Real Interest Rate and Growth Rate: Theory and Empirical Evidence

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Abstract

This study presents an assessment of the links between the real interest rate and the growth rate. In the first part of the paper we recall the theoretical foundations of these links. In the second part, we compare empirical data with the predictions of the theory. The real interest rate should exceed the growth rate in the long run. This hierarchy has important consequences on the public finances. The Solow-Ramsey-Cass model fits better to the data than the endogenous growth theory. Furthermore, the Sixties and Seventies have been a time of transitory dynamics with real interest rates lower than the growth rates.

Keywords: real interest rate, growth rate, time preference rate, productivity of capital, debt burden
JEL classification: E22, E43, E44, O41

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1- Introduction

Many financial institutions and institutional investors justify the international diversification of their portfolios by the rule according to which in each economy and in the long run the return of their assets is roughly equal to the growth rate. This is a direct application of the golden rule of accumulation (Phelps, 1961). But what are the foundations and the reliability of such a rule? In the economic theory, the relations between real interest rate and growth rate were often studied. However, whereas the determinants of the real rates in the short and medium terms (savings rate, capital goods profitability, monetary policy orientation, risk aversion etc) are rather well identified, the essential object of contemporary theoretical work is the monetary control with the use of an interest rate rule. The main question in this research is the connection between the short-term interest rate policy and the realization of an inflation target (Woodford, 2003). But nowadays, the nature of the links between the real interest rate and the growth rate seem secondary and remain at the very least dubious. The problem of the determination of the hierarchy between these rates is yet crucial: if the interest rate exceeds the growth rate in the long run, the public debt burden becomes unsustainable (Domar, 1944). This is why this paper proposes to point out the theoretical determinants of this hierarchy in the contemporary economic theory. The empirical checking of this theory in the recent period is the object of the second part of this study.

2 - Interest and growth rates in the contemporary theory

The two main approaches in this field are the Solow model and the endogenous growth theory.

2.1 The neo-classical model

In the neo-classical theory, we study an economy with labor force supposedly growing at a constant rate n. The labor productivity grows at the rate m. The production per worker, the consumption per capita c as well as the capital per worker k are measured in effective units of labor. The capital per worker increases at the natural rate $m + n$. The macroeconomic production function per capita "is well behaved" ($f'(k) > 0$; $f''(k) < 0$). It relates output per worker, y, to the capital per worker, k : $y = f(k)$. A percentage δ of the capital is replaced in each period. The firms maximize their profits. The real interest rate r is equal to the net marginal productivity of the capital, $f'(k) - \delta$. This rate r is thus a decreasing function $r(k)$ of k. According to this analysis, each Δk increase in the capital per worker in efficiency units makes it possible to obtain $r\Delta k$ additional finished product and implies an additional net investment equal to $(m + n)\Delta k$. Each positive variation Δk thus makes it possible to increase consumption per capita in a permanent path of $r\Delta k - (m + n)\Delta k$. As long as this difference is positive, it is rational to increase the value of k. In addition, r being a decreasing function of k, it inevitably exists a value of this variable that will cancel this difference. This value is that which corresponds to the equality of the natural rate $m + n$ with r. Such a situation is optimal because any additional increase in k would implies a negative value of $r\Delta k - (m + n)\Delta k$. Thus, the optimal value of r is that which corresponds to the long period growth rate (Von Neumann, 1945):

$$r = m + n \qquad (1)$$

From a technical point of view, this equality would have been easy to find by maximizing the function $c(k) = f(k) - sf(k)$. This expression indicates the share of the product which is not saved. In the Solow model, $sf(k)$ is equal to $(m + n + \delta)k$ on a balanced growth path. The function $c(k)$ reaches a maximum value if :

$$f'(k) = m + n + \delta \qquad (1')$$

By replacing $f'(k) - \delta$ by r, we thus obtain the equation (1). This equality also implies that the share of the profits is equal to the total propensity to save. To show this result, we multiply by the capital coefficient v $(= K/Y)$ and remember that the natural rate $m + n$ is itself equal to s/v. We obtain: $s = a$.

