Improving Exchange Rate Forecasting with a Kalman Filter: Using Less Information to Obtain Better Forecasts

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Abstract

There is empirical evidence for a time-varying relationship between exchange rates and fundamentals. Such a relationship with time-varying coefficients can be estimated by a Kalman filter model. A Kalman filter estimates the coefficients recursively depending on the prediction error of the examined model. Using a Taylor rule based exchange rate model, which in the literature was found to have promising forecasting abilities, it is possible to further improve the performance if the utilization of information from the prediction error is restricted. This is necessary as the bad performance of classic exchange rate models is not solely due to their neglect of the time varying relationship, but also due to missing explanatory information. So, if the Kalman filter used the entire information from the prediction error, it would overestimate the need for coefficient adjustment. With this calibration of the Kalman filter model the short-term out-of-sample forecasting accuracy can be enhanced for 10 out of 12 exchange rates compared to an OLS estimation. Without our adjustment of the calibration, the Kalman filter performs worse.

Key words: Exchange rates, forecasting, Kalman filter, state space model

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1 - Introduction

Exchange rates are important for many economic agents. Currencies need to be exchanged when goods are traded across borders, investors finance projects abroad or emigrants send parts of their wage home. In an increasingly globalized world this is of particular importance. However, all these economic agents face exchange rate risks. The costly consequences of these risks could be mitigated by reliable exchange rate forecasts. Performing reliable exchange rate forecasts is, however, one of the puzzles of the economic profession. Meese and Rogoff (1983) found in their seminal paper that classic exchange rate models were not able to forecast better than a naive random walk. Cheung et al. (2005) confirmed this finding for newer models and a longer sample.

A possible reason for the non-satisfactory performance of economic exchange rate models is that the relationship between exchange rates and their fundamentals varies over time. For example, if the output of a country decreases by just a small amount, this generally does not make a significant impact on the country’s exchange rate. In times of recession, however, the impact of exchange rate variation is much stronger. In such a situation the output is likely to have a stronger impact on the exchange rate than usual. Another reason for a time-varying relationship is provided by the scapegoat model of Bacchetta and van Wincoop (2004). In such a model the exchange rate can change due to unobservable factors such as non-speculative trading, but investors attribute this change to other observable explanatory variables which increases their impact on the exchange rate.

Furthermore, empirical evidence for such a relationship can be found in the literature. Fratzscher et al. (2015) tested the aforementioned scapegoat theory empirically using survey data of about 50 foreign exchange investors who were asked about their individual weighting of the importance of various fundamentals for 12 currencies. Additionally, they obtained order flow data from the UBS, a major player in the foreign exchange market, to model the unobservable factors that drive exchange rates. They found that the in-sample performance of an exchange rate model for the period of 2001-2009 including this time-varying weighting and the unobservable factors improves substantially for all examined currency pairs. Further in-sample evidence is provided by Beckmann et al. (2011), who confirmed that the relationship between the German mark/euro-US dollar exchange rate and its fundamentals incorporates breaks and depends on different fundamentals over time. Out-of-sample evidence is provided by Sarno and Valente (2009). They show, using a rolling forecasting exercise, that the best model to forecast exchange rates
consists of frequently changing fundamentals. In this paper we estimate a
time-varying relationship and use it then for out-of-sample forecasting, as
done by Rossi (2006).

For the underlying exchange rate model, a Taylor rule based model
which was found to have promising out-of-sample forecasting abilities for
the short-term is used (Rossi, 2013). Additionally, contrary to Meese and
Rogoff (1983), for example, true out-of-sample forecasts are performed.
True out-of-sample forecasts means that no rational expectations about
future realizations of the explanatory variables are needed, and only lagged
variables are used. Such Taylor rule models were first used for forecasting
by Molodtsova and Papell (2009). They evaluated such models as more
useful than classic exchange rate models as the purchasing power parity or
the monetary approach. A time-varying relationship can be inserted into the
models using a Kalman filter. The Kalman filter approach is chosen for two
reasons: Firstly, this framework allows for continuously changing
coefficients which are not restricted to certain historical or recurring values.
Secondly, a stochastic component is embedded. Those features justify our
choice ahead of other techniques such as Markov switching or smooth
transition models. As in Schinasi and Swamy (1987), Wolff (1987), Ng and
Heidari (1992) or Rossi (2006), it is assumed that the coefficients of the
exchange rate model behave like random walks and evolve over time. The
Kalman filter allows a recursive estimation of the unobservable time-varying
variation in the coefficients. Using an updated version of the data set of
Molodtsova and Papell (2009), it is then tested whether this modification
improves the forecasting performance compared to the baseline model.

The starting point for our calibration of the Kalman filter is Rossi
(2006). The coefficients are assumed to behave as a random walk. The
evolution of the time-varying coefficients is slowed down by introducing a
parameter smaller than one into the state equations’ error term of the Kalman
filter model. Our newly introduced feature is that also the considered
prediction error, which usually is very high for exchange rate models (Evans
and Lyons (2002)), is reduced by incorporating another parameter lower than
one into the signal equation’s error term. So, if the performance of exchange
rate models is weak not only because of non-time-varying coefficients, but
also due to missing information, then the Kalman filter would overestimate
the prediction error and thereby the need for coefficient adjustment. With this
calibration the short-term forecasting accuracy can be enhanced for 10 out of
12 exchange rates. This favorable result is also robust against other window
sizes of the rolling regressions. This is interesting as for lower window sizes a
time-varying relationship is already more strongly implied for the baseline
model. Comparisons against the random walk without drift show that the result of Meese and Rogoff (1983) prevails.

This paper’s contribution is to show that applying a time-varying coefficient approach to Taylor rule exchange rate models can further improve the short-term out-of-sample forecasting abilities compared to simple OLS forecasts. Using a very long sample, examining 12 different exchange rates and varying window sizes, the result is found to be robust. However, this is only possible if one adjusts the calibration of the Kalman filter model for the estimation of the underlying exchange rate model. This opposes the results of older papers such as Schinasi and Swamy (1987), Wolff (1987) or Ng and Heidari (1992) who found, using short samples of about 10 years, that a simple Kalman filter model is better in forecasting exchange rates than OLS. Other authors such as Rossi (2006), using longer and more recent samples, did not obtain such results. In our robustness chapter we also find that the nearer we get to the standard calibration of the Kalman filter, the worse the results become. Therefore, our new calibration adds value to the literature by showing that a Kalman filter can be capable of improving exchange rate forecasts - also for more recent data.

