The Dynamics of Market Efficiency: Testing the Random Walk Hypothesis in South Africa

Dr. Yudhvir Seetharam[1]

Prof. Christ Auret[2]

Prof. Turgay Celik[3]

Abstract

We examine the dynamics of market efficiency, with the aim of providing a framework for testing the Adaptive Market Hypothesis. We first begin by examining the return characteristics of shares and indices in South Africa. Our results show that: (1) the examination of market efficiency is dependent on the frequency of data; (2) by binning data into smaller sub-samples, one can obtain a pattern of whether the equity market is efficient or not; (3) by running a variety of tests, one provides robustness to the results; (4) analysis according to industries also adds to the result of efficiency, especially if markets have high concentration sectors. Examining five frequencies of data, 86% of the shares and indices exhibited a random walk under daily data, 78% under weekly data, 56% under monthly data, 22% under quarterly data and 24% under semi-annual data.

Keywords: Market efficiency, random walk, emerging markets, Hurst exponent
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[1] School of Economic and Business Science, University of the Witwatersrand, South Africa Yudhvir.Seetharam@wits.ac.za
[2] School of Economic and Business Science, University of the Witwatersrand, South Africa Christo.Auret@wits.ac.za
[3] School of Computer Science and Applied Mathematics, University of the Witwatersrand, South Africa, Turgay.Celik@wits.ac.za

1 – Introduction

Since the seminal work of Fama (1970) setting out the Efficient Market Hypothesis (EMH), empirical tests have been conducted to determine whether markets are efficient or not. In recent years, the status quo has shifted towards providing alternate hypotheses of market efficiency which are testable and can thus be proven to be true or false. We examine one such alternative, the Adaptive Market Hypothesis (AMH) of Lo (2004, 2005) with a focus on testing the baseline hypothesis of whether share returns are randomly generated. Given the development of the AMH, no formal means of testing cyclical efficiency has been established in the literature. In this particular article, it will be determined whether share returns are deterministic or not, so that we can then attempt to model the returns generating process in future research.

The present literature on testing the EMH focuses on tests of randomness in share returns. The literature groups these tests into predictability of security returns and profitability of trading strategies. Under the first grouping, the tests are further subdivided into constant or time-varying parameters with permutations to the full sample, non-overlapping samples and overlapping samples. While employing these tests on any sample is not necessarily unique, there is no South African literature on the comprehensive testing of the random walk behaviour of share returns from both a methodological and data frequency perspective. We therefore examine random walk behaviour on a selected group of equity and equity indices over five frequencies of data (ranging from daily to semi-annual). Further, for the market index (the J203), we divide the daily sample in non-overlapping sub-samples to add robustness to the results – this division allows insight as to whether the random walk behaviour can be present in sub-samples, yet absent overall (or vice versa).

Our hypothesis is simply set out as follows:

H0: Share price behaviour, in the South African market, does not follow a random walk.

H1: Share price behaviour, in the South African market, does follow a random walk.

As mentioned previously, this is the first article that aims to provide a framework for testing the AMH. The hypothesis of cyclical efficiency will be tested through three phases. Firstly, it is necessary to establish whether share price changes follow a random walk or not. If price changes are random, they cannot be predicted, thus enforcing the notion of weak form market efficiency. However, if price changes are not random, secondly, it is then viable to establish whether they can be modelled. In the simplest case, one can model current share prices based on prior values. If this model is found to be inadequate, then lastly, one can model share prices based on both prior values and exogenous factors. This lays the groundwork for our future research, as we begin by assessing whether returns are randomly generated.

This paper proceeds as follows. Chapter 2 provides an overview of the literature. Chapter 3 discusses the data and methodology, with Chapter 4 the results. Lastly, Chapter 5 concludes.

2 - Literature Review

The intrinsic value of a share is measured by the sum of future discounted cash flows accruable to investors. Any new information that can be expected to change a company's future performance must be immediately reflected in the share price as delays in this diffusion can be exploited by certain individuals to forecast future profitability. Thus, prices should only be able to respond to new information. Since this information arrives randomly, prices must fluctuate unpredictably. The Random Walk model of share prices is represented as follows.

{1}

Where is the price of a security at time *t+1* and is a random error term with zero mean and finite variance.

The equation above indicates that the future price of a security is based on the arrival of new and unpredictable information. This implies that price changes are independent of past price changes. Fama (1970) argues that the random walk model is an extension of the fair game model in that the latter indicates conditions of the market equilibrium that can be stated in terms of the return generating process of the former model. Tests of the weak form of the EMH consider the above three models in their hypothesis as the determination of whether the market is weak form efficient or not is a function of both the return generating process and of the tests employed.

The tests of the weak form of the EMH are synonymous with testing the random walk hypothesis; the notion that stock price changes are random and thus unpredictable. As with most literature on testing the EMH, tests done on the weak form of the EMH show conflicting results. Tests of the weak form are based on examining the interrelationship between current and past prices. Practically, the runs test, tests for autocorrelation and the variance ratio test have been used to test for weak form efficiency. Given the array of literature on the topic, a select few works will be discussed here.

A combination of approaches is adopted by Dickinson and Muragu (1994) who also find that share prices on the Nairobi Stock Exchange follow a random walk. Their study examines weekly and monthly data for a sample of 30 of the most liquid shares on the exchange. Employing correlation and runs tests, the authors find that the majority of share prices examined follow a random walk. While they then generalise this result to conclude that the overall Nairobi stock market follows a random walk, the authors are careful to place their results in the context of literature on the EMH. They explicitly state that while their results show evidence in favour of a random walk, they are cautious to imply that the Nairobi stock market is weak form efficient. A possible reason for this hesitation is that one cannot easily generalise a result of a particular sample period and data frequency to time periods and data frequencies not used in the study. Further, the methodology used needs to be robust enough to provide comprehensive evidence that can hold across out-of-sample data. Other researchers, such as Seddighi and Nian (2004) use spectral analysis and ARCH tests for detecting if the Chinese market is weak form efficient. The authors conclude on a particularly small sample of daily share returns from the Shanghai Stock Exchange that the Chinese stock market is weak form efficient. The frequency of the time series under observation has also been investigated to determine if a result that holds for a particular frequency will hold at other frequencies. For example, Groenewold (1997) uses daily, weekly and monthly data to determine if the Australian and New Zealand Stock Exchanges are weak and semi-strong efficient. The author employs the popular tests of autocorrelation, runs and cointegration on a 17 year sample period. The results however are mixed - the returns appear to have some predictability according to the autocorrelation coefficient but are stationary in the long run. This could possibly imply that if one uses higher frequency data to test market efficiency, one might find a short term "memory" of the series, which dissipates over lower frequency data. Thus, to examine this notion, five frequencies of data are used in our study to examine market efficiency. While much literature exists on tests of the weak form of the EMH, it becomes redundant to mention them as there was no conclusive evidence of whether emerging or developed markets are weak form efficient.