The equality $r = m + n$ is the "golden rule of accumulation" (Phelps, 1961). The golden rule makes it possible to assert that an economy is in a situation of capital under-accumulation if the real interest rate is higher than the growth rate in the long run. Conversely, an economy is over-saving if its real interest rate is lower than its long-term growth rate. It is interesting to compare the golden rule of accumulation with another very general formulation of the relation which links directly the variables r and g: the interest rate is, in a neo-classical perspective, equal to the net yield of the capital i.e. to the net profit rate ($P/K - \delta$), in which K is the capital and P the profit. If we consider that the share of the profit is stable and equal to a, the rate of profit is aY/K. Finally, the difference between the net rate of profit $P/K - \delta$ (itself equal in the long run to the real interest rate r) and the growth rate g (equal to the rate of capital accumulation $sY/K - \delta$) is :

$$r = g + (a - s) A \quad (2)$$

In this equation, A is the productivity of capital Y/K. It results from this formulation that if the propensity to save is equal to the share of profits, real interest rate is equal to growth rate. The preceding formulation has two implications:

- The difference between r and g is stable if A, s and a are fixed, a plausible assumption in the long run.
- Moreover, it is possible to find the equality of the two rates in the particular case where the savings rate is equal to the share of the profits.

However, this formula does not show up the hierarchy between the real interest rate and the growth rate in the general case. In other words, it does not indicate if $r > g$ or if $g < r$ or if $g = r$. The optimal theory of growth enables us to raise this uncertainty.

Indeed, the golden rule such as it has just been expressed is not any more the standard rule of the contemporary economic theory. The economic literature rather refers to a “modified golden rule” (Blanchard and Fischer, 1989). The latter consists in introducing into the reasoning a rate of time preference (noted by γ in this paper). In a more precise way, the modified golden rule is one of the main results of the optimal growth theory (Ramsey, 1928 and Cass, 1965). Let us recall that in this optimal growth model, the economy seeks to maximize its utility function not at a given moment but on several periods within the framework of its inter-temporal horizon. The assumption of a constant saving rate is abandoned. The advisable strategy is between two extreme choices. A first choice would be to consume almost nothing during the first periods to obtain a maximum consumption at a near term horizon. This attitude would reveal a rate of time preference equal to zero. With the choice of another extreme strategy, we can consider another policy by which the society consumes almost all during the first periods while deferring to later periods the necessary savings. In this case, the rate of time preference would be very strong. The economy has to determine at each period an optimal level of consumption. The assumption is that the labor force grows at a constant rate n and that the labor productivity increases at the rate m. We keep the research of the optimal path in the following reasoning. Each Δk increase in capital per worker enables to increase consumption per worker in a steady state by $r\Delta k$ because r is the marginal capital productivity. However, this extra effort involves an immediate sacrifice of consumption whose value is $\gamma\Delta k$. The net yield of this effort is thus $(r - \gamma) \Delta k$. Moreover, one additional investment equal to $(m + n)\Delta k$ must be carried out to reach the new permanent path. As long as the net yield of the operation implemented $(r - \gamma)\ \Delta k$ is higher than the additional investment equal to $(m + n)\ \Delta k$, it is advisable to increase the value of k. The optimal situation is thus reached when the following equality is verified:

$$r = m + n + \gamma \qquad (3)$$

Like in the case of the golden rule, this equality could have been found by maximizing the function $c(k) = f(k) - (m + n + \delta)k - \gamma k$.

However, the equation (3) has three major implications:

- This new rule indicates an optimal real rate of interest higher than that of the golden rule, which enables the economy to escape some form of over-saving by stopping the effort of investment before the net marginal capital productivity overtakes the value of the potential growth rate of the economy. In other words, the impatience reflected in the time preference rate g prevents the economy to sacrifice on current consumption the amount of goods necessary to reach the golden rule value (Barro and Sala-I-Martin, 1995).

- In addition, insofar as the rate of time preference is positive, the modified golden rule indicates a clear hierarchy: the real interest rate must be strictly higher than the long period growth rate.

- The difference between the real interest rate and the growth rate in the long run is positive in a balanced growth path since it is equal to the rate of time preference γ.