The outline of this work is as follows: In the first section the underlying Taylor rule exchange rate model, as used by Molodtsova and Papell (2009), is stated. Secondly, it is explained how the time-varying relationship is incorporated into the exchange rate model with a Kalman filter model. After this, the criteria to evaluate the forecasting performance, the data set and the forecasting methodology are described. Then, the results of a forecast performance comparison between a Taylor rule exchange rate model including a time-varying relationship and a benchmark model, either the baseline Taylor rule model of Molodtsova and Papell (2009) or the random walk, are presented. Lastly, the robustness of the previous results with regard to different window sizes of the rolling regressions and the Kalman filter calibration is examined.

2 - Taylor Rule Exchange Rate Model

The underlying exchange rate model for the forecast comparison between a model with and without a time-varying relationship is Taylor (1993) rule based. Such Taylor rule models have been applied to exchange rate economics by Engel and West (2005), Engel and West (2006) or Mark (2009), Engel et al. (2007), Wang and Wu (2012) and Lansing and Ma (2015) found that they are useful for exchange rate forecasting. Additionally,
Molodtsova et al. (2008), Molodtsova and Papell (2009), and Beckmann and Schüssler (2016) noted that the influence of the included variables varies over time. Also, the Taylor rule model itself has been estimated with time-varying coefficient model. See Yüksel (2013) for a survey.

Taylor’s rule relies on the idea that a central bank sets its short-term key interest rate as a function of inflation, targeted real key interest rate and output gap:

\[ i_t^* = \pi_t + \phi(\pi_t - \pi^*) + \gamma y_t + \tau^* \]  

An asterisk denotes the targeted level. This equation shows that the central bank raises its key interest rate target, \( i^* \), in case of inflation, \( \pi \), rising above the targeted level. It also sets the rate higher when the output gap, \( y \), increases. Because it is assumed that the target for the key interest rate is reached within one period, the actual interest rate is not included (Mishkin, 2004).

From this basic equation, we can get the nominal interest rate reaction function assuming that the central bank targets its currency’s purchasing power parity (PPP) to hold and that adjustments to the interest rate are only done successively. For details refer to Molodtsova and Papell (2009). So, the derived equation looks like the following:

\[ i_t = (1 - \rho)(\mu + \lambda \pi_t + \gamma y_t + \delta q_t) + \rho i_{t-1} + \nu_t \]  

The \( q \) represents the real exchange rate. For the United States \( \delta \) is equal to zero because it is the reference currency for all exchange rates examined later. To obtain an estimable equation, the US interest rate reaction function is subtracted by its foreign country’s counterpart:

\[ i_t - i^*_t = \alpha + \alpha_u \pi_t - \alpha_f \tilde{\pi}_t + \alpha_u \gamma y_t - \alpha_f \tilde{\gamma}_t - \alpha_q \tilde{q}_t + \rho u i_{t-1} - \rho f i^*_{t-1} + \eta_t \]  

A tilde denotes a foreign variable. The indices \( u \) and \( f \) display coefficients of the US and the foreign country, respectively. \( \alpha \) is a constant, \( \alpha_{\pi} = \lambda(1 - \rho) \) and \( \alpha_{\gamma} = \gamma(1 - \rho) \) for both countries, and \( \alpha_q = \delta(1 - \rho) \) for the foreign one. The error term of this two-country equation is denoted as \( \eta \).

The effect of a successively adjusted interest rate can be explained in the following way: If inflation rises, the central bank will increase the interest rate by \( (1 - \rho)\lambda \Delta \pi \) in the first period. In the second period, it will be raised by
(1 − \rho^2)\lambda \Delta \pi$ and so forth. Note that $\rho$ is assumed to be between zero and one. If one assumes uncovered interest rate parity (UIP), the interest rate differential is proportional to the expected exchange rate depreciation:

$$E(\Delta s_{t+1}) = \omega(i_t - \bar{i}_t), \quad (4)$$

Inserting this equation into Equation (3) and assuming rational expectations, a Taylor rule based exchange rate forecasting equation is obtained:

$$\Delta s_{t+1} = \omega + \omega_u \pi_t + \omega_f \bar{\pi}_t + \omega_u \bar{y}_t - \omega_f y_t - \omega_q q_t + \rho_u \bar{i}_{t-1} - \rho_f \bar{i}_{t-1} + \eta_t \quad (5)$$

However, the literature shows that the UIP does not hold in the short run. Chinn (2006) examines new empirical findings about the UIP and found that in the short-term the interest rate differential is still a biased predictor of ex post changes of exchange rates. Cheung et al. (2005) found that the UIP can forecast exchange rates only in the long-run. Recent carry trade literature, such as Burnside et al. (2007), even suggests that $\omega$ of Equation (4) can be negative. Still, this equation can be used for forecasting if the signs and values of the coefficients are not restricted.

Furthermore, Molodtsova and Papell (2009) made the prediction that an increase of the interest rate results in exchange rate appreciation. Gourinchas and Tornell (2004) presented a theoretical model in which the investors misperceive the persistence of an interest rate change. This allows for a mechanism in which an increase of the interest rate leads to an exchange rate appreciation. Additionally, they presented survey evidence which shows that investors underestimate the persistence of interest rate shocks to support their theory.

If interest rate smoothing is assumed, higher inflation not only raises the current interest rate, but also leads to the expectation that the interest rate will continue to rise in the future. Therefore, an increase of the inflation rate does result in immediate and expected future exchange rate appreciation. A further effect of interest rate smoothing, in the context of the Gourinchas and Tornell (2004) theory, is that investors are even more likely to underestimate the true nature of persistence of interest rate shocks because the initial change of the interest rate is smaller than the total change over time. This strengthens the effect of inflation on the exchange rate (Molodtsova and Papell, 2009).