Thus far, common tests for weak form efficiency include: the runs test, examining autocorrelation coefficients and the ADF test for stationarity. Poterba and Summers (1988) and Lo and MacKinlay (1988) provided the foundation for the variance ratio (VR) test of the random walk hypothesis. This test compares the variance of the stock return series against stationary alternatives, under the assumption that the variance of random walk increments will be linear across the sample. The VR test can be used to test secondary hypotheses of the random walk, specifically whether stock prices mean revert. While the concept of the test is straightforward, it is often difficult to implement in practice as the test relies on overlapping data in computing the variance of long term horizon returns. Lo and MacKinlay (1988) suggest this approach as it can improve the statistical power of the test and suggest that an asymptotic distribution be used instead of the exact distribution of the test. However, while other tests have been developed to remedy the shortcomings of the VR test, the VR test still remains popular in literature. Given an array of tests to use in examining the return generating process of share returns, one should also not assume that share returns follow certain pre-specified conditions. Three such assumptions are now discussed.

Prior to an alternative by Mandelbrot (1963), the assumption of normality in share returns was scarcely questioned. Mandelbrot (1963) conducted investigations into both the excess kurtosis and skewness of return distributions and thus developed an alternative Power Law hypothesis of distributions based on his findings. His position was later reiterated as he noted that "Bachelier's assumption, that the marginal distribution of $L(t,T)$ (returns) is Gaussian with vanishing expectation, might be convenient, but virtually every student of the distribution of prices has commented on their leptokurtic (i.e., very long-tailed) character." (Mandelbrot, 1966, p.396). Thus, while the normality assumption is required for the EMH, many practical tests of the EMH show that this assumption is violated. However, Fama (1965b) does not see this violation as evidence that the EMH does not hold.

Fama (1965b) studies the statistical properties of returns using shares on the Dow Jones Industrial Average (DJIA). He finds that a greater proportion of observations are centred around the mean as well as in the tails of the distribution. Further, when examining extreme tail observations (those that are beyond five standard deviations from the mean), he finds that they are almost 2000 times greater than that implied by a normal distribution. These findings indicate leptokurtic behaviour of the returns and Fama (1965b) concludes that a normal distribution is ill fitting to the data. Officer (1972) has similar findings over a longer time period (1926 to 1968) on data from the Centre for Research in Security Prices (CRSP) database. He finds that the distributions are reasonably stable across time, yet not across the sample of stocks used. Further, under differing frequencies, the stability of the distribution changes - daily returns produce a stable distribution only up to 20 days, whereas monthly distributions are stable up to 5 months. These results point towards examining efficiency using differing frequencies of data as the results may not always hold or be generalised if only one particular sampling frequency is used. This approach is adopted in this study..

Durbin and Watson (1950) describe independence as the serial correlation function of returns that should decay to zero. A market is thus described as efficient in the absence of linear serial correlation. Further, if serial correlation is present, then the anomaly is short lived. The assumption of independence can be viewed either from a statistical perspective or from an investor's perspective. If an investor finds that returns are not independent, then investors can theoretically use knowledge of past returns to increase future profits (Fama, 1965b).

Kendall (1953) studies the properties of returns and finds that the pattern of events in a price series is less systematic than what is generally accepted. He concludes that these price changes follow a random walk and are thus independent. Further, the author argues that it is generally difficult (at least at the time) to distinguish between a true random series and one where the systematic element is particularly weak. This implies that when testing any hypothesis, one should take caution to the results and model(s) used. Lastly, the author states that given his results on a lack of serial correlation in the sample of stock prices, he argues that it is near impossible to predict values, in their case one week ahead, without any additional information. While Fama (1965b) states that it is difficult to find a series that conforms to the independence assumption, statistical independence holds even if some level of dependence is present. Further, the simplest explanation for the assumption of independence is due to the arrival of new information, which does not follow any consistent pattern. After testing returns on the DJIA, he finds that most follow the independence assumption with the remainder being serially correlated but with the serial correlation decreasing at higher orders. When correlation is statistically significant, they are low enough to ignore any statistical or practical implications. Noting that the empirical evidence for market efficiency was publicised before the theory, one questions whether the results of Fama (1965b) were taken into consideration in developing higher hurdles for the EMH. Over the long run, Campbell *et al.* (1998) show that the independence assumption is violated. They test returns of shares on the CRSP value and equally weighted index and find that there is significant first order serial correlation in weekly and monthly returns. Further, the serial correlation decayed slower on the equally weighted index than the value weighted index. This implies that market capitalisation plays a role in efficiency. To test this hypothesis, the authors employ VR tests and find that indeed, market capitalisation plays a role in determining whether the aggregate stock market, comprising of individual stocks, is efficient.

The third assumption of returns is that of stationarity. According to Mandelbrot (1966), stationarity implies that the statistical moments of the distribution do not change from one sample to another. Taylor (1997) focused on the time varying property of variance in his study of share returns. The author shows that the absolute and square transformations of U.S. share returns are good proxies for volatility and exhibit high levels of first order serial correlation. Further, this correlation over an extended period can imply that there is a time varying structure in variances over the sample period of 1966 to 1976.

3 - Data and Methodology

3.1 Data

Closing prices for the local equity and local indices were obtained for the period September 1997 to October 2014 and each variable contains total returns (inclusive of corporate actions or dividends where applicable). Given the task of ensuring returns inclusive of dividends are correctly incorporated, the simplifying assumption of using the dividend yield (converted to the appropriate frequency) was used. Thus, the total return is the sum of the share price change and the dividend yield. With respect to the indices used on a monthly basis, the total return index (TRI) of those indices were obtained and used instead of the method outlined previously. The sample period was chosen so as to ensure full data were available (a longer sample period could have been used if a particular frequency or less shares were required). The number of observations ranges from 4480 for daily data, 896 for weekly data, 206 for monthly data, 68 for quarterly data and 34 for semi-annual data. While the different frequencies add an element of robustness to the study, the choice of frequencies is certainly not exhaustive. Indeed, one can extend the frequencies to include the highest (high-frequency data or tick-by-tick data), to lower ones (perhaps annual). Forty four local equity series were randomly selected from the top 100 shares by market capitalisation on the JSE, as of October 2014, along with six local equity indices. For the market index, the full sample period is split evenly into 10 sub-samples that do not overlap and span 21 months of data. These sub-samples consist of 448 daily observations.

3.2 Methodology

3.2.1 Preliminary Tests

As the primary focus of this study is to examine the potential random walk behaviour of share returns, it is important to conduct preliminary tests on the data which establish the nature of these return distributions. As such, we run tests for normality (Jarque Bera test, the Q-Q plot and the Kolmogorov Smirnov test), stationarity (the Augmented Dickey Fuller test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test) and linearity (the Brock-Dechert-Schenkman test). The range of tests cover parametric, non-parametric and graphical versions, which ensure robustness to the results. We provide the results of these tests in the Appendix.

3.2.2 Testing for random walk behaviour

Given the multitude of tests for random walk behaviour, one needs to be cognisant of which tests are used to accurately provide results. As such, four tests are considered, with each being an improvement on the prior.

