inverse F^{-1} to the distribution function F. For instance, if F is continuous then 1%–quantile is the number x such that $F(x) = 0.01$. In general,

Figure 1. QQ Plot for Daily Stock Market Returns from Brazil and Mexico January 2, 1970 – December 31, 2004





(a) Brasil **(b) Mexico**

Figure 1 clearly shows that both the Mexican and Brazilian returns distributions present heavier tails and sharper distribution forms than the normal distribution. If returns are distributed normally, all points from the empirical series should remain on the line depicting quantiles from the normal distribution. However, points from the empirical distribution deviate on the extremes from the straight line, that is, the extreme points show more variability than the points captured at the center. The typical S-shaped curve shows that the returns distributions from the Mexican and Brazilian exchange markets present a leptokurtic and asymmetrical behavior compared with the normal distribution.

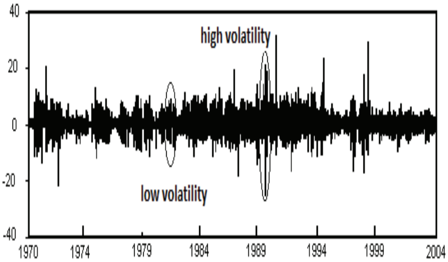
Finally, Figure 2 shows that the two emerging Latin American markets exhibit periods of relative tranquility; stock market prices are

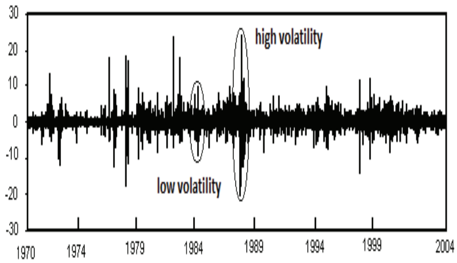
[17]As previously pointed out, this work benefits from a large set of data for each country (8772 observations). Susmal (2001) demonstrates that the lack of normality at emerging markets is less pronounced when low frequency data is used.

relatively stable, but they are followed by volatile periods of varied intensity; changes in the prices are large and generally take place in clusters. This implies that the series of these two markets are time dependent, that is, illustrated by a conditional heteroskedastic behavior. This is commonly known as the clustering effect, one of the more important characteristics of financial series, demonstrated in the seminal works by Engle (1982) and Bollerslev (1986).

Figure 2. Brazilian and Mexican Stock Market Returns

January 2, 1970–December 31, 2004





(a) Brazil **(b) Mexico**

Summing up, returns series from the Mexican and Brazilian stock markets constitute an excellent showcase to analyze the relationship between the behavior of extreme returns and financial risk in extremely volatile markets presenting distributions with heavy tails because these two markets present empirical distributions that are more leptokurtic than those of mature stock markets from industrialized countries.

4.2 GEVD Parameters for the Stock Markets from Brazil and Mexico

To analyze the behavior of extreme returns for the stock markets of Brazil and Mexico, this paper applies the models of block maxima.[18] As previously

[18] In classic extreme value theory two methods are used to identify the atypical values in a series, the block maxima technique and the peaks over a threshshold technique: both divide the data into consecutive blocks and focuses on the series on the maxima of these blocks. We have chosen the block maxima technique because its flexibility; the total sample can be divided into blocks of *n* different sizes allowing to evaluate the behavior of a markets in terms of daily, weekly, monthly, etc. periods. Additionally, overall, block maxima produces more accurate tail estimates that correspond well to the empirical ones both at traditional confidence levels and at

explained, this consists of dividing the total series sample in subsamples of a determined number of observations wherein the extreme maximum and minimum values are chosen. The number of extreme returns in each subsample depends on the determination of the size of the subsamples and, of course, on the total sample size. Here, extreme daily returns are chosen for different time horizons, one month (n=21), one quarter (n=63), one semester (n=126) and one year (n=252). Once the subsamples have been constructed, the parameters for the distribution of extreme values are estimated, for each of the tails of the return distribution. The parameters of location, scale and form are estimated by the modeling of maximum likelihood, independently for each tail of the returns distribution.