Three additional predictions in the context of the Taylor rule are made by Molodtsova and Papell (2009). Firstly, an increase in the US output gap
leads to a rise of the interest rate which then results in a depreciation of the exchange rate. Secondly, if the real exchange rate depreciates, the foreign central bank will raise its interest rate to revert the real exchange rate back to its PPP equilibrium. Lastly, assuming interest rate smoothing, a higher lagged interest rate leads to further immediate and expected future depreciation of the exchange rate.

This Taylor rule forecasting equation can be used in several different specifications of the variables on the right-hand side. Firstly, in his original paper Taylor (1993) stated for Equation (3) that the central bank sets the nominal interest rate only according to the inflation gap, the output gap and the equilibrium real interest rate omitting the lagged interest rate differential and the real exchange rate. If the foreign central bank behaves similarly, this model is called symmetric. However, following Clarida et al. (1998), it can be assumed that the foreign central bank aims at the difference between the actual and targeted exchange rate defined by PPP. In such an asymmetric model the real exchange rate is included in the equation, \( \omega_q \neq 0 \). Here, however, only symmetric models, for which Molodtsova and Papell (2009) found the best predictability, are examined for conciseness. Alba et al. (2015) examine asymmetric models for inflation-targeting emerging markets for which the real exchange rate is likely to be an important determinant.

The next specification adds the lagged interest rate differential to the right-hand side to account for a successively adjusted interest rate, as suggested by Clarida et al. (1998). In this case the model is denoted as with smoothing, \( \rho_u \neq 0 \), or otherwise as with no smoothing.

Lastly, if both central banks respond equally to changes of the right-hand side variables so that \( \omega_u = \omega_f \), \( \omega_y = \omega_y \) and \( \rho_u = \rho_f \), the model is called homogeneous. If they are not identical, each appears solely on the right-hand side and the model is called heterogeneous. Therefore, the models which are considered are with or without smoothing and include homogeneous or heterogeneous coefficients.

3 - Modeling the Time-Varying Relationship

Section 1 outlined empirical evidence for a time-varying relationship between exchange rates and fundamentals. In this section the estimation of such a relationship with a Kalman (1960) filter model is described. Other authors, for example, Canova (1993), Carriero et al. (2009) or Beckmann and Schüssler (2016) used other models which allow for a time-varying
relationship. See Rossi (2013) for a further review.

First of all, a short introduction to a Kalman filter model is given. A Kalman filter model is a state space model which consists of signal and state equations. The signal equation describes the relationship between the observed and unobserved variables. The state equations represent the evolution of the unobserved variables, here the time-varying coefficients, $\beta_t$. The Kalman filter recursively estimates the unobserved variables conditional on the available information. With some chosen starting values the first signal equation is estimated in $t = 0$. In $t = 1$ the prediction error is calculated by comparing the actual dependent variable with the estimated one. This prediction error is then used to update the beliefs about $\beta_{t=0}$. Using the updated coefficient the next signal equation is estimated and so forth.

Kalman filter models with a time-varying relationship have already been used in exchange rate forecasting. Schinasi and Swamy (1987) allowed the coefficients of their monetary exchange rate models to vary over time in the following way:

$$s_t = f_t' \beta_t + \epsilon_t$$  \hspace{2cm} (6)
$$\beta_t = \beta_{t-1} + \eta_t$$  \hspace{2cm} (7)
$$\eta_t = \phi u_{t-1} + \nu_t$$  \hspace{2cm} (8)
$$E(\nu_t) = 0$$  \hspace{2cm} (9)

The first equation of this system is the signal equation with $f_t$ containing the explanatory variables. Equation (7) is the state equation which describes the development of the coefficients. As the error term $\eta_t$ of the state equation is assumed to be of autoregressive order, the coefficients follow an autoregressive process. Comparing their forecasts with the random walk, they found favorable results if a lagged dependent variable is included.

Wolff (1987) used the Kalman filter to estimate such a time-varying parameter model for similar monetary models. Contrary to Schinasi and Swamy (1987), he assumed that the coefficients behave as a random walk so that $\phi = 0$. He found that such a model is often able to forecast more accurately than the random walk. However, a newer study by Rossi (2006) was not able to obtain similar findings for autoregressive models, with or without further economic variables, and using a longer sample. Rossi (2013), giving an overview of further studies, comes to the conclusion that the empirical evidence is mixed. Furthermore, Bacchetta et al (2010) conclude that time-varying parameters cannot explain the puzzle of Meese and Rogoff (1983) which, though, is debated by Chinn (2009) and Giannone (2009).
Here the time-varying relationship between exchange rates and fundamentals is modeled comparable to Wolff (1987) and Rossi (2006). However, it is newly applied to models with Taylor rule fundamentals for short-term forecasting and a longer sample.

\[
\Delta s_{t+1} = \omega + \omega_{w,t} \tilde{\pi}_t - \omega_{f,t} \tilde{\pi}_t + \omega_{u,t} \tilde{\pi}_t - \omega_{f,t} \tilde{\pi}_t + \rho_{u,t} \tilde{\pi}_t - \rho_{f,t} \tilde{\pi}_t - 1 + \epsilon_t \\
+ \omega_{c,t} = \omega_{c,t-1} + \eta_t \text{ for } c = (f, u) \text{ and } \nu = (\pi, y) \quad (10) \\
\rho_{c,t} = \rho_{c,t-1} + \eta_t \text{ for } c = (f, u) \quad (11)
\]

This model can be formulated more concisely:

\[
\Delta s_{t+1} = f_t \beta_t + \epsilon_t \\
\beta_t = \beta_{t-1} + \eta_t \quad (13) \quad (14)
\]

where \( \epsilon_t \sim iid N(0, k^2 \sigma^2) \), \( \eta_t \sim iid N(0, \lambda^2 \sigma^2 \eta Q) \), \( E(\eta_t \epsilon_t) = 0 \) and \( Q = E(f_t f_t')^{-1} \) \( \forall t \) and with vector \( f_t \) including the \( k \) fundamentals.