3.2.2.1 Runs test

The runs test is a non-parametric test for detecting whether a series is random. If a series is random, then the observed number of runs should be close to the expected number of runs. A run is defined as a sequence of consecutive (price) changes with the same sign. Thus, there are three categories of run: upward, downward and flat. Under the null hypothesis of independence in share returns, the total expected number of runs (m) is estimated as:

$$m = \frac{N(N+1) - \sum_{i=1}^{3} n_i^2}{N} \qquad \{2\}$$

where N is the total number of observations and n_i is the number of price changes in each of the three categories. For a large number of observations, the sampling distribution of m is approximately normal and the standard error of m is given by

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{3} n_i^2 \left[\sum_{i=1}^{3} n_i^2 + N(N+1)\right] - 2N\sum_{i=1}^{3} n_i^3 - N^3}{N^2(N-1)}} \qquad \{3\}$$

Standard normal Z statistics can be used to test whether the hypothesis of independence is rejected. A disadvantage of the Runs test shown above is that it can only detect randomness at a lag order of one only. While other versions of the Runs test have been developed, more powerful tests examine the decomposition of variance.

3.2.2.2 Variance ratio test

The variance ratio test of Lo and MacKinlay (1988) is shown by many authors to be an adequate test of the weak form of the EMH. The test assumes that the variance of increments in the random walk series is linear in the sample interval - the variance should be proportional to the sample interval. Specifically, if a series q follows a random walk, the variance of its *q-differences* should be a q multiple of the first difference.

$$Var\left(p_t - p_{t-q}\right) = qVar(p_t - p_{t-1}) \qquad \{4\}$$

The Variance Ratio (VR) is then calculated as

$$VR(q) = \frac{\frac{1}{q}Var\left(p_t - p_{t-q}\right)}{Var(p_t - p_{t-1})} = \frac{\sigma^2(q)}{\sigma^2(1)} \qquad \{5\}$$

For a sample size of *n(q+1)* observations the formulae for computing the variances are modified as follows

$$\sigma^2(1) = \frac{\sum_{t=1}^{nq}(p_t - p_{t-q} - q\hat{u})^2}{(nq-1)} \qquad \{6\}$$

$$\sigma^2(q) = \frac{\sum_{i=q}^{nq}(p_t - p_{t-q} - q\hat{u})^2}{h} \qquad \{7\}$$

where $h = q(nq + 1 - q)(1 - \frac{q}{nq})$
and $\hat{u} = \frac{1}{nq}\sum_{t=1}^{nq}(p_t - p_{t-1}) = \frac{1}{nq}(p_{nq} - p_0)$

Under the assumption of either homoscedasticity or heteroscedasticity, two standard normal statistics, *Z(q)* and *Z*(q)*, can be used.

$$Z(q) = \frac{VR(q) - 1}{\sqrt{\emptyset(q)}} \qquad \{8\}$$

$$Z^*(q) = \frac{VR(q) - 1}{\sqrt{\emptyset^*(q)}} \qquad \{9\}$$

A shortcoming of the Lo and MacKinlay (1988) VR test is that the lag order q is required to be specified. Thus, a modified version of this test is employed, by Chow and Denning (1993) as this tests for multiple lags of order q. As both the single and multiple order VR test statistics have shortcomings in their reliance to an approximated distribution, these tests can often give rise to size distortions or low power (Wright, 2000).

Thus, the modification of Wright (2000) is used as this provides a non-parametric version of the Lo-MacKinlay test, displaying results for the variance decomposition based on ranks (*R1*, *R2* variables in the test) and sign (*S1*). Assume that $r(Y_t)$ is a rank of return Y_t among $T_1, T_2, ..., T_r$, then $r(Y_t)$ is the number from *1* to *T* given by

$$r_{1,t} = \frac{r(Y_t) - \frac{T+1}{2}}{\sqrt{\frac{(T-1)(T+1)}{12}}} \qquad \{10\}$$

$$r_{2,t} = \Phi^{-1}\left(\frac{r(Y_t)}{T+1}\right) \qquad \{11\}$$

where Φ is the standard normal cumulative distribution function and Φ^{-1} is its inverse. The series $r_{1,t}$ is a linear transformation of the ranks that is standardised to have a sample mean of 0 and a sample standard deviation of 1. The *R1* and *R2* test statistic are defined as

$$R_1 = \left(\frac{\frac{1}{Tk} \sum_{t=k}^{T} \left(r_{1,t} + r_{1,t-1} + \cdots + r_{1,t-k+1} \right)^2}{\frac{1}{T} \sum_{t=1}^{T} r_{1,t}^2} \right) \left(\frac{2(2k-1)(k-1)}{3kT} \right)^{-0.5} \qquad \{12\}$$

$$R_2 = \left(\frac{\frac{1}{Tk} \sum_{t=k}^{T} \left(r_{2,t} + r_{2,t-1} + \cdots + r_{2,t-k+1} \right)^2}{\frac{1}{T} \sum_{t=1}^{T} r_{2,t}^2} \right) \left(\frac{2(2k-1)(k-1)}{3kT} \right)^{-0.5} \qquad \{13\}$$

Similarly, Wright (2000) defines a sign statistic, s_t, by being equal to 0.5 if the return Y_t is positive and -0.5 otherwise. The sign based variance ratio test statistic, *S1*, is thus defined as:

$$S_1 = \left(\frac{\frac{1}{Tk} \sum_{t=k}^{T} \left(s_{1,t} + s_{1,t-1} + \cdots + s_{1,t-k+1} \right)^2}{\frac{1}{T} \sum_{t=1}^{T} s_{1,t}^2} \right) \left(\frac{2(2k-1)(k-1)}{3kT} \right)^{-0.5} \qquad \{14\}$$

Therefore, the Chow and Denning as well as Wright modifications of the VR test are used as the former examines multiple variances and the latter ranks and signs. In addition, a graphical plot of the variance decomposition over time would reveal if the series follows random walk behaviour or not. If the variance decomposition is not within acceptable confidence intervals, then it implies that the variance does not decompose over time as expected. Considered a more sophisticated version of the variance ratio test, the Hurst exponent provides a measure of long term memory in a time series and as such is discussed below.

3.2.2.3 Hurst Exponent

To test for non-linear dependence, the Hurst exponent is used. Zunino *et al.* (2009) argue that the exponent measures the long range dependence in stock market indices, where an existence of autocorrelation between distant observations will imply market inefficiency. The exponent provides a measure of memory and fractality of a time series. Ranging from values between 0 and 1, the Hurst exponent can identify if a time series follows a random walk or is persistent. A value of 0 indicates that the series is anti-persistent (mean-reverting); a value of 1 indicates that the value is persistent and a value of 0.5 indicates that the series is random. Further, there are various permutations of calculating the Hurst exponent, leading one to be cautious in the preference of one calculation over another. Taqqu, Teverovsky and Willinger (1995) conduct simulations of the different methods of the Hurst exponent on data of differing sample sizes to empirically determine the best method to use for a particular sample size. The authors find that for series that have between 4000 and 7000 data points, the Peng estimator should be used; for series with 700 to 1000 data points, the Whittle Estimate be used; and for series less than 700 data points, the R/S method be used. The first considers analysing both the mean and standard deviation of a time series; the second on detrending the time series and then analysing the variance to determine the Hurst exponent; whereas the third relies on a periodogram fit.