Table 2 presents the results of the maximum-likelihood estimators for the unknown parameters for the GEVD, for maximum and minimum extreme values for the stock markets under analysis; it also shows their standard errors. The evidence shows that the location parameter increases as the size of the block increases. In the case of the Brazilian index, the location parameter presents a change in absolute terms from 3.928 to 8.235 for positive returns and from -3.486 to -8.354 for negative returns. In the case of the Mexican stock exchange, the parameter increases from 2.066 to 5.382 for positive returns and from -1.716 to -5.174 for negative returns. This significantly increases the average of extreme positive and negative returns derived from both financial instabilities as well as from the explosive growth experienced by those two emerging markets, which can be attributed to internal and external shocks, liberalization and deregulation, and the hyperinflation rates which characterized Brazil and Mexico on different occasions during the period under analysis, affecting nominal returns. Nonetheless there is a significant difference between the exchange markets from Brazil and Mexico in relation to the value of the location parameter. Considering a block of 126 days, the estimated parameter for positive returns for the Sao Paulo stock market is approximately 1.59 times greater that the Mexican stock market parameter; for negative returns the location parameter for the Brazil is 1.63 times grater that the Mexican parameter.

Estimations for the scale parameter show similar tendencies. An increase in the parameters can be observed as the size of the subsample increases. Considering the right tail of the returns distribution, the scale

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more extreme confidence levels, 99.0%, 99.9% (Byström, 2001) which we use to compare alternative VaR models in our work parameter for the Brazilian stock exchange changes from 2.349 to 3.337; the Mexican stock exchange presents a lower change, from 1.285 to 2.984. The absolute change for the scale parameter for the right tail for both countries is slightly lower: for Brazil the change is from 2.10 to 2.981 and for Mexico it ranges from 1.133 to 2.686. These results indicate that these emerging markets are subject to high risks, which is a characteristic of economies with fragile financial and economic structures. Comparing further the Brazilian and Mexican stock markets, considering semester subsamples, it can be observed that the scale parameters from Brazil are approximately 1.29 and 1.20 times greater than the values obtained for the Mexican markets, for the left and right tails, respectively.

The tail index is the parameter that best explains the historical behavior of the tails of the generalized extreme-values distribution of returns of the financial series under analysis. The estimated tail indexes for different sample sizes for positive returns from the Brazilian and Mexican markets changes from –0.121 to –0.337; and –0.119 to –0.278, and from –0.235 to –0.351, respectively.

Analyzing Sherman's goodness-of-fit test, in both markets, it can be observed that the hypothesis that maximum and minimum returns follow a generalized extreme-values distribution cannot be rejected at a confidence level of five percent, except for a block size equal 20 daily observations, that is, approximately monthly observations. These findings confirm the fact that the extreme-value distribution explains adequately the asymptotic behavior of maximum and minimum returns, even though the tail indexes show increasing negative values, with high standard errors as the subsample size increases.

Table 2: Maximum Likelihood Estimates for GEVD for Daily Maximum and Minimum Returns for the Stock Markets from Brazil and Mexico