Thus, the Kalman filter is calibrated in the following way: Starting values for the coefficients are set to zero for every regression. For all state equations the error terms are assumed to be equal, and are normalized by a \( k \times k \) covariance matrix which is specified by \( Q = E(f_t f_t')^{-1} \). Furthermore, \( \lambda < 1 \) is the ratio between the variance of the signal and state equations’ error terms. A smaller \( \lambda \) leads to a slower coefficient evolution. Until now the calibration of the Kalman filter is the same as in Rossi (2006) or Stock and Watson (1996).

The difference to previous approaches is that a parameter \( k < 1 \) is inserted into the signal equation’s error term. The effect is that the prediction error, \( \Delta s_t - \Delta s_{t-1} \), which usually is very high in exchange rate models (Evans and Lyons (2002)), has less of an effect on the updating of the coefficients. If the bad performance of exchange rate models is not solely due to non-time-varying coefficients, but also due to missing explanatory information, then the Kalman filter would adjust the coefficients too strongly. Henceforth, \( k \) is set to 0.001 and \( \lambda \) is set to \( k \cdot 0.1 \). The value of \( k \) is chosen to be very small to ensure that only a very limited amount of information of the prediction error is used. The value of \( \lambda \) is set comparable to Rossi (2006) and slows the evolution of the coefficients down. In Section 5.1 it will be examined whether the upcoming results are robust to other calibrations of these parameters. We
refrain from adding further variables as it is already difficult for a Kalman filter to estimate a model with 6 variables and account the information of the prediction error towards a specific variable.

For the upcoming forecast exercise rolling regressions are used. See Section 4.2 for details. Rolling regressions already implicitly allow coefficients to change at every step forward. Molodtsova and Papell (2009) showed in their paper how the coefficients evolve across their sample. Here it is shown that it is still worthwhile to use models which allow for time-varying parameters in addition to rolling regressions. In Figure 1 the path of the coefficients across the sample for rolling regressions estimated with OLS and a Kalman filter is shown. For the Kalman filter model the final state values are reported. All equations are estimated for a symmetric Taylor rule model with smoothing and heterogeneous coefficients for the Pound-US dollar exchange rate. In general, it can be seen that for both estimation methods the coefficients follow a similar path, but the coefficients of the Kalman filter model incorporate more abrupt adjustments. For further explanations about the path of the coefficients, refer to Molodtsova and Papell (2009).
Recent studies also employed other methods to model the time-varying relationship. See, for example, Fratzscher et al. (2015) who obtained actual data about the weighting investors put on fundamentals. Finding favorable results, their sample is, however, too short to conduct an out-of-sample forecasting exercise.
4 - Comparison of Forecast Performance

In this section the forecasting performance of the Taylor rule models will be examined with a special focus on whether it is possible to improve forecasting by taking the time-varying relationship between exchange rates and fundamentals into account.

1.1 Evaluating Forecasting Performance

The forecasting performance of the models is evaluated by calculating the mean squared prediction error (MSPE). For a sample with $T$ observations the MSPE of $P$ forecasts is calculated as follows:

$$
\hat{\sigma}_{m}^2 = P^{-1} \sum_{t=T-P}^{T} (\hat{e}_{m,t+1})
$$

(15)

Where $\hat{e}_{m,t+1}$ is the one-period ahead forecast error of the respective model $m$. The forecast for the random walk is that the exchange rate does not change, $\hat{e}_{rw,t+1} = \Delta s_{t+1}$. With the Diebold-Mariano test (1995), it is possible to test whether the MSPE of a model is different from a competing model. The null hypothesis is that both models’ MSPE are equal which is tested against the alternative that model $m$ has a higher MSPE:

$$
H_0: \sigma_m^2 - \sigma_n^2 = 0
$$

(16)

$$
H_1: \sigma_m^2 - \sigma_n^2 > 0
$$

(17)

For the construction of the Diebold-Mariano (DM) test statistic, the average difference between the forecasts’ errors of the competing models is defined as:

$$
\bar{f} = P^{-1} \sum_{t=T-P}^{T} (\hat{e}_{m,t+1} - \hat{e}_{n,t+1}) = \hat{\sigma}_m^2 - \hat{\sigma}_n^2
$$

(18)

Then the DM test statistic can be computed as follows:
The denominator represents the standard error of the difference between the competing model’s forecast errors. The test statistic of the Diebold-Mariano test can easily be gained by regressing the difference of the forecast errors on a constant. To estimate the standard error consistently, a Newey-West covariance estimator which is consistent in the presence of both heteroskedasticity and autocorrelation is used (Elliot et al., 2006). For the covariance estimator the lag length is chosen by the automatic bandwidth selection method for kernel estimators of Newey-West (1994). Because the DM statistic has an asymptotic normal distribution, the resulting t-statistic of the regression can be used to test the above mentioned hypothesis (Diebold and Mariano, 1995). Note that the DM test in this setup is one-sided. Therefore, a significant and positive DM test statistic implies that model \( m \) outperforms \( n \) because \( \sigma_m^2 \) is significantly higher than \( \sigma_n^2 \). The critical values for a one-sided test and \( P \to \infty \) are 1.282, 1.645 and 1.960 for the significance levels 0.10, 0.05 and 0.01, respectively.

Newer test statistics for a test which compares nested models also exist and were used, for example, by Molodtsova and Papell (2009), Gourinchas and Rey (2007) or Engel et al. (2007). In nested models, one model is contained within the other. This is, for example, given for a comparison between the random walk and any other model. Note that this is not the case if Taylor rule models with and without time-varying relationships are compared as the models are different due to the time-varying coefficients. The tests of Clark and West (2006, 2007) and Clark and McCracken (2001, 2005) compensate for the fact that the DM test statistic is undersized under the null hypothesis when comparing nested models. In the context of exchange rates, both tests take into account that, given the null hypothesis, the exchange rate follows a random walk. However, as Rogoff and Stavrakeva (2008) showed, these tests are no real minimum MSPE tests. Rather, they show if the true model is a random walk instead of whether the random walk has a lower MSPE than the structural model. So, a DM test compares forecast ability while the newer tests look at predictability (Molodtsova and Papell, 2009). Here it is stuck to the DM test for a
comparison between nested models.