2.2 Endogenous growth

In pure economic theory, the most recent version of the analysis of the relations between the real interest rate and the growth rate is the endogenous growth model (Romer, 1986, Barro and Sala-I-Martin, 1995, and for a synthesis of the literature, Aghion and Howitt, 2009). In the simplest formulation of this analysis, the marginal capital productivity is not decreasing any more. We start with an economy comparable with that of the Ramsey-Cass model. We suppose however that the labor force is constant ($n = 0$) and that there is no exogenous technological progress ($m = 0$). Thus, any increase in the production is endogenous since it appears without any external contribution of demography or of the technical progress. The production function depends only on the capital k used and has as an expression: $y = Ak$. The variable y is the production per worker and A the constant capital productivity. A crucial assumption of the model is that the capital marginal productivity does not decrease, even for “infinite” levels of k. Lastly, the conditions of maximization of the corporate profit imply the equality between the real interest rate and the net marginal productivity of the capital:

$$r = A - \delta \tag{4}$$

With the intertemporal optimum, the economy is indifferent between the assignment of a unit of product to consumption (called *c*) or to investment. The real yield (noted *r* like previously) of savings (and of the additional investment) is thus equal to the satisfaction from marginal consumption. In other words, to persuade the economy to save an additional unit of the current product, which enables to increase the later consumption of *g %*, it is necessary to propose as return of its effort not only the rate of time preference γ but also the compensation of the decline in the marginal utility of consumption due to a later increase in *c* at the rate *g*. This reduction in the marginal utility is calculated by using its elasticity σ with respect to *c*. If the consumption and the product grow at the rate *g*, the marginal utility of *c* decreases by σg. In a steady state, the equality between the real yield of the additional investment and the marginal consumption is thus expressed in the following way: $A - \delta = \gamma + \sigma g$. This equality can be written as:

$$g = \frac{r - \gamma}{\sigma} \tag{5}$$

This result is equivalent to say that *g* is such that the product *Ak* is just sufficient to compensate $(\delta_{.}\ \gamma + \sigma g)k$ which includes the capital depreciation, the impatience due to the time preference and the marginal utility decline. Furthermore, the productivity *A* of capital being independent of the level of *k*, *g* is such that $A - \delta - .\gamma - \sigma g = 0$.

The equation (5) has three consequences:

- In this analysis, the growth exists only if the real interest rate is higher than the rate of time preference. Insofar as we continue to suppose that this real interest rate is equal to the net marginal productivity of the capital, the model of endogenous growth enables us to assert that there is an all the more high growth rate as the difference *r* - *g* will be large.

- In addition, the equation (5) confirms the traditional hierarchy of the superiority of the real interest rate on the growth rate only if the condition $r < \gamma + r\sigma$ is satisfied, which is always true if $\sigma \geq 1$. This condition seems a reasonable one from an empirical point of view (part 3 of the paper).

- Lastly, in the case (considered to be plausible by many authors) where σ is close to the unit, the difference between the rates *r* and *g* expresses the extent of the time preference.

To sum up, the neo-classical theory and the endogenous growth theory consider that the real interest rate tends to be higher than the growth rate in the long run. Such a conclusion has important consequences for the public finances. A well known result of the economic theory is indeed that the debt/GDP ratio in each period t is such that[1]:

$$\frac{B_t}{Y_t} = \left[1 + \frac{(r_t - g_t)}{1 + G_t}\right]\frac{B_{t-1}}{Y_{t-1}} + \frac{D_t}{Y_t} \tag{6}$$

In this equation, B is the public debt, Y is the GDP, D is the public deficit and G is the nominal growth rate. Let us assume that the economy is in a steady growth path. If the difference $r - g$ is 1 % with G = 4 %, the debt/GDP ratio will increase for instance by 0,96 % in each period even when there is no public deficit. After twenty periods, the debt/GDP ratio will have grown by 21% and after fifty years by 61% … Furthermore, with a constant deficit of 3 % of GDP and a public debt ratio of 60 % of GDP in the first period, the value of B/Y is double after only 18 periods. In other words, when the hierarchy $r > g$ is stable in the long run, the mere economic growth cannot stabilize the debt/GDP ratio. In a steady growth path, this latter ratio increases at a rate slightly lower than g at each period[2].

3 – Empirical evidence

In this section, we compare the inferences of the pure theory with the empirical data.