Subsample Size	Location Parameter	Scale Parameter	Tail Index	Sherman Statistic
Maximum Returns				
Brazil				
$n = 20$, $m = 420$	3.928 (0.040)	2.349 (0.031)	-0.121* (0.039)	1.966(2.46%)
$n = 60$, $m = 140$	5.346 (0.086)	2.898 (0.067)	-0.159* (0.069)	1.203(11.4%)
$n = 120$, $m = 70$	6.700 (0.131)	3.085 (0.106)	-0.222* (0.108)	-0.350(36.3%)
$n = 252$, $m = 35$	8.235 (0.243)	3.378 (0.154)	-0.337 (0.159)	-1.127(87.0%)
Mexico				
$n = 20$, $m = 420$	2.066 (0.033)	1.285 (0.027)	-0.284* (0.043)	1.984(2.36%)
$n = 60$, $m = 140$	3.130 (0.054)	1.804 (0.045)	-0.313* (0.074)	1.158(12.4%)
$n = 120$, $m = 70$	4.207 (0.102)	2.386 (0.088)	-0.365* (0.122)	-0.152(56.0%)
$n = 252$, $m = 35$	5.382 (0.233)	2.948 (0.129)	-0.388 (0.174)	-1.119(86.9%)
Minimum Returns				
Brazil				
$n = 20$, $m = 420$	-3.486(0.036)	2.100 (0.028)	-0.119* (0.040)	1.982(2.37%)
$n = 60$, $m = 140$	-5.349(0.074)	2.479 (0.057)	-0.134* (0.068)	1.320(9.33%)
$n = 120$, $m = 70$	-6.696(0.114)	2.505 (0.073)	-0.221 (0.098)	-0.296(38.3%)
$n = 252$, $m = 35$	-8.354(0.185)	2.981 (0.135)	-0.278 (0.152)	-1.504(93.4%)
Mexico				
$n = 20$, $m = 420$	-1.712(0.029)	1.138 (0.023)	-0.235* (0.043)	1.826(3.39%)
$n = 60$, $m = 140$	-2.835(0.046)	1.531 (0.037)	-0.251* (0.073)	1.477(6.98%)
$n = 120$, $m = 70$	-4.099(0.087)	2.096 (0.067)	-0.320* (0.096)	-0.336(36.8%)
$n = 252$, $m = 35$	-5.174(0.179)	2.686 (0.128)	-0.351 (0.176)	-0.803(78.9%)

Values in parenthesis represent standard errors for the estimates of maximum likelihood for the GEVD, derived from the diagonal of the variance-covariance matrix and calculated with a subroutine in C++ language. The last column shows results from the test of goodness of fit developed by Sherman; the *p* value is within parenthesis. The value of Sherman statistic is compared with a threshold value at a 5% confidence level, which is equal to 1.645. If the value of the statistic is smaller than the threshold value, then the hypothesis that returns follow a generalized extreme value distribution cannot be rejected. The tail index estimates marked with an asterisk are significantly different from zero at a confidence level of one percent.

Results from the likelihood ratio test indicate that the tail indexes are significantly different than zero for a confidence level of one percent, for monthly, quarterly, and semester subsamples of the maximum and minimum returns for both emerging markets (with the exception of semester block for negative returns for the Brazilian market). This indicates that the asymptotic distribution is at the maximum domain of attraction of the Fréchet distribution (distribution of heavy tails) generally used to model real financial data. Because the emerging Latin American economies experienced dramatic booms and crises, the returns of these markets are incompatible with normality. These results are consistent with other empirical studies that have applied classic extreme value theory to returns from other stock markets (Longin, 2000; Ho et al 2000, da Silva and Mendes, 2003).

The Brazilian and Mexican findings also show that the distributions of daily returns have a light asymmetrical distribution. The right tail of the Brazilian returns is slightly heavier for periods from one month to one semester, but for a one-year period the right tail is markedly more stable and heavier than the left tail, although the tail index is not significantly different from zero (–0.337 vs. –0.278). Considering the Mexican stock market tail index it can be observed that the right tail tends to be heavier than the left tail for any subsample period. Therefore, the Mexican market behaves contrary to the behavior from stock markets from industrialized countries where the left tail tends to be much heavier than the right tail (Longin, 2000). Similarly estimations for the tail index for positive and negative returns for the Mexican stock market are more stable than the estimates for the Sao Paulo stock market; the former has lower estimates.

Two important facts determining this differentiated behavior are the world stock market crash that took place with a lag in November 16, 1987 in Mexico (-20.24% return) and the Brazilian crash (-25.20% return) which took place on March 21, 1990. Taking out these observations from the statistics of extreme order, their impact can be detected immediately; the value of the tail indexes change from -0.320 to -0.267 for Mexico and from –0.221 to –0.159 for Brazil, whereas the estimators for the localization and scale parameters change only slightly. The same effect takes place for maximum returns when the maximum values are omitted; this is an indication that the distributions seemingly have less heavy tails for one-semester periods; another interesting finding concerning the tail estimators is that the distribution from the Mexican stock exchange present heavier tails than the tails of distribution from the Brazilian stock market for any time interval, even though the magnitude of extreme returns for Brazil is greater and adjacent, whereas Mexican extreme returns are smaller and more disperse.