4.2 - Data Set and Forecasting Methodology

The data used in this work was obtained from David Papell’s homepage and has been updated. Besides the update it is the same as in Molodtsova and Papell (2009).\(^3\) It ranges, in general, from March 1973 to December 1998 for the euro area countries and to May 2015 for the others.\(^4\) Due to missing data, the sample starts in January 1975 for Canada, September 1975 for Switzerland and January 1983 for Portugal. The main source for the data is the International Monetary Fund’s *International Financial Statistics* database.\(^5\) Because GDP data is available only quarterly, a seasonally adjusted monthly industrial production index is used for the calculation of the output gap. The economies’ price levels are measured by the consumer price index and inflation is then calculated as the 12-month difference of the price index. For the interest rates set by the central banks the money market rates are taken. The exchange rates are obtained from the *Federal Reserve Bank of Saint Louis’* database.

For the construction of the output gap, a Hodrick and Prescott (1997) filter with the standard smoothing parameter of 14,400 for monthly frequencies was used, with a sample beginning in January 1971, to obtain the potential output. The output gap is then calculated by the percentage of deviation of the potential and actual output. Because rolling regressions are used for forecasting, the output gap is recalculated for each step with data excluding the forecasted month. So, no ex post data is used for the calculation. Thereby, it is simulated that the central bank decision to set the interest rate is based only on the information available at that time. However, this only mimics real-time data because revised data is still used. Another shortcoming is that it is not known how exactly the central banks calculate the output gap for their decision making. For example, Orphanides (2001), estimating Taylor rules for the USA, used output gap data generated by the US central bank itself, but such data is not available for the full sample or

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\(^3\) Compare [http://www.uh.edu/~dpapell/papers2.htm](http://www.uh.edu/~dpapell/papers2.htm).

\(^4\) The euro area currencies are French franc, German mark, Italian lira, Dutch guilder and Portuguese escudo. The others are Canadian dollar, Australian dollar, Danish kroner, Japanese yen, Swedish kronor, Swiss franc and British pound.

\(^5\) The only exception is that the German consumer price index is taken from the *OECD Main Economic Indicators* database.
other countries. Nevertheless, Orphanides and van Norden (2002) stated that most of the difference between revised and real-time data of output gap estimation comes from using ex-post data, not from revisions.

With this data the Forecasting Equations (5) and (10) are estimated. The regressions are then used for the one-month ahead forecasts of the exchange rates. As in Molodtsova and Papell (2009), a sample of 120 observations is taken for the rolling regressions. Thus, the first estimation is done with data from March 1973 until February 1983 and the exchange rate is out-of-sample forecasted for March 1983. The out-of-sample forecast is done with data for the explanatory variables excluding the forecast period contrary to Meese and Rogoff (1983). Thus, rational expectations about their development are not needed and the forecasts are truly out-of-sample. For the next estimation the first data point is dropped and an additional one is added at the end of the sample. Then the next out-of-sample forecast follows. So, the regressions roll over the full sample. The full sample, however, differs for the countries. As previously mentioned, the euro area countries’ samples end in December 1998 and data is missing for Portugal, Switzerland and Canada. So, for the euro area exchange rates 190 instead of 388 forecasts are performed, for Portuguese, Swiss and Canadian exchange rates only 72, 358 or 365, respectively. Note that also the output potential is calculated again with each step, but the beginning of the sample is not dropped.

Because the various specifications for Equations (5) and (10) differ in their forecasting performance, only symmetric specifications for which Molodtsova and Papell (2009) found the best predictability are evaluated. First, the MSPE for each exchange rate is calculated. Secondly, the Diebold-Mariano test statistic for a comparison between the model of Equation (10) and the benchmark model, which is either the random walk or the corresponding model without a time-varying weighting, Equation (5), is determined.
Figure 2: Forecast Comparison between Equation (5) and (10) for the Pound- US Dollar Exchange Rate

An example comparison between Equation (5) and (10) for a symmetric model with interest rate smoothing of the Pound-US dollar exchange rate is shown in Figure 2. The graph shows the difference between the forecast errors of both models. A positive value represents that the model with a time-varying relationship has a smaller forecasting error. The differences fluctuate around zero and it cannot be said that one model performs better in a certain period.

4.3 - Results

In this section the results of the forecasting equations are presented. The finding of Meese and Rogoff (1983), or Cheung et al. (2005) for newer models and a longer sample, which states that exchange rate models are not able to forecast better than a random walk without drift which is the toughest benchmark to beat according to Rossi (2013) still prevails. Therefore, the focus will be on whether including a time-varying relationship between exchange rates and fundamentals improves the forecasting performance of Taylor rule based exchange rate models. In Table 1 the MSPE of symmetric specifications of the baseline Equation (5) and Equation (10) with a time-
varying relationship are presented. Symmetric models do not include that the foreign central bank is targeting the real exchange rate. A model denoted as smoothing incorporates the lagged interest rate. Furthermore, it is necessary to distinguish between specifications with heterogeneous or homogeneous coefficients. If a MSPE value is written in bold, then the model has a MSPE lower than the random walk’s one. This is given for 7 of the 12 currency pairs. While for Italy most models perform better than the random walk, this is only given for one of the time-varying ones for Japan, Sweden and the United Kingdom. For five currencies no specification forecasts better than the random walk. For all currency pairs, it cannot be generalized that models with a time-varying relationship perform better. Additionally, no model is able to significantly beat the random walk as can be seen in Table 2, which is comparable to the findings of Rogoff and Stavrakeva (2008) for Taylor rule exchange rate models.

To evaluate whether there is a significant difference between the Forecasting Equations (5) or (10), Table 3 is presented. The signs of the DM test statistics vary across currency pairs and Taylor rule model specifications, but are in general positive. Recall that the critical values for a one-sided test and $P \to \infty$ are 1.282, 1.645 and 1.960 for the significance levels 0.10, 0.05 and 0.01, respectively. Therefore, the time-varying Taylor rule model outperforms the baseline one for 10 of the 12 currencies. For 2 currency pairs no significant statistic was obtained. Furthermore, as the MSPE is sensitive to outliers, we also use another criterion to evaluate our forecast. The mean absolute error (MAE) incorporates no squared loss function and is thus more robust to outliers. We show the results for this criterion in Table 4. The results are only slightly weaker as we can improve forecasting only for 9 out of 12 currencies.