3.1 A standard calibration of optimal real interest rates

A first method is to calibrate the standard models to confront the preceding analytical results with the value of real interest rate. Robert Barro and Xavier Sala-I-Martin (1995) propose for example like parameterization: m = 2%, n = 0.5% or 1% and γ = 1% or 2%. Philippe Dareau (2003) applies also this method in his study of the contemporary growth models. Such a calibration implies that the optimal real interest rate ($m + n + \gamma$) of the examined countries would be in a range of 3.5% to 5% in the long run. However, this estimate appears particularly high, even if it corresponds to the levels of the 1980-89 period and to a part of the statistics in the past decade. Indeed, on the whole 1960-2008 period, the observed real interest rate is

[1] We denote by i the nominal rate of interest. We can write: $B_t = (1 + i_t)B_{t-1} + D_t$. Hence $\frac{B_t}{Y_t} = \frac{(1 + i_t)B_{t-1}}{(1 + G_t)Y_{t-1}} + \frac{D_t}{Y_t}$ and $\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = \frac{(i_t - G_t)B_{t-1}}{(1 + G_t)Y_{t-1}} + \frac{D_t}{Y_t}$. Now, $i - G = r - g$.

[2] The exact value of this rate is $\frac{\gamma}{1 + G}$.

lower than 3.5%, except in Germany (3.7%): it reaches only 2.6% in the United Kingdom, in the United States and in Japan. In France, this rate is 3.0%. Except in the case of Japan where the real interest rate was of 3.7% between 1960 and 1969, it is not plausible to deduce from these estimates that there was an over accumulation of the capital in the second half of the twentieth century owing to the strong deceleration of the investment effort at that time. Moreover, in the case of Germany and France during the Sixties, such a diagnosis would be false because the needs for rebuilding and modernizing of the two countries were then considerable. Lastly, such a judgment runs counter to the opinion often expressed by macroeconomists according to which the period of the Sixties was the statistical illustration of a balanced growth.

From an empirical point of view, one can also observe that the application of the golden rule would imply a savings rate of about 30% in the developed countries since this rule corresponds with an equality of the share of profits and of the savings rate. Such a recommendation appears excessive (except, historically, in the case of Japan). It undoubtedly comes from the limited character of the golden rule theory that neglects the role of the preference for the present by economic agents. Moreover, the golden rule theory is based on the assumption of a constant saving rate. The results of our first test are therefore not in accordance with the economic theory. To continue the research, we will thus use another method in the remainder of the paper.

3.2. Real interest, effective growth rates and potential growth rates

In this second method, the real interest rates are directly calculated starting from statistical data before we compare them with observed growth rates. To confront the theory with the recent facts, we however have a methodological problem to overcome. Indeed, the models analyzed in the first part of this paper refer to a steady state whereas the empirical data can express transitory trajectories. Nevertheless, a means to overcome this difficulty is to check the stability of the hierarchy during a transition period. Indeed, the neoclassical theory gives us indications into these matters.

Let us denote by s the propensity to save, by a the constant elasticity of the product per worker with respect to capital per capita and $f(k)$ the production function per worker (the marginal productivity $f'(k)$ is positive but decreasing, which implies $f''(k) < 0$). We then calculate the difference $r - g$[3] :

$$r - g = f'(k) - a\frac{sf(k)}{k} - (1 - a)(m + n + \delta) \qquad (7)$$

This formula is useful to determine the transitory dynamics of the difference between r and g. It can be easily calculated that[4]:

$$\frac{d}{dk}(r - g) = f''(k) + \frac{(1 - a)sf'(k)}{k} \qquad (8)$$

Moreover, we have assumed that the elasticity of the output with respect to the capital was constant, which is not very restrictive within the neo-classical framework. We thus have: $y = Ak^a$, with $0 < a < 1$. It can then be written that:

$$\frac{d}{dk}(r - g) = a(a - 1)(1 - s)Ak^{a-2} < 0 \qquad (9)$$

In other words, the difference $r - g$ is a decreasing function of the capital per worker. When the economy approaches its steady state, this difference must thus diminish. In the neo-classical model, the hierarchy $r > g$ is accompanied by a decreasing difference between the two variables when the capital per worker increases.