While the Hurst exponent (and its various methods) are considered powerful tests of random walk behaviour, the method in general suffers from a lack of distribution theory to correctly allocate confidence intervals to interpret the results. In other words, faced with an answer of 0.49 for the Hurst exponent, one does not have a clearly defined interval to determine if 0.49 is statistically close (or not) from 0.5. As such, authors have proposed three avenues to determine the significance of the Hurst exponent. The first relies on conducting the test using a variety of methods and simply choosing the consensus. The other relies on simulating data to obtain confidence intervals that can be applied in general to a sample of finite observations; and the final considers a simple case of the inverse of the number of observations in the sample (this provides a point estimate as opposed to a confidence interval). We rely on the second method and use robust estimates obtained from the literature, shown in Table 1 below. Weron (2002) provides equations based on simulations to estimate the confidence intervals for the Peng and Whittle estimators. Rasheed and Qian (2004) provide a confidence interval for the traditional R/S method used in this study, also based on simulations.

Table 1 - Hurst exponent confidence intervals

Frequency	Method	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
Daily	Peng	0.4508	0.5432	0.4429	0.5515	0.4260	0.5685
Weekly	Whittle	0.2492	0.7241	0.2027	0.7630	0.0913	0.8471
Monthly/Quarterly/ Semi-annual	R/S	0.4656	0.6252	0.4503	0.6405	0.4205	0.6703

4 – Results

4.1 Market Index Results

In determining whether the frequencies of return data of the ALSI follow a random walk, various parametric and non-parametric tests were performed. These results, for the overall sample, are provided in Table 2 below. All tests for normality concluded that the daily, weekly and monthly ALSI return series are non-normal. Similarly, the BDS test showed that all frequencies of data exhibit non-linear behaviour and are stationary according to the ADF and KPSS tests. A simple measure of rolling autocorrelation (RA) depicted a cyclical trend in all of the return series, indicating that the ALSI was efficient over periods of time, but also inefficient over other periods of time – a preliminary affirmation towards the notion of cyclical market efficiency.

Examining the results of the Runs test and Hurst exponent, the daily ALSI series appears to not be randomly generated under the Runs test, whereas it appears to be randomly generated under the Hurst exponent. Similarly, the monthly ALSI series appears to be randomly generated under the Runs test and not randomly generated under the Hurst exponent. These contradicting results can be explained however. Recall that the Runs test examines randomness at a lag order of 1 only, in contrast to the Hurst exponent which examines randomness along a rolling window. Therefore, the Hurst exponent may detect randomness or non-randomness over smaller intervals that may not be apparent over the entire sample period. As such, the results of the Hurst exponent are considered more reliable than that of the Runs test.

Attention is now turned to the results of the variance tests - the Chow Denning (CD) test, Wright test and variance decomposition (VD) plot. The Chow Denning method of the variance ratio test pointed towards only the weekly and monthly series following a random walk; whereas the Wright modification for the variance ratio test led to all three series following a random walk. As the Chow and Denning modification is seen as a superior method compared to the Wright modification (the former tests for multiple variances compared to the latter), the results of the Chow and Denning modification hold more significance. Further, the variance decomposition plot offers the same conclusion as the Chow Denning test. However, this result conflicts with those of the Hurst exponent.

Summarising the above two paragraphs produces conflicting results when comparing the Hurst exponent test to the Chow and Denning test. In other words, according to the Hurst exponent, the ALSI appears randomly generated using daily and weekly data and not randomly generated using monthly data. In contrast, the Chow Denning test shows that the ALSI appears not randomly generated under daily data and is randomly generated under weekly and monthly data. Reconciling these results is arguably quite straight forward. Recall that the Hurst exponent examines random walk behaviour using a rolling window approach; whereas the variance ratio tests used in this study used a "fixed" window approach. Thus, the Hurst exponent can be considered superior to that of the variance ratio test. In order for the results to be truly comparable, one would need to use a rolling window approach when implementing the variance ratio test.

Table 2 - Summary of results for the random walk hypothesis

	Daily	Weekly	Monthly	Quarterly	Semi-Annual
Jarque Bera	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
Q-Q Plot	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
K-S test	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
BDS test	Non-linear	Non-linear	Non-linear	Non-linear	Linear and Non-linear
ADF test	Stationary	Stationary	Stationary	Stationary	Stationary
KPSS test	Stationary	Stationary	Stationary	Stationary	Stationary
RA	Cyclical	Cyclical	Cyclical	Cyclical	Cyclical
Runs test	Not Random	Random	Random	Random	Random
CD test	Not Random	Random	Random	Random	Random
Wright test	Random	Random	Random	Random	Random
VD plot	Not Random	Random	Random	Random	Random
Hurst exponent	Random	Random	Random	Not Random	Not Random

Similarly, the results for each sub-sample are summarised below in Table 3 and Table 4. The tests for normality all point towards each sub-sample not being drawn from a normal distribution as well as being stationary. The test for linearity produces mixed results throughout the sample period. There are sub-samples that are linear, sub-samples that are non-linear and sub-samples that are both linear and non-linear. This interesting result shows that while the overall daily sample is non-linear, there are components of the daily data that have both linear and non-linear behaviour. Within samples where there were both evidence of linear and non-linear behaviour, one can arguably divide these samples further. Further, as per Kaboudan (1999), if a series' data generating process is a combination of linear and non-linear or linear, non-linear and stochastic processes, then the predictability of the series decreases significantly.

As the results of the Chow and Denning and Wright variance ratio tests differ, the logic outlined previously will be employed. Therefore, according to the Chow and Denning modification of the variance ratio test, all of the sub-samples are randomly generated. This shows that a non-randomly generated series can consist of series that are randomly generated.

Reconciling the difference in results between the Runs test, variance ratio plot and Hurst exponent is somewhat more difficult. It was previously mentioned that the Runs test examines randomness at a single lag order. Therefore, the results of Runs test are not considered superior to that of the variance ratio plot and Hurst exponent. The variance ratio plot examines the fraction of known and unknown variance against lag orders. Each of the plots was compiled to a maximum of 10 lags. Thus, any long term memory would theoretically not be captured in these plots. Therefore, according to the Hurst exponent, one of the sub-samples exhibit non-random behaviour, in contrast to the overall daily sample results in which the exponent showed random behaviour. This implies that a series that is randomly generated can consist of sub-series that are both random and non-randomly generated. Further, given that the Hurst exponent employs a sliding window approach, it is considered more sophisticated than the other tests employed in detecting randomness in a series. It is possible, however, that the window used in the Hurst exponent, which differed per frequency, could have affected the results. In other words, if a longer (shorter) sliding window was used, the test result could have differed. The interaction between this element along with the frequency of data used presents an interesting avenue to explore as future research – the impact of data frequency in determining the optimal sliding window.