In these first results it is shown that estimating a Taylor rule exchange rate model with a Kalman filter to take time-varying coefficients into account can improve the forecasting accuracy. Nevertheless, these improvements are not strong enough to beat the random walk so that the result of Meese and Rogoff (1983) prevails.

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6 The following abbreviations are used henceforth in tables: Can for Canadian dollar, Aus for Australian dollar, Dan for Danish kroner, Fra for French franc, Ger for German mark, Ita for Italian lira, Jap for Japanese yen, Dut for Dutch guilder, Por for Portuguese escudo, Swe for Swedish kronor, Swi for Swiss franc and UK for British pound.
The table reports the MSPE for symmetric specifications of the baseline Equation (5) and (10) with a time-varying relationship. The values were calculated by forecasts for the months March 1983 to December 1998 for euro area currencies and to May 2015 for the other ones. Bold values denote that the model has a lower MSPE than the random walk.
Table 2: DM Statistics for MSPE Comparisons between Symmetric Specifications of Forecasting Equations (5) and (10) Against the Random Walk

<table>
<thead>
<tr>
<th>Specification</th>
<th>Can</th>
<th>Aus</th>
<th>Dan</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heterog. Baseline</td>
<td>-1.51</td>
<td>-0.75</td>
<td>-2.48</td>
<td>-1.73</td>
<td>-1.78</td>
<td>-0.61</td>
</tr>
<tr>
<td>Time-varying</td>
<td>-1.26</td>
<td>-1.23</td>
<td>-1.75</td>
<td>-0.96</td>
<td>-1.65</td>
<td>0.22</td>
</tr>
<tr>
<td>Homog. Baseline</td>
<td>-1.96</td>
<td>-1.53</td>
<td>-1.40</td>
<td>0.49</td>
<td>-1.24</td>
<td>0.87</td>
</tr>
<tr>
<td>Time-varying</td>
<td>-2.06</td>
<td>-0.41</td>
<td>-0.76</td>
<td>0.72</td>
<td>-1.37</td>
<td>0.92</td>
</tr>
<tr>
<td>Heterog. Baseline</td>
<td>-0.90</td>
<td>-0.91</td>
<td>-3.95</td>
<td>-2.00</td>
<td>-1.91</td>
<td>-1.26</td>
</tr>
<tr>
<td>Time-varying</td>
<td>-1.25</td>
<td>-0.51</td>
<td>-1.75</td>
<td>-0.93</td>
<td>-1.24</td>
<td>-0.24</td>
</tr>
<tr>
<td>Homog. Baseline</td>
<td>-1.46</td>
<td>-1.65</td>
<td>-2.20</td>
<td>0.05</td>
<td>-1.31</td>
<td>0.10</td>
</tr>
<tr>
<td>Time-varying</td>
<td>-1.14</td>
<td>-0.53</td>
<td>-0.86</td>
<td>-0.26</td>
<td>-1.56</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specification</th>
<th>Jap</th>
<th>Dut</th>
<th>Por</th>
<th>Swe</th>
<th>Swi</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heterog. Baseline</td>
<td>-1.17</td>
<td>-0.51</td>
<td>0.06</td>
<td>-2.09</td>
<td>-2.87</td>
<td>-1.82</td>
</tr>
<tr>
<td>Time-varying</td>
<td>-0.73</td>
<td>-0.31</td>
<td>-0.43</td>
<td>-1.65</td>
<td>-3.29</td>
<td>-0.41</td>
</tr>
<tr>
<td>Homog. Baseline</td>
<td>-1.16</td>
<td>-0.59</td>
<td>-0.03</td>
<td>-2.71</td>
<td>-0.96</td>
<td>-0.75</td>
</tr>
<tr>
<td>Time-varying</td>
<td>-0.86</td>
<td>0.23</td>
<td>1.28</td>
<td>-1.40</td>
<td>-2.36</td>
<td>-0.25</td>
</tr>
<tr>
<td>Heterog. Baseline</td>
<td>-0.71</td>
<td>-1.32</td>
<td>-0.73</td>
<td>-2.46</td>
<td>-3.52</td>
<td>-1.05</td>
</tr>
<tr>
<td>Time-varying</td>
<td>-0.45</td>
<td>-0.33</td>
<td>-0.46</td>
<td>-0.40</td>
<td>-4.20</td>
<td>-0.44</td>
</tr>
<tr>
<td>Homog. Baseline</td>
<td>-0.02</td>
<td>-0.89</td>
<td>-0.41</td>
<td>-2.48</td>
<td>-1.89</td>
<td>-1.03</td>
</tr>
<tr>
<td>Time-varying</td>
<td>0.30</td>
<td>0.00</td>
<td>0.46</td>
<td>0.50</td>
<td>-2.93</td>
<td>0.06</td>
</tr>
</tbody>
</table>

The table reports the DM statistics for a test of the equality of forecast accuracy for symmetric specifications of the baseline Equation (5) and (10) with a time-varying relationship against the random walk. The values were calculated by forecasts for the months March 1983 to December 1998 for euro area currencies and to December 2013 for the other ones. The critical values for $P \rightarrow \infty$ are 1.282, 1.645 and 1.960 for the significance levels 0.10, 0.05 and 0.01, respectively.
Table 3: DM Statistics for MSPE Comparisons between Different Specifications of Forecasting Equations (5) and (10)