In the endogenous theory of growth, the assumption of a decreasing marginal productivity of the capital is abandoned. The latter is from now on constant and equal to $A - \delta$. Taking into account the value of the growth rate in this theory ($g = \frac{r - \gamma}{\sigma}$), we can write:

$$r - g = \left(\frac{\sigma - 1}{\sigma}\right)(A - \delta) + \frac{\gamma}{\sigma} \qquad (10)$$

It results from the equation (10) that $\frac{d}{dk}(r - g) = 0$. The transitory dynamics of the endogenous growth model is thus different from the preceding one. The variation $r - g$ must be constant (except we assume that the rate of time preference is a decreasing function of k, which is not a standard assumption). Let us notice in any event that the definition of a

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[3] For we have seen that : $r = f'(k) - \delta$. Furthermore,: $\dot{y} = f'(k)\dot{k} \Rightarrow \frac{\dot{y}}{y} = \frac{f'(k)k}{f(k)}\frac{\dot{k}}{k} = a\frac{\dot{k}}{k}$. Now, $\frac{\dot{y}}{y} = g - (m+n)$ and $\frac{\dot{k}}{k} = \frac{sf(k)}{k} - (m+n+\delta)$.

[4] Now, a is the elasticity of $f(k)$ with respect to k. Then : $\frac{asf(k)}{k^2} = \frac{f'(k)k}{f(k)}\frac{sf(k)}{k^2} = \frac{sf'(k)}{k}$.

steady state in the endogenous growth model is not obvious since any reference to a natural rate of growth disappears in this analysis. To evaluate the real interest rates, we will use the nominal rates in the ten years government bonds and the increase in the consumer price index. These prices are indeed those that are appropriate for the calculation of the savers' purchasing power when they fix the interest that remunerates their abstinence. This is why the majority of the empirical studies use the consumer price index to evaluate real interest rates (see for example Fitoussi and Phelps, 1988, and, for a recent study, Laubach and Williams, 2003). Taking into account the topic of this paper, one also notes that the long-term rates are the good references for the accumulation of capital. Furthermore, the natural growth rate in the theory can be evaluated in an approximate way starting from the potential growth rate of each economy. The average real interest rates and average growth rates are geometric averages. We use OECD statistics. The table 1 shows the values of r - g.

Table 1 **Values of $r – g$ by sub-periods**

Country	1960-1969	1970-1979	1980-1989	1990-1999	2000-2008
UK	- 0.2 %	- 3.2 %	0.9 %	2.5 %	0.4 %
United States	- 1.5 %	- 1.6 %	2 %	0.6 %	-0.7 %
Germany	- 0.8 %	0.1 %	2.8 %	2.1 %	1.1 %
France	- 3.3 %	- 2.6 %	2.2 %	3.4 %	0.4 %
Japan	- 8.1%	- 4.5 %	0 %	1 %	0.4 %
Euro Area					0.1 %

This table confirms one important point of the neo-classical theory of growth. The essence of the convergence process towards a balanced growth path in the neo-classical theory finds here a statistical illustration. In the Eighties and after, the hierarchy between r and g is in conformity with that of the pure economic theory. Moreover, in the long run, the economies of the studied countries seem to verify the hierarchy $r > g$ if one makes the calculations from 1970 and not from 1960 (table 2).

Table 2 **Values of $r - g$ in the long run**

Country	1960-2000	1960-2008	1970-2000	1970-2008
United Kingdom	0 %	0.2 %	0.1 %	0.1%
United States	- 0.5 %	- 0.5 %	- 0.3 %	1.4 %
Germany	1.3 %	1.2 %	1.8 %	1.7 %
France	- 0.1 %	0.02 %	1 %	1.4 %
Japan	- 2.3 %	-1.8 %	- 0.9 %	- 0.5 %

The table 1 shows likewise that during the Sixties and Seventies, the growth rates were very high. They thus exceeded the real rates. The Sixties were only a transition.

However that may be, the preceding results raise also several questions: the table 1 shows that the difference r - g is characterized by some instability. After the Sixties and Seventies, the real interest rates become higher than the growth rates, but the difference between the two rates becomes lower and lower during the 2000s, becoming even negative in the United States. These results are not easily interpretable within the framework of the Solow model. In this kind of analysis, as we already have seen, we rather expect a monotonous and negative variation of r - g during the increase process of the capital per worker (which is a stylized fact of the growth).

For the recent periods, one can however confront more directly the facts with the theory by using estimates of the potential growth rate. This concept corresponds to the natural growth rate $m + n$ of the pure theory. The reference to the potential growth rate thus enables us to carry out an empirical analysis for the long run. We thus can compare the real interest rate with its "optimal" value. We use the potential growth rates derived from the OECD statistics. On such a basis, it can be observed (table 3) that during the two last decades, the real interest rate was always higher than its level in the strict golden rule in the large developed countries. In the United States, the two rates are approximately equal during this period: such a result corresponds to the strict golden rule. It could suggest up to a point an over-accumulation of capital in this country during the 1990-2008 period.