Another relevant result, derived from the tail index estimators, that must be pointed out is that there exist second moments for the superior and inferior tails of the returns distributions for both stock market indexes. In other words these distributions have finite variances, but with different characteristics for each one of the tails as the estimated value for the tail indexes for the extreme positive and negative returns is greater than –0.50 for any time interval; this indicates that the log-likelihood function is regular and that all the maximum likelihood estimators have the usual asymptotic properties demonstrated by Smith (1985). The fact that the returns distribution presents heavy tails does not mean that higher moments do not exist. For instance the first four moments exist for the returns distribution of the Brazilian stock market when extreme returns are chosen for subsamples of less than one year because the estimated tail index is above -0.25. In the case of the Mexican stock market index the first four moments only exist for extreme negative returns for one-month periods. Similarly the first three moments exist for extreme positive returns for monthly periods as well.

4.3 VaR and CVaR evidence

Studies dealing with measuring risk have mainly focused on the negative returns captured along the left tail of probability distribution. However, empirical analysis based on EVT yields information from both tails of the distribution. Therefore, VaR and CVaR can be expressed in percentage terms for long and short positions. In the first case an investor could take losses when prices decrease because of liquidity needs; in the second case an investor can experience losses when the assets prices increase. This generally takes place when investors sell short. In this context VaR and CVaR estimations for a long position and a short position are concentrated in the left tail where positive and negative returns are captured, respectively.

Finally considering daily positive and negative returns selected on a 126-day block length,[19] both VaR and CVaR under EVT are estimated for confidence levels of 95%, 97.5%, 99% and 99.9%. The results are then compared with results from conventional first-generation parametric and non-parametric models. Parametric models include the delta-normal model with unconditional normal distribution, GARCH(1,1) and the model of weighted moving averages with conditional normal distribution. Non-parametric

[19] Note that the procedure of block maxima represents a trade-off problem between low bias and minimum variance. In other words, a large block size leads a better fit of the GEVD to the extreme returns and unbiased estimates, but with high variance in the parameter estimates. Conversely, a small block size provides minimum variance estimates, but the extreme returns will not be governed by the GVED.

models include the method of historical simulation and Monte Carlo simulation. In this work only the historical simulation method is used. The Monte Carlo methodology is not used to measure market risk because the risk factors considered are linear.

Results for the five VaR and CVaR models that include the tails of the returns distributions for the stock markets from Brazil and Mexico are included in Table 3. In both markets, it can be observed that the parametric estimates based on conditional and unconditional returns distributions with normal innovations, yield more conservative VaR results for both the long and short positions, that is, greater losses than the parametric estimates based on an unconditional extreme-value distribution, for smaller confidence levels. For instance at a confidence level of 95% the maximum delta-normal VaR loss on a $100.00 long position[20] for a long position taken in the Mexican market is $2.39; applying the conditional GARCH and weighted moving averages leads to losses figures of $1.93 and $1.89, respectively. However the value-at-risk estimated using EVT equals only $1.21. In the case of a long position at the Sao Paulo market estimate losses according to the conditional and unconditional return distributions equal $3.47, $2.44 and $2.41; using the asymptotic extreme-values distribution the loss equals $2.95. Therefore, estimates using VaR based on EVT underestimate losses for low confidence levels.