<table>
<thead>
<tr>
<th>Symmetric</th>
<th>Can</th>
<th>Aus</th>
<th>Dan</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Smooth. Heterog.</td>
<td>0.44</td>
<td>0.01</td>
<td>0.75</td>
<td><strong>1.40</strong></td>
<td><strong>1.38</strong></td>
<td>1.16</td>
</tr>
<tr>
<td>Homog.</td>
<td>0.58</td>
<td><strong>1.62</strong></td>
<td>1.21</td>
<td>-0.13</td>
<td>0.91</td>
<td>0.08</td>
</tr>
<tr>
<td>Smooth. Heterog.</td>
<td>-0.44</td>
<td>1.02</td>
<td>0.40</td>
<td><strong>2.33</strong></td>
<td><strong>1.59</strong></td>
<td><strong>1.82</strong></td>
</tr>
<tr>
<td>Homog.</td>
<td>0.29</td>
<td><strong>1.71</strong></td>
<td><strong>1.82</strong></td>
<td>-0.45</td>
<td>0.75</td>
<td>0.26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symmetric</th>
<th>Jap</th>
<th>Dut</th>
<th>Por</th>
<th>Swe</th>
<th>Swi</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Smooth. Heterog.</td>
<td><strong>1.28</strong></td>
<td>0.52</td>
<td>-0.55</td>
<td>0.92</td>
<td>0.27</td>
<td><strong>2.79</strong></td>
</tr>
<tr>
<td>Homog.</td>
<td>0.37</td>
<td>0.86</td>
<td>0.67</td>
<td><strong>2.17</strong></td>
<td>-0.78</td>
<td><strong>1.44</strong></td>
</tr>
<tr>
<td>Smooth. Heterog.</td>
<td><strong>1.39</strong></td>
<td><strong>2.15</strong></td>
<td>0.99</td>
<td><strong>1.51</strong></td>
<td><strong>1.31</strong></td>
<td><strong>1.33</strong></td>
</tr>
<tr>
<td>Homog.</td>
<td>0.56</td>
<td>1.05</td>
<td>0.64</td>
<td><strong>1.80</strong></td>
<td>-0.69</td>
<td><strong>2.59</strong></td>
</tr>
</tbody>
</table>

The table reports the DM statistics for a test of the equality of forecast accuracy for symmetric specifications of the baseline Equation (5) and (10) with a time-varying relationship. The values were calculated by forecasts for the months March 1983 to December 1998 for euro area currencies and to May 2015 for the other ones. The critical values for $P \rightarrow \infty$ are 1.282, 1.645 and 1.960 for the significance levels 0.10, 0.05 and 0.01, respectively.
Table 4: DM Statistics for MAE Comparisons between Different Specifications of Forecasting Equations (5) and (10)

<table>
<thead>
<tr>
<th>Symmetric</th>
<th>Can</th>
<th>Aus</th>
<th>Dan</th>
<th>Fra</th>
<th>Ger</th>
<th>Ita</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Smooth. Heterog.</td>
<td>0.11</td>
<td>1.19</td>
<td><strong>1.29</strong></td>
<td>1.42</td>
<td>1.24</td>
<td><strong>1.41</strong></td>
</tr>
<tr>
<td>Homog.</td>
<td>-0.01</td>
<td><strong>2.86</strong></td>
<td>0.24</td>
<td>-0.19</td>
<td>0.77</td>
<td>0.19</td>
</tr>
<tr>
<td>Smooth.    Heterog.</td>
<td>-0.23</td>
<td><strong>2.38</strong></td>
<td>0.79</td>
<td><strong>2.56</strong></td>
<td><strong>1.91</strong></td>
<td><strong>2.96</strong></td>
</tr>
<tr>
<td>Homog.</td>
<td>-0.13</td>
<td><strong>2.55</strong></td>
<td><strong>1.36</strong></td>
<td>-0.40</td>
<td>0.45</td>
<td>0.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symmetric</th>
<th>Jap</th>
<th>Dut</th>
<th>Por</th>
<th>Swe</th>
<th>Swi</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Smooth. Heterog.</td>
<td>1.02</td>
<td><strong>1.67</strong></td>
<td>-0.87</td>
<td>1.09</td>
<td>-0.61</td>
<td><strong>2.93</strong></td>
</tr>
<tr>
<td>Homog.</td>
<td>0.53</td>
<td>0.65</td>
<td>0.42</td>
<td><strong>1.87</strong></td>
<td>-1.53</td>
<td>0.00</td>
</tr>
<tr>
<td>Smooth.    Heterog.</td>
<td>1.11</td>
<td><strong>3.07</strong></td>
<td>0.46</td>
<td><strong>1.91</strong></td>
<td>0.03</td>
<td><strong>1.46</strong></td>
</tr>
<tr>
<td>Homog.</td>
<td>0.54</td>
<td><strong>1.38</strong></td>
<td>0.17</td>
<td><strong>2.70</strong></td>
<td>-1.47</td>
<td><strong>1.56</strong></td>
</tr>
</tbody>
</table>

The table reports the DM statistics for a test of the equality of forecast accuracy for symmetric specifications of the baseline Equation (5) and (10) with a time-varying relationship. The values were calculated by forecasts for the months March 1983 to December 1998 for euro area currencies and to May 2015 for the other ones. The critical values for $P \to \infty$ are 1.282, 1.645 and 1.960 for the significance levels 0.10, 0.05 and 0.01, respectively.
5 - Robustness Analysis

This section details the robustness of the previously presented results. Some of the robustness issues Rossi (2013) mentioned for exchange rate studies are already dealt with as 12 different exchange rates and a very long sample are examined. Nevertheless, issues such as the calibration of the Kalman filter model or the rolling regressions’ window size remain to be examined.

5.1 - Calibration of the Kalman Filter Model

In this section an alternative calibration of the Kalman filter model will be examined. Recall the Kalman filter model of Section 3:

\[ \Delta s_{t+1} = f' \beta_t + \epsilon_t \]  \hspace{1cm} (20)

\[ \beta_t = \beta_{t-1} + \eta_t \]  \hspace{1cm} (21)

where \( \epsilon_t \sim iid \, N(0, k^2 \sigma^2_\epsilon) \), \( \eta_t \sim iid \, N(0, \lambda^2 \sigma^2_\eta Q) \), \( E(\eta_t \epsilon_t) = 0 \) and \( Q = E(f_t f_t')^{-1} \forall t \) and with vector \( f_t \) including the \( k \) fundamentals.