The table 3 data also suggest that unless we suppose that the rate of time preference is approximately 2% or more, the real interest rate was higher than its optimal value (in the sense of the modified golden rule) in the euro area. This fact could be the consequence of some under-accumulation of the capital. But if we choose the highest value of the standard estimations of the rate of time preference, i.e. $\gamma = 2\%$, we can be led to assert that the real rate was lower than its value of the modified golden rule. It is therefore difficult to conclude on this point.

Table 3 **Real interest rates and potential growth rates since twenty years**

Country	Long run real interest rates 1990-2008	Potential growth rates 1990-2008	Differe nce
France	3.8 %	1.9 %	1.9 %
Germany	3.4 %	2.3 %	1.1 %
Euro Area	3.6 %	2.3 %	1.3%
United Kingdom	3.4 %	2.5 %	0.9 %
United States	2.8 %	3.0 %	- 0.2 %
Canada	3.9 %	2.8 %	1.1 %
Japan	2.2 %	1.5 %	0.7 %

3.3 Interpreting empirical data with endogenous growth theory

Let us recall that in this analysis, the growth exists only if the real interest rate is higher than the time preference rate. Insofar as we continue to suppose that this real interest rate is equal to the capital net marginal productivity, the endogenous growth model leads to the conclusion that there will exist an all the more high growth rate as this difference between the two rates will be large. Such a conclusion is not very plausible in the case of the United States. The rate of time preference in the U.S. is indeed certainly very high givens low savings levels. The gross national savings rate in the U.S. is according to OECDs 12.1% in 2008, whereas in Canada it is 23.7%, 18,9% for France, 25,8% for Germany and 18,2% for Italy… And the American households' savings rate was one of lowest in the Western world with 1.7% in 2007 and 2.7% in 2008! Unless supposing that the capital marginal productivity in the U.S. is much higher than in the other developed economies (and we will reconsider this point below), that should result, according to the endogenous growth model, in a lower growth rate than in the other countries. It is obviously not what we can observe in looking at statistical data… Conversely, Japan showed a weak preference for the present. The savings gross rate in Japan was still 27% in 2007. That did not prevent a very low growth rate in this country from 1992 to 2003. These trends are actually not new. Over the 1991-2000 period, the United States had already a very low national savings rate and Japan on the contrary made a greater effort than the other developed countries. This suggested a strong value of the coefficient γ in the United States and a low value of this coefficient in the second economy of the world. Nevertheless, the growth rate was lower in Japan than in the American economy.

Moreover, the question of the savings rate is one of the most obvious deficiencies of the standard endogenous growth model. A prominent limit of this theory consists in focusing on the dynamics of a closed economy that gives a central role to the savings rate, therefore to the time preference, in the determination of growth rate. However, at present, a low savings level by no means implies a relatively low investment rate (and thus a weak growth rate). For in an open economy, the equilibrium condition is written:

$$\frac{I}{K} = \frac{S}{K} + \frac{B}{K} - \delta \qquad (11)$$

where B is the balance on current account. Thanks to the net capital outflows, a low savings rate due to a strong preference for the present which can be observed in the United States does not involve a weak capital accumulation. Japan is naturally in the opposite situation. The per worker output will not be higher in an economy with a high savings rate than in an economy with a low saving rate (Weil, 2008). The focus of the endogenous growth theory on the time preference rate is thus not suitable to the weak empirical relevance of this rate.

If, in spite of this flaw, we keep the framework of this analysis, we could object that it is in the United States that the new information technologies were the most used. This observation could justify the assumption of a differential in the capital productivity at the advantage of the first economy in the world. Consequently, one can wonder whether the strong time preference for the present in the United States were not compensated by a stronger value of A, which is plausible. From this point of view, the high American growth rate in the Nineties would be consistent with the endogenous growth model. In the same way, we could explain a low capital profitability in Japan via a considerable accumulation of capital. For this reason, the difference between $A - \delta$ and γ (which determines the endogenous growth rate) could be weak in this country. To try to obtain a clear result, it is desirable to test the conclusions of the endogenous growth theory in another way. A very simple possibility is to calibrate the model. Let us suppose that the elasticity of marginal utility σ is equal to 1, which seems to be realistic. According to Olivier-Jean Blanchard and Stanley Fischer (1989, page 44), σ