For the short position similar results were obtained for both emerging markets. On the contrary for tighter confidence levels the value-at-risk estimated based on ETV are greater, that is EVT models yield more conservative estimates than the conventional models. For a confidence level of 99.5% the VaR estimates for both markets based on EVT for a short position are approximately two times greater than the VaR estimates obtained with the delta-normal model; coming to $11.85 vs. $4.43 for the Mexican stock market and $15.06 vs. $7.21 for the Brazilian stock market. For a long position, VaR estimates derived from the alternative models under consideration are similar to the short position contrasts previously examined: $10.46 vs $4.49 (Mexico) and $13.47 vs $6.51 (Brazil). This difference can be explained by the way in which the distribution of extreme returns is derived because the estimated tail index is always negative for both markets.

[20] Assuming a U.S. investor takes long and short positions at several emerging markets. We report losses in dollars, rather than in percentages. Empirical analysis performed in this work, based on EVT, yields information from both tails of the distribution; VaR and CVaR can be expressed in percentage terms for long and short positions.

That is, the evidence clearly justifies the advantage of estimating VaR based on a generalized extreme-value distribution, for high confidence levels.

A comparative analysis between the historical-simulation methodology and the extreme-value estimates clearly show that models based on empirical distributions yield more conservative VaR estimates for any confidence level and any financial position in the market; that is, risk is overestimated.[21] This is more clearly reflected in the Sao Paolo market than in the Mexican market, for negative returns, with values produced equaling \$14.03 vs. \$13.47 for Brazil, and \$11.61 vs \$10.46 for Mexico. The main reason for this fact is that the empirical distribution is usually heavier at its interior because extreme returns are clearly discrete. This implies that VaR estimates that depend on tails are calculated in a discrete way, with a high variance, inducing overestimations or underestimations by the non-parametric model. The historical method cannot yield good estimates beyond the sample where estimates based on EVT guarantee more precise risk estimates.

Examining the empirical CVaR results reported in Table 3 it can be observed that there are also differences in the estimates for each country, particularly for the simulation and extreme-value models. CVaR based on extreme-values for a \$100 position and a confidence level of 99% equals \$5.47 for a long position and \$5.95 for a short position at the Mexican market; in the case of Brazil the results are \$8.01 for the long position and \$7.89 for the short position. The historical VaR estimates are \$7.62 and \$8.32 (Mexico) and \$10.95 and \$11.01 (Brazil). Based on a normal unconditional distribution the estimates are \$3.88 y \$3.82 (Mexico) and \$5.62 and \$6.22 (Brazil). The conditional methods yield smaller and identical CVaR estimates. For a confidence level of 99.9% CVaR figures are more conservative, implying that potential losses are greater, particularly for positive returns for the case of both markets, because of their explosive growth. This means that risk in emerging markets, because their instabilities are severe and persistent, present behavior opposed to the primary one characteristic of mature markets.

[21] When the product between the confidence level and the simple size (cT) is not a whole number the technique of linear interpolation is used to obtain more precise VaR estimates or the maximum loss by the historical simulation modeling.

Table 3: VaR and CVaR Estimates for Short and Long Positions Lineal Daily Returns

Long Position Losses

Model	Delta-Normal		GARCH		WMA		Extreme Value		Historical Simulation	
Mexico	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR
C=0.95	2.39	3.00	1.93	2.41	1.89	2.37	1.21	1.78	2.73	4.51
C=0.975	2.85	3.40	2.29	2.74	2.25	2.68	2.14	3.15	3.61	5.59
C=0.99	3.38	3.88	2.72	3.12	2.67	3.06	3.72	5.47	4.77	7.62
c=0.999	4.49	4.90	3.62	3.94	3.55	3.87	10.46	15.37	11.61	16.58
Brazil	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR
c=0.95	3.47	4.35	2.44	3.06	2.41	3.02	2.95	3.79	5.44	7.81
c=0.975	4.13	4.93	2.91	3.47	2.88	3.43	4.23	5.43	6.73	9.03
c=0.99	4.90	5.62	3.45	3.96	3.41	3.91	6.24	8.01	8.44	10.95
c=0.999	6.51	7.10	4.59	5.00	4.53	4.94	13.47	17.31	14.03	19.98