Table 3 - Summary of results for each sub-sample for the random walk hypothesis (1)

	S1	S2	S3	S4	S5
Jarque Bera	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
Q-Q Plot	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
K-S test	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
BDS test	Non-linear	Non-linear	Non-linear	Non-linear	Non-linear
ADF test	Stationary	Stationary	Stationary	Stationary	Stationary
KPSS test	Stationary	Stationary	Stationary	Stationary	Stationary
RA	Cyclical	Cyclical	Cyclical	Cyclical	Cyclical
Runs test	Not Random	Not Random	Not Random	Not Random	Not Random
CD test	Random	Random	Random	Random	Random
Wright test	Not Random	Not Random	Not Random	Not Random	Not Random
VD plot	Not Random	Not Random	Not Random	Not Random	Not Random
Hurst exponent	Random	Random	Random	Random	Random

Table 4 - Summary of results for each sub-sample for the random walk hypothesis (2)

	S6	S7	S8	S9	S10
Jarque Bera	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
Q-Q Plot	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
K-S test	Non-Normal	Non-Normal	Non-Normal	Non-Normal	Non-Normal
BDS test	Non-linear	Non-linear	Non-linear	Non-linear	Non-linear
ADF test	Stationary	Stationary	Stationary	Stationary	Stationary
KPSS test	Stationary	Stationary	Stationary	Stationary	Stationary
RA	Cyclical	Cyclical	Cyclical	Cyclical	Cyclical
Runs test	Random	Random	Random	Random	Random
CD test	Random	Random	Random	Random	Random
Wright test	Random	Random	Random	Random	Random
VD plot	Random	Random	Random	Random	Random
Hurst exponent	Random	Random	Random	Random	Random

4.2 Summary of equity and indices results

To examine cyclical efficiency on the South African equities market, a range of tests and modelling was conducted on 50 returns series - 44 equities and six indices. This provided a holistic view of market efficiency at both an individual share level and aggregated index level. It was found that 18 of the 44 shares had returns that were randomly generated under daily and weekly data, but not randomly generated under monthly data. Further, as the frequency of data lowered (from daily to monthly), more shares appeared to not follow a random walk. These two results indicate that the aggregated share market appears to not follow a random walk under monthly data.

The results of the BDS test show that the majority of shares follow a mix of a linear and non-linear distribution; there is little insight gained from industries. In most instances, the distribution remains the same under monthly, quarterly and semi-annual data. There are two cases, where the distributions became linear under semi-annual data but were linear and non-linear under quarterly data. From a trading perspective, the non-linear nature of the return distributions points towards some form of mean reversion in share prices, which implies that if one can time the market, it is possible to consistently earn abnormal profits.

When examining the results of the Runs test, there is no particular industry which stands out, apart from two financial shares, which are randomly generated. These shares had runs that were less than expected by pure chance, which infers that there were trends in these shares' returns over the sample period on a daily frequency. Nine shares were non-randomly generated under weekly data. These shares came from various sectors in the market – mining, consumer goods, consumer services, financials and industrials and their runs are a mix of greater than and less than expected by pure chance. However, those shares in the consumer goods (services) sector as well as the financial sector had runs that were greater than expected by pure chance, implying higher trading activity. There was a single share, ASR, that had a consistent lower number of runs under these three frequencies. In a related study, we examine the trading performance of these shares against the backdrop of these results. Last, when comparing those shares that had a greater number of runs, they were more likely to have a non-linear distribution under the BDS test – implying that they can be modelled. Six shares were non-randomly generated under monthly data. The mining industry features strongly again in the monthly data. The runs are predominantly greater than that expected by chance, implying higher volatility, which is generated by higher trading volumes. When viewed alongside the results from the JB test, it was found that mining shares are typically normally distributed. If returns are non-random according to the Runs test, yet are normally distributed, it is plausible that no consistent abnormal profits are made, especially so when the frequency of data is monthly. The results of the Runs test are equivalent to 12%, 82% and 88% of securities being randomly generated from daily, weekly and monthly data respectively. Last, under quarterly data, 11 shares are non-randomly generated and five are non-randomly generated under semi-annual data. The Healthcare and industrials sectors show up in both quarterly and semi-annual frequencies.

From the Chow Denning test results, 43 shares were non-randomly generated using daily data, 20 shares and one index (the J213) were non-randomly generated under weekly data. The most heavily represented sectors of mining, consumer goods and services, financials and industrials appear to have non-randomly generated returns. Most of these results are in line with the Runs test, yet there are some shares which were found to be non-random under the CD test, but random under the Runs test. Given that the CD test examines variances, as opposed to the Runs test which examines "level" data, it is possible that heteroscedasticity is the cause of non-randomness in these shares. This is equivalent to 14%, 56% and 94% being randomly generated from daily, weekly and monthly data respectively. Using quarterly data, eight securities are non-randomly generated and three under semi-annual data. The financial sector shows up strongly in both frequencies (indeed under all frequencies).

The results of the Wright test show that all securities were non-randomly generated under daily data, 28 shares and four indices were non-randomly generated under weekly data; and 15 shares and two indices (the J200 and J203) were non-randomly generated under monthly data. Consumer goods and services, as well as industrials stand out as being non-randomly generated from the monthly data. In contrast, financials show up as non-randomly generated under weekly data but randomly generated under monthly data. This is roughly in line with the CD test and is equivalent to 0%, 36% and 66% of securities being randomly generated under daily, weekly and monthly data respectively. Using quarterly data, two shares are non-randomly generated, whereas under semi-annual data, none of the shares are strictly non-randomly generated (in other words, there is not enough statistical evidence to conclude non-randomness).

When examining the variance decomposition test results, 35 shares and six indices were non-randomly generated under daily data, 23 shares were non-randomly generated under weekly data and one share was non-randomly generated under monthly data. Consumer goods and services appear to be randomly generated under both daily and weekly data, along with shares in the healthcare and industrial sectors. Financials are randomly generated when examining Variance Decompositions, but do appear to be randomly generated under more sophisticated tests. This result points towards a possibly complex return generating process in those shares, which could also be coupled with peculiarities in their trading compared to other sectors. For example, with spikes in trading volumes for financial shares, it is possible that multiple variances (the CD test) would be detected as opposed to variances at a particular lag (the variance decomposition test). In summary, the results are equivalent to 18%, 46% and 98% being randomly generated under daily, weekly and monthly data respectively. Last, under quarterly data, two shares are non-randomly generated, whereas all shares are randomly generated under semi-annual data.

Lastly, from the Hurst exponent results, 8 shares were non-randomly generated under daily data. This is in line with previous test results where the mining, consumer goods, financials and industrial sectors show strongly. No shares were non-randomly generated under weekly data, which is largely due to the confidence intervals given by the Whittle test estimate; and 22 shares and 4 indices were non-randomly generated under monthly data. This is equivalent to 84%, 100% and 56% being randomly generated under daily, weekly and monthly data respectively. From the trend, it would be presumable to say that the number of shares randomly generated under weekly data should lie between the number of daily and yearly shares. Further, the industrials and financial sector shares show up strongly under monthly data to be non-randomly generated. In general, if one were to examine whether the non-random trend is persistent or anti-persistent (mean reverting), the majority of shares under these three frequencies appear to be mean reverting; with industrial shares under monthly data being persistent in their trend (non-mean reverting). Under quarterly data, 33 shares and 6 indices are non-randomly generated, whereas under semi-annual data, 32 shares and 6 indices are non-randomly generated. In both of these frequencies, the majority of financial shares are non-randomly generated, along with healthcare and industrials. All of the indices show up as non-random. Further, of the 39 shares that are non-randomly generated under quarterly data, 10 are mean reverting. This trend is particularly strong in the financial sector. Similarly, of the 38 shares under semi-annual data, 12 are mean reverting. However, there is no particular trend across industries for this frequency of data. Looking across all five frequencies, all of the shares that are non-randomly generated are mean reverting under daily data, with some also mean reverting under monthly data – particularly those in the financial sector. As the frequency lowers, more shares are found to be non-randomly generated, and more appear to not be mean reverting. However, there are certain shares that remain mean reverting even under semi-annual data, yet there is no discernible industry pattern across the frequencies. This points towards share specific effects that affect the results as opposed to market effects.