is probably close to the unit. In the same way, Robert Barro, Xavier Sala-I-Martin (1995, page 79) and Jean-Olivier Hairault (2000, pages 301-335) assume that σ is generally lower than 2. However, Philippe Darreau uses a benchmark value of 2 for *s*. According to the *Ak* model, if the value of σ is equal to 1, the growth rate of the economy is: $g = r - \gamma$. We can deduce from this equality the value of the time preference rate by replacing *g* and *r* by their actual value: $\gamma = r - g$. The table 4 shows these estimates.

Table 4 **Estimations of the γ parameter for the 1990-2008 period**

Euro Area	Germany	France	United Kingdom	United States	Canada	Japan
1.7 %	2.0 %	2.1 %	1.0 %	0.1 %	1.3 %	0.9 %

The estimates obtained from this calibration of the endogenous growth model lead to dubious results that the time preference would be the weakest in the United States! However, we have previously seen that such an assertion is not plausible.

Secondly, the estimates of the table 2 have showed the instability of the difference $r - g$. But if it could be admitted that this difference has changed during the Sixties because of a technological shock, the posterior instability of this variation remains problematic.

Thirdly, the calibration of the endogenous growth model becomes completely unacceptable if we adopt a value of the parameter σ higher than 1. If we choose for instance a value of 2 ($\gamma = r - 2g$), one finds not only the preceding conclusion concerning the United States but also negative values for the rate of time preference in all the economies except in France and Germany!

4 – The case of the Sixties and Seventies

The preceding exercise was an interpretation of the empirical data by using the framework of the contemporary theory. We can consider that the data in the Sixties and Seventies correspond to a big and transitory raise of growth rates due to a productivity shock (rise of *m*). From this point of view, the variation $r - g$ in this period would have been negative because of the strong growth rate and/or of the low level of *r* (table 5).

Table 5 **Real interest rates and growth rates by sub-periods**

	United Kingdom	United States	Germany	France	Japan
1960-1979	r = 0.9% g = 2.4 %	r = 1.6 % g = 3.5 %	r = 3.6 % g = 3.6 %	r = 1.7 % g = 4.4 %	r = 2.2 % g = 7.4 %
1980-2008	r = 3.8 % g = 2.5 %	r = 3.5 % g = 3.0 %	r = 3.6 % g = 1.8 %	r = 4.1 % g = 2.1 %	r = 2.9 % g = 2.3 %

As Lawrence Summers showed it in a seminal paper (1983), nominal interest rates reflected only partially the permanent increase of the inflation rate during the Sixties and the Seventies. The observed real interest rates thus gradually joined their long term values. The variation r - g thus became negative because of the opposite movements of r and g. On the contrary in the Eighties, the rise in real interest rates is explained by three factors:

- the deceleration of inflation;
- the increase in profits (primarily in Europe where they increased approximately 10 basis points);
- the decline in the savings rate (particularly in Japan and the United States where the savings rate lost approximately five points).

5- Conclusion

The economic theory has not raised entirely the veil of uncertainty on the determinants of the real interest rate in a growing economy. The theory of the golden rule should be rejected by the fact that it has neglected the central role of the time preference for the present. The optimal growth model gives however more plausible results if it is considered that the rate of time preference in the largest OECD countries has been about 1%. This model shows that the real interest rate should be higher than the growth rate in the long run and the empirical evidence confirms this hierarchy. But the instability of the difference between interest rates and growth rates does not correspond what can be suggested by the theory.

On the other hand, the empirical implications of the endogenous growth model are not very convincing because the dynamics of this analysis are basically that of a closed economy. According to this analysis where the savings rate is the determining variable of investment rate, the calibration of the model suggests the existence of a weak time preference in the United States. Such a conclusion is not realistic. Moreover, the strong statistical variability of the difference $r - g$ does not seem in keeping with the theoretical analysis. Lastly, the calibration of the endogenous growth model is plausible only on the assumption of a stable unit value of the parameter σ. This hypothesis confers to this analysis a very fragile position.

However that may be, from an empirical point of view, it seems that the strongest proposition of the theory is that the interest rate tends to exceed the growth rate in the long run. This conclusion entails important consequences for the dynamics of public finance.

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