Short Position Losses

Model	Delta-Normal		GARCH		WMA		Extreme Value		Historical Simulation	
Mexico	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR
c=0.95	2.36	2.95	2.00	2.51	1.90	2.38	1.04	1.64	2.40	4.46
c=0.975	2.81	3.35	2.39	2.85	2.33	2.70	2.03	3.20	3.35	5.82
c=0.99	3.33	3.82	2.83	3.25	2.77	3.08	3.78	5.95	5.07	8.32
c=0.999	4.43	4.83	3.76	4.11	3.68	3.90	11.85	18.67	15.28	18.44
Brazil	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR
c=0.95	3.84	4.81	2.80	3.51	2.77	3.47	2.09	2.69	4.55	7.16
c=0.975	4.57	5.46	3.34	3.98	3.29	3.93	3.66	4.71	6.05	8.68
c=0.99	5.43	6.22	3.96	4.54	3.91	4.48	6.13	7.89	7.96	11.01
c=0.999	7.21	7.87	5.26	5.74	5.19	5.66	15.06	19.36	14.46	20.86

This empirical characteristic can be better appreciated considering the difference between VaR and CVaR. For instance, applying the extreme-value approximation for confidence levels of 99.0% and 99.9% it can be observed that the difference increases; in absolute terms these values are from $1.75 to $4.91 for the long position and from $2.17 to $6.82 for the short position in the case of Mexico, and from $1.78 to $3.84 for the long position and from $1.76 to $4.30 for the short position in the case of Brazil. From a statistical point of view, this difference in absolute terms can be attributed to the tail index value because a more negative tail index implies greater is impact on the value of conditional risk. This effect is also present for the application of the historical model, except for the Mexican case where the difference diminishes in absolute terms from \$3.25 to \$3.16 for the short position, respectively. Causing this significant increase is the fact that all observations that exceed the value-at-risk are considered. In relation to the delta-normal model, the difference also decreases from \$0.50 to \$0.41 and from \$0.49 to \$0.40 for the long and short positions at the Mexican market; and from \$0.72 to \$0.59 and from \$0.80 to \$0.66 for the long and short positions in the Brazilian market.

The above results clearly show that CVaR estimates based on EVT and the historical simulation tend to deviate from conventional VaR as the confidence level becomes more restrictive, whereas the CVaR estimated on a normal distribution tends to converge to the VaR estimate. This is, any model assuming normality not only yields incorrect information about the tail risks of the returns distributions, but also underestimates the value of conditional risk. Therefore, the application of EVT is very important to correctly model the tails from the distributions of returns at financial markets.[22]

Finally, VaR and CVaR estimates based on extreme values clearly show that the Sao Paulo stock market was riskier during the period under analysis, for both the short and long positions and for any confidence level. This finding is not fully supported by the value of the tail index, as both the left and right tails of the IPC distribution of returns are seemingly heavier than the tails from the BOVESPA returns distribution. For high-frequency (daily) data, tail indexes for the Mexican stock exchange are smaller or negative when compared with the tail indexes from the Sao Paolo stock market: –0.320 vs. –0.221 for daily positive returns and; –0.365 vs. –0.222 for daily negative returns. This situation can be also observed comparing the EVT value-at risk estimates for the left and right tails of the returns distributions for both Latin American markets, particularly for the case of Brazil market. For confidence levels of 95% y 97.5%, potential losses for the long position are greater than potential losses for the short position; for in the case of Brazil, \$2.95 vs \$2.09 and \$4.23 vs \$3.66, for the stated confidence levels, even though the estimated tail indexes are very similar. Considering the case of Mexico's potential losses are for the 95% and 97.5% potential losses are \$1.21 vs \$1.04 and \$2.14 vs \$2.03, for the long and short positions, respectively. These results imply that heavier tails do not necessarily mean higher risk. Therefore, VaR and CVaR estimates seemingly are not exempt from tail risk when the

[22] This analysis was also carried out for low frequency data for a ten days investment horizon. Results are even more conservative. They can be obtained contacting the authors.

asymptotic behavior of stock market indexes is described by a distribution of extreme values or heavy tails.[23]