5 – Conclusion

In the journey towards cyclical efficiency, the random walk hypothesis was examined. The results confirmed that in the time period under investigation, the changes in the daily ALSI returns were random. An interesting result emerged in that by investigation of five frequencies of ALSI returns, the frequency chosen by the researcher has a significant impact on the results. In particular, it was found that lower frequency ALSI returns series did not follow a random walk, indicative of market inefficiency; whereas the daily and weekly ALSI return series did follow a random walk. Thus, the hypothesis of share returns following a random walk can be rejected under lower frequency data, but not rejected under daily and weekly frequency data, with respect to the ALSI. Extending that analysis to incorporate a sample of 50 securities for robustness, it was found that 86% of the shares and indices exhibited a random walk under daily data, 78% under weekly data, 56% under monthly data, 22% under quarterly data and 24% under semi-annual data. While there is a slight increase between the number of randomly generated securities in quarterly and semi-annual data, the overall trend points towards higher frequency data being randomly generated, and lower frequency data being non-randomly generated. This is in line with the ALSI specific results in that it appears that the JSE can be considered weak form efficient on a daily and weekly basis but not the remaining frequencies. This result highlights that concluding whether markets are efficient or not, according to the EMH, is a function of the data frequency chosen as well as the sample of assets used. Further, it is intuitive that as the frequency of data decreases from daily to monthly, a fewer number of shares exhibit random walk behaviour as the series have less “noise”.

By comprehensively examining the behaviour of equity prices in South Africa, there are stylised facts that emerged. First, studies which examine the efficiency of markets are dependent on the frequency of data. If one were to only use a single frequency of data, one might obtain conflicting conclusions. Second, by binning data into smaller sub-samples (for example, splitting daily data into yearly sub-samples), one can obtain an interesting pattern of whether the equity market is efficient or not. Here, it is often the case that the sum of the parts is not equal to the whole – in other words, one might get a conclusion of, say, randomness, over the entire sample period of daily data, but there may be pockets of non-randomness with the daily data. Third, by running a battery of tests, one provides robustness to the results. This is a somewhat debateable issue as one could either run a variety of tests (each being an improvement over the other) or argue the theoretical merits of each test before selecting the more appropriate one. Fourth, analysis according to industries also adds to the result of efficiency, if markets have high concentration sectors (such as the JSE). One might be tempted to conclude that the entire JSE exhibits, say, randomness, where it could be driven by the resources sector as opposed to any other sector.

As with all studies, one must be cognisant of generalising a result that held over a particular sample period to that over any sample period. As a natural extension of future research, one can apply the framework to different sample periods, different countries and most importantly, to more frequencies of data - one might call it striving to be "comprehensive". Further, as returns for lower frequencies are calculated by using the beginning and end of that particular time period (such as the first and last day of the month), it would be interesting to employ the methods used here across an average of daily return data points, such that a weekly or monthly return represents the average daily return. This might present volatility not inherent during the first and last day of the observed prices. Similarly, the use of a particular test will always have proponents and opponents in the field. An attempt was made to circumvent this issue by employing, as far as possible, parametric, non-parametric and graphical versions of tests to ensure that the results obtained are consistent. There is slight favour towards non-parametric tests, as they do not rely on the underlying return distribution to be specified. Given the non-normality of returns found in these five frequencies, this supports the notion of using non-parametric tests in studies of market efficiency. If one were to use a parametric test, one needs to first establish the distribution of that particular security's returns. When applied across multiple frequencies and multiple securities, this becomes cumbersome, along with the comparability of results being compromised. However, within each of the three categories, numerous tests do exist and it is quite possible that a superior test can be employed.

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Appendix

Table 5 – BDS test results for all shares (1)

Share	Sector	BDS		
		Monthly	Quarterly	Semi-Annual
AFE	Basic materials	Linear and Non-linear	Linear and Non-linear	
SAP	Basic materials			Linear
BIL	Mining			Linear and Non-linear
AGL	Mining	Linear	Linear	Linear and Non-linear
IMP	Mining		Linear and Non-linear	Linear and Non-linear
ANG	Mining	Linear	Linear and Non-linear	Linear and Non-linear
GFI	Mining		Linear and Non-linear	Linear
ASR	Mining	Linear and Non-linear	Linear and Non-linear	Linear
SAB	Consumer goods	Linear and Non-linear	Linear and Non-linear	Linear and Non-linear
RCL	Consumer goods			Linear and Non-linear
ILV	Consumer goods	Linear	Linear	Linear and Non-linear
OCE	Consumer goods		Linear and Non-linear	Linear and Non-linear
GRT	Consumer goods		Linear	Linear and Non-linear
FBR	Consumer goods		Linear and Non-linear	
PIK	Consumer services	Linear and Non-linear	Linear and Non-linear	Linear and Non-linear
CLS	Consumer services	Linear and Non-linear	Linear and Non-linear	Linear and Non-linear

Table 6– BDS test results for all shares (2)

Share	Sector	Frequency		
		Monthly	Quarterly	Semi-Annual
MPC	Consumer services	Linear and Non-linear	Linear and Non-linear	Linear and Non-linear
TFG	Consumer services		Linear and Non-linear	Linear and Non-linear
NPN	Consumer services			Linear and Non-linear
SUI	Consumer services	Linear and Non-linear		Linear and Non-linear
FSR	Financials		Linear and Non-linear	Linear and Non-linear
SBK	Financials		Linear and Non-linear	Linear and Non-linear
BGA	Financials	Linear	Linear and Non-linear	Linear and Non-linear
RMH	Financials	Linear and Non-linear	Linear and Non-linear	
INP	Financials	Linear and Non-linear	Linear and Non-linear	Linear and Non-linear
INL	Financials	Linear and Non-linear	Linear and Non-linear	Linear and Non-linear
MMI	Financials			Linear and Non-linear
SNT	Financials		Linear and Non-linear	Linear and Non-linear
HYP	Financials		Linear	Linear and Non-linear
FPT	Financials			Linear and Non-linear
SAC	Financials	Linear and Non-linear		Linear and Non-linear
MDC	Healthcare			Linear and Non-linear
NTC	Healthcare			Linear

Table 3– BDS test results for all shares (3)