Now, the case where \( k \) is increased to a value of 0.1 and \( \lambda \) remains as \( k \cdot 0.1 \) is examined. Thus, we use more information of the prediction error of the model. The results for this modified Kalman filter model are not presented as it can be said in general that it now performs worse than the baseline OLS model for all exchange rates. To examine the reasons for this, Figure 3 shows how the final state estimates for the Pound-US dollar exchange rate change due to this adjustment of the error term. It can be seen that the final state coefficient estimates of the adjusted Kalman filter model, drawn in green, have a larger variance than the baseline Kalman filter or OLS ones. This is in particular the case for the years between 1995 and 2003. This shows that if the Kalman filter model uses a higher share of the prediction error as information for the updating of the coefficients, then the estimates get out of hand. The problem gets worse if \( k \) is increased further. Setting \( k \) smaller than in the baseline Kalman filter model alters the results only slightly. Changes of \( \lambda \) have no significant impact.
Figure 3: Coefficients Estimated with Different Kalman Filter Models or OLS

The previous paragraph showed that using too much information of the prediction error leads to worse results. A reason for this can be that the Kalman filter cannot distinguish whether the model performs badly because of a constant relationship between exchange rates and fundamentals, or missing explanatory information. Compare for example the order flow model of Evans and Lyons (2002). If the bad performance is mainly due to the latter, then the Kalman filter should not use the entire information of the prediction error to estimate the time-varying coefficients.

5.2 - Window Size of the Rolling Regressions

In the exchange rate forecasting literature window sizes are often chosen arbitrarily. For example, Meese and Rogoff (1983) used a window of
93 observations for monthly data, Molodtsova and Papell (2009) took 120 observations, and Gourinchas and Rey (2007) used 104 for quarterly data, while Cheung et al. (2005) took 42 and 59. This can lead to two shortcomings. Firstly, the ad hoc chosen window may not find a significant difference between the forecasting ability of the competing models, though it exists for shorter or longer window sizes. Secondly, the researcher could compare models for different window sizes until he finds satisfactory results. Addressing the first point, Inoue and Rossi (2012), using the same data and methodology as Molodtsova and Papell (2009), found improved evidence for predictability at smaller windows.

To analyze whether the favorable results of the baseline Kalman filter model are robust to the chosen window size of the rolling regressions, the regressions were estimated for windows between 48 and 120. For comparability, the sample is held constant across window sizes although a shorter window would allow using a longer sample. The starting and ending points of the window sizes are chosen such that between 4 and 10 years of data are incorporated in the regressions.

The results across different window sizes for the Pound-US dollar exchange rate can be seen in Figure 4. The MSPE of different specifications of Equations (5) and (10) are grouped in one graph. The Kalman filter model’s MSPE is drawn in red. For all groups it can be said that smaller window sizes lead to slightly higher MSPE. To get a clearer view, the DM statistics for a comparison between Equations (5) and (10) are also shown for the Pound-US dollar exchange rate in Figure 5. Lines are drawn in the graphs at -1.282 and 1.282 for the 10% significance levels of the DM statistic. While for the right hand models the previous observation holds, it is less clear for the left hand side ones. Therefore, a comparison of Equations (5) and (10) depends substantially on the chosen window size. The figures, only for the non-euro countries to save space, can be found in the appendix beginning with Figure 6. In general, the favorable results for the Kalman filter are robust to the choice of the window size, and for some currencies and specifications even strengthened. This result is particularly interesting as for lower window sizes a time-varying relationship is already more strongly implied for the baseline model.
Figure 4: MSPE across Different Window Sizes of Equations (5) and (10) for the Pound-US Dollar Exchange Rate

Figure 5: DM Statistics across Different Window Sizes of Equations (5) and (10) for the Pound-US Dollar Exchange Rate

**Figure 6:** DM Statistics across Different Window Sizes of Equations (5) and (10) for the Canadian Dollar-US Dollar Exchange Rate

**Figure 7:** DM Statistics across Different Window Sizes of Equations (5) and (10) for the Australian Dollar-US Dollar Exchange Rate
Figure 8: DM Statistics across Different Window Sizes of Equations (5) and (10) for the Danish Kroner-US Dollar Exchange Rate

Figure 9: DM Statistics across Different Window Sizes of Equations (5) and (10) for the Swedish Kronor-US Dollar Exchange Rate
6 - Conclusion

Although exchange rates are very important, it is not yet possible to perform reliable forecasts. A solution for this puzzle of the economics profession could be to incorporate a time-varying relationship into exchange rate models. For example, Sarno and Valente (2009) were able to beat the random walk in out-of-sample forecasting if they chose the model with the lowest forecast error out of a selection with all combinations of six different fundamentals ex post for every forecast. Here it is examined whether estimating a time-varying relationship with a Kalman filter model using ex ante information can improve forecasting compared to a model with a constant relationship. Using a Taylor rule exchange rate model, which is found to have promising out-of-sample forecasting abilities for the short-term (Molodtsova and Papell, 2009; Rossi, 2013), it is possible to further enhance the forecasting performance if the information used to update the time-varying coefficients is restricted. This is necessary as classic exchange rate models do not perform badly only due to a constant relationship between the exchange rate and its fundamentals, but also because of missing information. See Evans and Lyons (2002) to compare. Thus, without the restriction of the information, the Kalman filter would adjust the coefficients
too strongly. We thereby show that the Kalman filter is still a useful technique for exchange rate forecasting compared to OLS. We further note for future research that such an adjustment to the calibration could also improve forecasting with other models which allow for a time-varying relationship as Markov switching or smooth transition models.

Still, this approach is not able to beat the random walk in forecasting performance so that the result of Meese and Rogoff (1983) prevails. However, exchange rate models which incorporate more explanatory information, such as the order flow model of Evans and Lyons (2002), would allow an improvement of the estimations of the Kalman filter. Until now, the Kalman filter cannot distinguish whether a high prediction error is due to missing information or a constant relationship between the exchange rate and the fundamentals. Thus, with further explanatory information, the restriction on the information used by the Kalman filter to estimate the time-varying coefficients can be eased as not only the prediction error is smaller, but also because the size of the error is (more) exclusively determined by the negligence of the time-varying relationship. With this enhanced estimation of the time-varying coefficients, forecasting accuracy could be improved.

References


and Finance, 60, pp. 267-288.


Yüksel, E., Metin-Ozcan, K. and Hatipoglu, O. (2013): A survey on time-