4.4 Backtesting Results for VaR Models

A backtesting procedure is performed for delta-normal, historical-simulation, and extreme-value VaR Models at different confidence levels. The statistical evaluation period is from January 2, 2001 to December 30, 2004, totaling 1000 daily returns from the two emerging stock markets under analysis. Therefore, the relative performance of each model is determined in terms of the failure rate and *p*-values of the Kupiec test statistic. A failure occurs when a realized return is greater than the VaR estimate. The failure rate is defined as the total number of failure, divided by the sample size T. A failure rate at c-quantile greater than α% indicates that the model consistently underestimates the risk of the tail. Otherwise, a failure rate in the c-quantile less than α% indicates that the model consistently overestimates the tail risk. Finally, a *p*-value less than or equal to 0.05 will be sufficient evidence for rejecting the null hypothesis of unconditional coverage

Table 4 reports for both long and short trading positions and for Brazilian and Mexican stock markets, the failure rate and the *p*-values of the LR statistic to test the null hypothesis of correct unconditional coverage, using various VaR confidence levels. The results show that delta-normal and extreme-value models perform better than historical simulation model for long positions in the Brazilian stock market at the 95%, 97.5% and 99% confidence levels but significantly underestimate and overestimate risk at the 99.9% quantile. On the contrary for short positions the three models' performance deteriorates, with the models tending to overestimate the true risk. In fact, the null hypothesis of unconditional coverage is rejected for all models except for delta-normal and extreme value models at 0.1% and 2.5% significance levels with failure rates of 0.1% and 2.3%, respectively, which are statistically equal to the expected failure rates with *p*-values greater than 5% percent. In the case of long position in the Mexican stock market, the models based on the normal and generalized extreme-value distributions also provide better VaR estimates at the 97.5% and 99% confidence levels than a measure of risk based on the empirical distribution with failure rates of 1.6%,

[23] Examples about the impact of tail risk on VaR and CVaR estimates can be found in Yamai y Yoshima (2002a, 2002b).

3.1% 1.2% and 0.6%, respectively, which amount to 0.9% of risk overestimation and 0.6% of risk underestimation at the 2.5% level and 0.2% of risk underestimation and 0.4% of risk overestimation at the 1% level, respectively.

Assuming the short position, the delta-normal model performs better than extreme-value and historical- simulation at the 97.5%, 99% and 99.9% confidence levels. These models tend to overestimate the right-tail risk at higher quantiles, especially in emerging stock markets that have recently been prone to experiencing extraordinary market movements because of economic booms. In the context of risk management, some financial institutions may then not prefer such models because they would have to allocate more capital to meet regulatory requirements.

However, despite the fact that VaR model based on the generalized extreme value distribution overestimates the true risk for both long and short trading positions at 99% and 99.9% confidence levels. It is important to emphasize that extreme value theory provides a useful tool for evaluating tail-related risk in emerging stock markets under extreme market conditions. This finding is mainly drawn from fact EVT technique captures more accurate statistical properties on the behavior of tails of return distributions than those obtained by conventional models, namely the delta-normal and historical-simulation models, as it can incorporate not only asymmetric characteristics but also leptokurtosis into the unconditional returns distributions.