Share	Sector	Frequency		
		Monthly	Quarterly	Semi-Annual
APN	Healthcare		Linear and Non-linear	Linear
PPC	Industrials	Linear and Non-linear	Linear and Non-linear	Linear and Non-linear
MUR	Industrials	Linear and Non-linear		Linear and Non-linear
WBO	Industrials		Linear and Non-linear	Linear and Non-linear
NPK	Industrials		Linear	Linear and Non-linear
IPL	Industrials	Linear	Linear	Linear
GND	Industrials		Linear and Non-linear	Linear
SPG	Industrials	Linear and Non-linear	linear	Linear and Non-linear
SOL	Oil		Linear and Non-linear	Linear
MTN	Teleco		Linear and Non-linear	Linear
J150	JSE Gold Mining Index	Linear		Linear
J200	JSE Top 40			Linear and Non-linear
J203	JSE All Share Index (ALSI)			Linear and Non-linear
J211	JSE Industrial 25	Linear and Non-linear	Linear	Linear and Non-linear
J213	JSE Financial and Industrial 30	Linear and Non-linear	Linear	Linear and Non-linear
J177	JSE Mining Index	Linear and Non-linear	Linear	Linear and Non-linear

Table 4 – Runs test results for all shares (1)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
AFE	Basic materials	Less		Less	Less	Greater
SAP	Basic materials	Less				
BIL	Mining			Greater		
AGL	Mining	Less				
IMP	Mining	Less			Less	
ANG	Mining	Less				
GFI	Mining	Less				
ASR	Mining	Less	Less	Less		
RCL	Consumer goods	Less				
ILV	Consumer goods	Less				
OCE	Consumer goods	Less				
GRT	Consumer goods	Less	Greater			
FBR	Consumer goods	Less			Less	Less
PIK	Consumer services	Less	Greater	Greater	Greater	
CLS	Consumer services	Less			Less	
MPC	Consumer services	Less				
TFG	Consumer services	Less		Less		
NPN	Consumer services	Less				
SUI	Consumer services	Less				
FSR	Financials	Less				
SBK	Financials		Greater			
BGA	Financials	Less				
INP	Financials	Less				
INL	Financials	Less				
MMI	Financials	Less				
SNT	Financials	Less		Greater	Less	
HYP	Financials	Less				

Table 57 – Runs test results for all shares (2)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
FPT	Financials	Less	Greater		Greater	
SAC	Financials	Less	Greater			
MDC	Healthcare	Less				Greater
NTC	Healthcare	Less		Greater		Less
APN	Healthcare	Less			Less	
PPC	Industrials				Greater	
MUR	Industrials	Less				
WBO	Industrials	Less	Less			
RLO	Industrials	Less				
NPK	Industrials		Greater		Greater	
IPL	Industrials	Less				
GND	Industrials	Less			Less	Less
SPG	Industrials	Less				
SOL	Oil	Less	Greater	Greater	Less	
J150	JSE Gold Mining Index	Less				
J200	JSE Top 40	Less				
J203	JSE All Share Index (ALSI)	Less				
J211	JSE Industrial 25	Less				
J213	JSE Financial and Industrial 30	Less				
J177	JSE Mining Index	Less				

Table 6 – Chow Denning test results for all shares (1)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
AFE	Basic materials	Non-Random				Non-Random
SAP	Basic materials	Non-Random				
BIL	Mining	Non-Random	Non-Random			
AGL	Mining	Non-Random				
IMP	Mining	Non-Random	Non-Random		Non-Random	
ANG	Mining	Non-Random	Non-Random			
GFI	Mining	Non-Random	Non-Random			
ASR	Mining	Non-Random				
SAB	Consumer goods	Non-Random	Non-Random			
RCL	Consumer goods	Non-Random				
ILV	Consumer goods	Non-Random	Non-Random			
OCE	Consumer goods	Non-Random	Non-Random		Non-Random	
GRT	Consumer goods	Non-Random				
FBR	Consumer goods	Non-Random	Non-Random			
PIK	Consumer services			Non-Random	Non-Random	
CLS	Consumer services	Non-Random	Non-Random			
MPC	Consumer services	Non-Random	Non-Random			

Table 7 – Chow Denning test results for all shares (2)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
TFG	Consumer services	Non-Random		Non-Random		
NPN	Consumer services	Non-Random				
SUI	Consumer services	Non-Random				
FSR	Financials	Non-Random	Non-Random			Non-Random
SBK	Financials	Non-Random	Non-Random			
BGA	Financials	Non-Random	Non-Random		Non-Random	
RMH	Financials	Non-Random			Non-Random	Non-Random
INP	Financials	Non-Random				
INL	Financials	Non-Random				
MMI	Financials	Non-Random				
SNT	Financials				Non-Random	
HYP	Financials					
FPT	Financials	Non-Random	Non-Random			
SAC	Financials	Non-Random	Non-Random			
MDC	Healthcare	Non-Random		Non-Random	Non-Random	
NTC	Healthcare					
APN	Healthcare	Non-Random				
PPC	Industrials					

Table 8 – Chow Denning test results for all shares (3)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
MUR	Industrials	Non-Random				
WBO	Industrials	Non-Random				
RLO	Industrials					
NPK	Industrials	Non-Random	Non-Random			
IPL	Industrials	Non-Random				
GND	Industrials	Non-Random	Non-Random		Non-Random	
SPG	Industrials	Non-Random	Non-Random			
SOL	Oil	Non-Random	Non-Random			
MTN	Teleco	Non-Random	Non-Random			
J150	JSE Gold Mining Index	Non-Random				
J200	JSE Top 40	Non-Random				
J203	JSE All Share Index (ALSI)	Non-Random				
J211	JSE Industrial 25	Non-Random				
J213	JSE Financial and Industrial 30	Non-Random	Non-Random			
J177	JSE Mining Index	Non-Random				

Table 9 – Wright test results for all shares (1)

Share	Sector	Frequency			
		Daily	Weekly	Monthly	Quarterly
AFE	Basic materials	Non-Random	Non-Random		
SAP	Basic materials	Non-Random		Non-Random	
BIL	Mining	Non-Random	Non-Random		
AGL	Mining	Non-Random			
IMP	Mining	Non-Random	Non-Random	Non-Random	Non-Random
ANG	Mining	Non-Random	Non-Random		
GFI	Mining	Non-Random	Non-Random		
ASR	Mining	Non-Random	Non-Random	Non-Random	
SAB	Consumer goods	Non-Random	Non-Random	Non-Random	
RCL	Consumer goods	Non-Random	Non-Random		
ILV	Consumer goods	Non-Random			
OCE	Consumer goods	Non-Random	Non-Random		
GRT	Consumer goods	Non-Random	Non-Random		
FBR	Consumer goods	Non-Random	Non-Random	Non-Random	
PIK	Consumer services	Non-Random	Non-Random		
CLS	Consumer services	Non-Random	Non-Random	Non-Random	
MPC	Consumer services	Non-Random	Non-Random		

Table 10 – Wright test results for all shares (2)

Share	Sector	Frequency			
		Daily	Weekly	Monthly	Quarterly
TFG	Consumer services	Non-Random	Non-Random	Non-Random	
NPN	Consumer services	Non-Random			
SUI	Consumer services	Non-Random			
FSR	Financials	Non-Random	Non-Random		
SBK	Financials	Non-Random	Non-Random		
BGA	Financials	Non-Random	Non-Random		
RMH	Financials	Non-Random			
INP	Financials	Non-Random			
INL	Financials	Non-Random	Non-Random		
MMI	Financials	Non-Random			
SNT	Financials	Non-Random			
HYP	Financials	Non-Random			
FPT	Financials	Non-Random	Non-Random		
SAC	Financials	Non-Random	Non-Random		
MDC	Healthcare	Non-Random		Non-Random	
NTC	Healthcare	Non-Random	Non-Random	Non-Random	
APN	Healthcare	Non-Random			

Table 11 – Wright test results for all shares (3)

Share	Sector	Frequency			
		Daily	Weekly	Monthly	Quarterly
PPC	Industrials	Non-Random			
MUR	Industrials	Non-Random		Non-Random	
WBO	Industrials	Non-Random	Non-Random	Non-Random	
RLO	Industrials	Non-Random			
NPK	Industrials	Non-Random	Non-Random		
RLO	Industrials	Non-Random			
NPK	Industrials	Non-Random	Non-Random		
IPL	Industrials	Non-Random	Non-Random		
GND	Industrials	Non-Random	Non-Random	Non-Random	Non-Random
SPG	Industrials	Non-Random		Non-Random	
SOL	Oil	Non-Random	Non-Random	Non-Random	
MTN	Teleco	Non-Random	Non-Random	Non-Random	
J150	JSE Gold Mining Index	Non-Random	Non-Random		
J200	JSE Top 40	Non-Random		Non-Random	
J203	JSE All Share Index (ALSI)	Non-Random	Non-Random	Non-Random	
J211	JSE Industrial 25	Non-Random	Non-Random		
J213	JSE Financial and Industrial 30	Non-Random	Non-Random		
J177	JSE Mining Index	Non-Random			

Table 12 – Variance Decomposition test results for all shares (1)

Share	Sector	Frequency			
		Daily	Weekly	Monthly	Quarterly
AFE	Basic materials	Non-Random	Non-Random		
SAP	Basic materials	Non-Random	Non-Random		
BIL	Mining	Non-Random	Non-Random		
AGL	Mining	Non-Random			
IMP	Mining	Non-Random	Non-Random		Non-Random
ANG	Mining	Non-Random	Non-Random		
GFI	Mining		Non-random		
ASR	Mining	Non-Random			
SAB	Consumer goods	Non-Random	Non-Random		
RCL	Consumer goods	Non-Random			
ILV	Consumer goods	Non-Random	Non-Random		
OCE	Consumer goods		Non-random		
GRT	Consumer goods	Non-Random	Non-Random		
FBR	Consumer goods	Non-Random	Non-Random		
CLS	Consumer services	Non-Random	Non-Random		
MPC	Consumer services	Non-Random			
TFG	Consumer services	Non-Random			

Table 13 – Variance Decomposition test results for all shares (2)

Share	Sector	Frequency			
		Daily	Weekly	Monthly	Quarterly
NPN	Consumer services	Non-Random			
SUI	Consumer services	Non-Random			
FSR	Financials	Non-Random	Non-Random		
SBK	Financials	Non-Random	Non-Random		
BGA	Financials	Non-Random	Non-Random		
RMH	Financials	Non-Random			
INP	Financials	Non-Random			
INL	Financials	Non-Random	Non-Random		
MMI	Financials	Non-Random			
FPT	Financials	Non-Random	Non-Random		
SAC	Financials	Non-Random	Non-Random		
APN	Healthcare	Non-Random	Non-Random		
MUR	Industrials	Non-Random		Non-Random	
WBO	Industrials	Non-Random			
NPK	Industrials	Non-Random			
IPL	Industrials	Non-Random			
GND	Industrials	Non-Random	Non-Random		Non-Random

Table 14– Variance Decomposition test results for all shares (3)

Share	Sector	Frequency			
		Daily	Weekly	Monthly	Quarterly
SPG	Industrials	Non-Random	Non-Random		
SOL	Oil	Non-Random	Non-Random		
MTN	Teleco	Non-Random	Non-Random		
J150	JSE Gold Mining Index	Non-Random			
J200	JSE Top 40	Non-Random			
J203	JSE All Share Index (ALSI)	Non-Random			
J211	JSE Industrial 25	Non-Random			
J213	JSE Financial and Industrial 30	Non-Random			
J177	JSE Mining Index	Non-Random			

Table 15 – Hurst exponent results for all shares (1)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
AFE	Basic materials				Anti-Persistent	Anti-Persistent
SAP	Basic materials				Persistent	
BIL	Mining				Persistent	Persistent
AGL	Mining	Anti-Persistent			Persistent	Persistent
IMP	Mining					Anti-Persistent
ANG	Mining				Persistent	
GFI	Mining				Persistent	Persistent
ASR	Mining			Persistent		Anti-Persistent
SAB	Consumer goods				Persistent	Persistent
RCL	Consumer goods				Anti-Persistent	Persistent
OCE	Consumer goods			Persistent		
GRT	Consumer goods					Anti-Persistent
FBR	Consumer goods	Anti-Persistent			Persistent	
PIK	Consumer services				Anti-Persistent	Persistent
CLS	Consumer services					Persistent
MPC	Consumer services			Anti-Persistent	Persistent	Anti-Persistent
TFG	Consumer services					Anti-Persistent
NPN	Consumer services				Persistent	

Table 16– Hurst exponent results for all shares (2)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
SUI	Consumer services			Anti-Persistent		Persistent
FSR	Financials			Anti-Persistent	Anti-Persistent	
SBK	Financials	Anti-Persistent		Anti-Persistent	Anti-Persistent	
BGA	Financials			Anti-Persistent	Anti-Persistent	
RMH	Financials	Anti-Persistent			Anti-Persistent	
INP	Financials			Anti-Persistent	Anti-Persistent	Persistent
INL	Financials			Anti-Persistent	Anti-Persistent	Persistent
MMI	Financials				Anti-Persistent	Anti-Persistent
SNT	Financials				Persistent	Persistent
HYP	Financials				Persistent	Persistent
FPT	Financials	Anti-Persistent		Persistent	Persistent	Persistent
SAC	Financials	Anti-Persistent		Persistent		
MDC	Healthcare				Persistent	Persistent
NTC	Healthcare				Persistent	Persistent
APN	Healthcare			Persistent	Persistent	Persistent
PPC	Industrials			Persistent	Persistent	Anti-Persistent
MUR	Industrials			Persistent	Persistent	
WBO	Industrials				Persistent	Anti-Persistent
RLO	Industrials			Persistent	Persistent	Persistent

Table 17– Hurst exponent results for all shares (3)

Share	Sector	Frequency				
		Daily	Weekly	Monthly	Quarterly	Semi-Annual
NPK	Industrials				Persistent	Anti-Persistent
IPL	Industrials					Persistent
GND	Industrials	Anti-Persistent			Persistent	Persistent
SPG	Industrials			Anti-Persistent		Anti-Persistent
SOL	Oil	Anti-Persistent		Persistent	Persistent	Persistent
MTN	Teleco			Persistent	Persistent	Persistent
J150	JSE Gold Mining Index				Persistent	Anti-Persistent
J200	JSE Top 40			Anti-Persistent	Persistent	Persistent
J203	JSE All Share Index (ALSI)			Anti-Persistent	Persistent	Persistent
J211	JSE Industrial 25			Anti-Persistent	Persistent	Persistent
J213	JSE Financial and Industrial 30			Anti-Persistent	Persistent	Persistent
J177	JSE Mining Index				Persistent	Persistent