Table 4: Daily VaR Backtesting Results for both Emerging StockMarkets

Period: 01/02/01-12/30/04
Number of daily returns: 1000

Index	Brazil				Mexico			
Expected failure rate (α)	5%	2.5%	1%	0.1%	5%	2.5%	1%	0.1%
Long position								
Delta-Normal								
Failure rate	3.9	2.2	0.9	0.4	2.6	1.6	1.2	0.4
p-values	0.097	0.535	0.746	0.023	0.0001	0.051	0.537	0.039
Extreme Value								
Failure rate	5.7	2.1	0.5	0	10.6	3.1	0.6	0
p-values	0.320	0.405	0.078	na	0	0.241	0.169	na
Historical Simulation								
Failure rate	0.5	0.3	0.1	0	1.8	0.8	0.3	0
p-values	0	0	0.0002	na	0	0	0.009	na
Short position								
Delta-Normal								
Failure rate	2.0	1.0	0.3	0.1	3.0	1.9	1.2	0.3
p-values	0	0.0005	0.009	0.999	0.0017	0.2047	0.537	0.1071
Extreme Value								
Failure rate	12.7	2.3	0.3	0	17.9	4.6	0.3	0
p-values	0	0.6814	0.009	na	0	0.0001	0.009	na
Historical Simulation								
Failure rate	1.0	0.3	0	0	2.9	1.1	0.2	0
p-values	0	0	na	na	0.0009	0.0014	0.0019	na

This table reports real failures rates and p-values of the unconditional coverage test for competing models. The models are successively a delta-normal model, the extreme value model and the historical simulation model. For example, a higher failure rate than the expected value corresponds to an underestimation of the risk while a smaller failure rate than expected value indicates an overestimation of risk by VaR models. Finally, a p-value greater than 5 percent indicates that the null hypothesis based on the likelihood ratio test of unconditional coverage o backtesting is not rejected, that is, the VaR model is adequate for measuring the extreme risk.

5 - Conclusions

This paper has shown that EVT offers several benefits in terms of a more precise estimation for high confidence levels and in terms of potential losses because of tail-risk from the returns distributions for the emerging stock markets from Brazil and Mexico. This is particularly true for the stock market in Sao Paulo where market fluctuations have been large and adjacent due to an explosive growth, coupled with hyper-inflation rates.

The first empirical main finding of this study is that both stock markets are characterized by heavy tails resulting from excess kurtosis; this can be explained by the recurrent crisis that the Mexican and Brazilian economies experienced during the last decades. This fact is also supported by the negative tail index values of the generalized extreme-value distribution of these markets, demonstrating that the asymptotic distribution for extreme values is at the maximum domain of attraction of the Fréchet distribution, also known as a distribution of heavy tails, which is generally used to model real financial returns. Also, the right and left tails present different characteristics due to existing bias or asymmetry; for this reason the level of risk shows a different behavior than mature markets because in emerging markets atypical movements are fairly frequent reflecting weak private and public institutions, frail public and market fundamentals, and recurrent high disequilibria and disparities.

Second, the empirical evidence clearly shows that VaR and CVaR estimates based on EVT yield more precise and robust information about financial risk than conventional parametric approximations, for confidence levels of 99% and 99.9%. This derives from the fact that the GEVD correctly models the magnitude of extreme movements found in the right and left tails of the probability distribution. It is worth mentioning that values based on empirical distributions yield more conservative risk estimates than those based on heavy-tail distributions; however they are affected by a high variance derived from the discrete behavior of returns found outside the tails. Further, estimates based on EVT give better information about risk outside the sample for more conservative confidence levels (99%). Also, VaR and CVaR estimates based on EVT are more conservative for short and long positions taken in the Brazilian stock market than those taken in the Mexican stock market; however, they are not statistically supported, as both tails for the Mexican market are heavier than the tails for the Sao Paulo market, which results in model risk, particularly for the VaR estimate, as the CVaR estimate is characterized by better properties in terms of tail risk.

Therefore, VaR and CVaR estimates based on EVT can be used effectively to manage financial risk in a univariate context; they allow investors to have a better perspective about the risks assumed because of extreme movements, and to make better buy and sell decisions under uncertainty in both mature and emerging markets.

To sum up, VaR and CVaR based on EVT can become very valuable tools to manage risk from portfolio holdings. Furthermore, risk analysis based on EVT also offers valuable information for the banking industry and its regulatory authorities for the purpose of more efficiently determining capital needs. Like any other model, EVT has some shortcomings; the most important is that risk is estimated in an individual and static way; it does not consider the stochastic volatility present in most financial series. VaR should be also applied to asses the impacts of the recent 2007-2009, with a wider sample of emerging markets, revisiting EVT and CVaR applications and dynamic modeling as well. These two problems are left for future financial research.

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